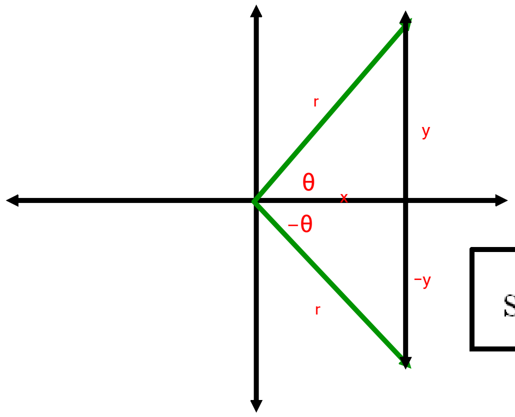
Negative Angles



$$\sin\theta = \frac{y}{r} \qquad \cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r} \qquad \cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta} \qquad \boxed{\cos(-\theta) = \cos\theta}$$

Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2(-\csc 8x)(-\cot 8x)$

$y' = -2\csc(8x)\cot(8x)$

Ex 7.2

① o) $y = \sin(\cos x)$

$y' = \cos(\cos x)(-\sin x)$

$y' = -\sin x \cos(\cos x)$

p) $y = \cos^3(\sin x) = [\cos(\sin x)]^3$

$y' = 3[\cos(\sin x)]^2(-\sin(\sin x))\cos x$

$y' = -3\cos^2(\sin x)\sin(\sin x)\cos x$

q) $y = x\cos\frac{1}{x} = (x)(\cos x^{-1})$

$y' = \cos x^{-1} + x(-\sin x^{-1})(-x^{-2})$

$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$

s) $y = \frac{1+\sin x}{1-\sin 2x}$

$y' = \frac{\cos x(1-\sin 2x) - (1+\sin x)(-\cos 2x \cdot 2)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x\sin 2x + 2\cos 2x(1+\sin x)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x\sin 2x + 2\cos 2x + 2\sin x\cos 2x}{(1-\sin 2x)^2}$

Ex 7.3

① a) $y = 3\tan 2x$

$y' = 3\sec^2(2x) \cdot 2$

$y' = 6\sec^2(2x)$

g) $y = \sec\sqrt[3]{x} = \sec(x^{1/3})$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3}x^{-2/3}$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3x^{2/3}}$

$y' = \frac{\sec\sqrt[3]{x}\tan\sqrt[3]{x}}{3\sqrt[3]{x^2}}$

7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2\csc(8x)\cot(8x)$

*Negative Angle Identity

m) $y = 2x(\sqrt{x} - \cot x)$

$y = 2x^{3/2} - 2x\cot x$

$y' = 3x^{1/2} - [2\cot x + 2x(-\csc^2 x \cdot 1)]$

$y' = 3\sqrt{x} - 2\cot x + 2x\csc^2 x$

$u = x^2$

Ex: $y = \sin(x^2)$

$y' = \cos(x^2) \cdot 2x$

$y' = 2x\cos(x^2)$

$u = \tan x$

① n) $y = \sin(\tan x)$

$y' = \cos(\tan x) \cdot \sec^2 x$

$y' = \sec^2 x \left[\cos(\tan x)\right]$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = -\frac{3}{2}(\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$

$y' = \dfrac{-3\sec(2x)\tan(2x)}{(\sec 2x - 1)^{5/2}}$

$y' = -\dfrac{3\sec(2x)\tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$

① a) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$

$y' = -2\sin x[\tan(\cos x)][\sec^2(\cos x)]$

b) $y = \dfrac{x^2 \boxed{\tan x}}{\boxed{\sec x}} = x^2\left(\dfrac{\sin x}{\cos x}\right) \cdot \left(\dfrac{\cos x}{1}\right) = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x(\sin x)$

$y' = x^2\cos x + 2x\sin x$

$y' = x[x\cos x + 2\sin x]$

Final Review

① b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$ $f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

$$f'(x) = \lim_{h \to 0} \frac{(x+3)\cancel{(x+h+3)}\dfrac{2x+2h-2}{\cancel{x+h+3}} - \dfrac{2x-2}{\cancel{x+3}}\cancel{(x+3)}(x+h+3)}{h(x+3)(x+h+3)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2}+\cancel{2xh}-\cancel{2x}+\cancel{6x}+6h-\cancel{6}-(\cancel{2x^2}+\cancel{2xh}+\cancel{6x}-\cancel{2x}-2h-\cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h \to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \boxed{\frac{8}{(x+3)^2}}$$

h) $y = (x^2)(\csc x)$

$y' = (x^2)(-\csc x \cot x)(1) + 2x\csc x$

$y' = -x^2 \csc x \cot x + 2x\csc x$

$y' = x\csc x(-x\cot x + 2)$

$y' = x\csc x(2 - x\cot x)$

$u = (1-2x)^2$
$du = 2(1-2x)(-2)$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 3\cot^2(1-2x)^2[-\csc^2(1-2x)^2][-4(1-2x)]$

