① $f(x)=x^4-3x^3+3x^2-x$

$f'(x)=4x^3-9x^2+6x-1$

$f''(x)=12x^2-18x+6$

② y int: $(x=0)$

$f(0)=(0)^4-3(0)^3+3(0)^2-0=0$

$(0,0)$

③ xint: $(y=0)$

$0=x^4-3x^3+3x^2-x$ ← Synthetic Sub.

$0=x(x^3-3x^2+3x-1)$

1	1	-3	3	-1
		1	-2	1
	1	-2	1	

$(x-1)(x^2-2x+1)$ ← simple trinomial

$(x-1)(x-1)(x-1)$

$0=x(x-1)(x^2-2x+1)$

$0=x(x-1)^3$

$x=0,1$

$(0,0)$ & $(1,0)$

④ No asymptotes

⑤ Intervals of Inc/Dec.

$f'(x)=4x^3-9x^2+6x-1$ ← syn. sub.

$f'(x)=(x-1)^2(4x-1)$

1	4	-9	6	-1
		4	-5	1
	4	-5	1	

$(x-1)(4x^2-5x+1)$ ← Decomp

$(x-1)[(4x^2-4x)(x+1)]$

$(x-1)[4x(x-1)-1(x-1)]$

$(x-1)(x-1)(4x-1)$

CV: $x=1, \frac{1}{4}$ (neither, min)

− ↓ + ↓ +

(0) $\frac{1}{4}$ (0.5) 1 (2)

Increasing on $(\frac{1}{4},\infty)$

Decreasing on $(-\infty,\frac{1}{4})$

⑥ Local Max/Min.

$f(\frac{1}{4})=(\frac{1}{4})^4-3(\frac{1}{4})^3+3(\frac{1}{4})^2-(\frac{1}