$y' = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

Final Review

$$= \frac{2(x+h)-2}{(x+h)+3}$$

① b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$ $\quad f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

$$f'(x) = \lim_{h\to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2} \cancel{+2xh} \cancel{-2x} \cancel{+6x} + 6h \cancel{-6} - (\cancel{2x^2} \cancel{+2xh} \cancel{+6x} \cancel{-2x} - 2h \cancel{-6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

④ b) $y = \dfrac{\sqrt{x}}{3+x^2} = \dfrac{x^{1/2}}{3+x^2}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x(x^{1/2})}{(3+x^2)^2}$$

CD: $2x^{1/2}$

$$y' = \frac{\frac{3+x^2}{2x^{1/2}} - \frac{2x^{3/2}}{1}}{(3+x^2)^2}$$

$$y' = \frac{3+x^2-4x^2}{2x^{1/2}(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Final Review:

① a) $y = \left[\dfrac{2x+1}{x-1}\right]^5$

$$y' = 5\left[\frac{2x+1}{x-1}\right]^4 \left[\frac{\overset{2x-2\;-\;2x-1}{2(x-1) - 1(2x+1)}}{(x-1)^2}\right]$$

$$y' = 5 \cdot \frac{(2x+1)^4}{(x-1)^4} \cdot \frac{-3}{(x-1)^2} = \frac{-15(2x+1)^4}{(x-1)^6}$$

⑤ Find the equation of the tangent line

$y = (x^2-3)^8$ at $x = \underline{2}$, $y = 1$

$(2,1)$

① Find y

$y = ((2)^2-3)^8$

$y = (4-3)^8 = 1$

② Find y'

$y' = 8(x^2-3)^7(2x)$

$y' = 16x(x^2-3)^7$

③ Find m

$y'(2) = 16(2)[(2)^2-3]^7$

$y'(2) = 32(1) = 32$

$\boxed{m = 32}$

④ Find equation

$y - y_1 = m(x - x_1)$

$y - 1 = 32(x-2)$

$y - 1 = 32x - 64$

$y = 32x - 63$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$

① $x + 4y = 1$

$4y = -x + 1$

$y = -\frac{1}{4}x + \frac{1}{4}$

$m = -\frac{1}{4}$

② $y = \frac{x}{x-1}$

$x-1 \quad - x$

$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$

$y' = \frac{-1}{(x-1)^2}$

③ $\frac{-1}{(x-1)^2} = -\frac{1}{4}$

$-(x-1)^2 = -4$

$(x-1)^2 = 4$

$x^2 - 2x + 1 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad | \quad x + 1 = 0$

$x = 3 \quad | \quad x = -1$

④ $y = \frac{x}{x-1}$

$y = \frac{3}{3-1} \quad | \quad y = \frac{-1}{-1-1}$

$y = \frac{3}{2} \quad | \quad y = \frac{1}{2}$

$\boxed{(3, 3/2)} \quad | \quad \boxed{(-1, 1/2)}$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

① $6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6$

② $y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ $\frac{3\sqrt{x}}{2} = \frac{6}{1}$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$x = 16$

④ $y = x\sqrt{x}$

$y = (16)\sqrt{16}$

$y = 64$

$(16, 64)$

Do Ex. 2.9 on page 112 of Text
Questions 1, 4, 6, 9, 11, 12, 14

a, d

Page 103

③ Find $\left.\frac{dy}{dx}\right]_{x=4}$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$

① Find u

$u = x - \sqrt{x}$

$u = (4) - \sqrt{4}$

$\boxed{u = 2}$

② Find $\frac{dy}{du}$

$y = u^2 - 2u^5$

$\frac{dy}{du} = 2u - 10u^4$

③ Find $\frac{du}{dx}$

$u = x - x^{1/2}$

$\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$

$$\left.\frac{dy}{dx}\right]_{x=4} = (2u - 10u^4)\left(1 - \frac{1}{2\sqrt{x}}\right)$$

$$= (2(2) - 10(2)^4)\left(1 - \frac{1}{2\sqrt{4}}\right)$$

$$= (4 - 160)\left(1 - \frac{1}{4}\right)$$

$$= (-156)\left(\frac{3}{4}\right)$$

$$= -\frac{468}{4}$$

$$= -117$$

⑨ If $F(x) = f(g(x))$, where $\boxed{g(2) = 4}$ $\boxed{g'(2) = 3}$ $\boxed{f'(4) = 5}$ find $F'(2)$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'(g(2)) \cdot g'(2)$

$F'(2) = \boxed{f'(4)} \cdot \boxed{g'(2)}$

$F'(2) = 5 \cdot 3 = \underline{15}$

/

Page 112

④ e) $y=\sqrt{x}(5-\sqrt{x}) = 5\sqrt{x}-x = 5x^{1/2}-x$

$y' = \frac{5}{2}x^{-1/2}-1$

$y' = \frac{5}{2\sqrt{x}}-1 = \frac{5-2\sqrt{x}}{2\sqrt{x}}$

Using Product:

$y=\sqrt{x}(5-\sqrt{x})$ $\quad y' = \frac{1}{2}x^{-1/2}(5-\sqrt{x})+x^{1/2}\left(-\frac{1}{2}x^{-1/2}\right)$

$y' = \frac{1}{2\sqrt{x}}(5-\sqrt{x}) + \sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right)$

$y' = \frac{5}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}} - \frac{\sqrt{x}}{2\sqrt{x}}$

$y' = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}}$

$y' = \frac{5}{2\sqrt{x}} - 1$

f) $y = \frac{x^2-2x}{\sqrt{x}}$

$y' = \frac{x^{1/2}(2x-2) - \frac{1}{2}x^{-1/2}(x^2-2x)}{x}$

$y' = \frac{2x^{3/2}\cdot 2x^{1/2} - 2x^{1/2}\cdot 2x^{1/2} - \frac{1}{\cancel{2\sqrt{x}}}(x^2-2x)\cdot \cancel{2x^{1/2}}}{x\cdot 2x^{1/2}}$

$y' = \frac{4x^2-4x-x^2+2x}{2x^{3/2}} = \frac{3x^2-2x}{2x^{3/2}}$

$= \frac{x(3x-2)}{2x^{3/2}}$

$= \frac{x^{-1/2}(3x-2)}{2}$

$= \boxed{\frac{3x-2}{2\sqrt{x}}}$

Page 112

9 2, $y = x\sqrt{x^2+5}$, $(-2,-6)$ $x_1 = -2$ $y_1 = -6$

① Find y'

$y' = 1\sqrt{x^2+5} + x\left[\frac{1}{\cancel{2}}(x^2+5)^{-1/2} \cdot \cancel{2}x\right]$

$y' = \sqrt{x^2+5} + \frac{x^2}{\sqrt{x^2+5}}$

② Find m or $y'(-2)$

$y'(-2) = \sqrt{(-2)^2+5} + \frac{(-2)^2}{\sqrt{(-2)^2+5}}$

$y' = 3 + \frac{4}{3} = \boxed{\frac{13}{3}}$

③ Find equation:

$y + 6 = \frac{13}{3}(x + 2)$

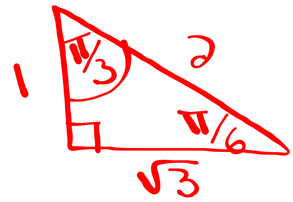
$y + 6 = \frac{13x}{3} + \frac{26}{3} - 6$

$\boxed{y = \frac{13x}{3} + \frac{8}{3}}$

$3y = 13x + 8$

$\boxed{0 = 13x - 3y + 8}$

Page 314



③ c) Find equation of tangent →

$$y = \frac{1}{\cos x} - 2\cos x \quad \text{at} \quad \left(\frac{\pi}{3}, 1\right) \qquad x_1 = \frac{\pi}{3} \quad y_1 = 1$$

① find y'

$$y = (\cos x)^{-1} - 2\cos x$$

$$y' = -1(\cos x)^{-2}(-\sin x) - 2(-\sin x)$$

$$y' = \frac{\sin x}{\cos^2 x} + 2\sin x$$

② Find m or $y'\left(\frac{\pi}{3}\right)$

$$y'(\pi/3) = \frac{\sin(\pi/3)}{\cos^2(\pi/3)} + 2\sin(\pi/3)$$

$$y'(\pi/3) = \frac{\left(\sqrt{3}/2\right)}{\left(1/2\right)^2} + \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$y'(\pi/3) = \frac{\sqrt{3}}{\cancel{2}_1} \cdot \frac{\cancel{4}^2}{1} + \sqrt{3}$$

$$y'(\pi/3) = 2\sqrt{3} + \sqrt{3} = \boxed{3\sqrt{3}}$$

③ Find equation:

$$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$y - 1 = 3x\sqrt{3} - \pi\sqrt{3}$$

$$0 = 3x\sqrt{3} - y - \pi\sqrt{3} + 1$$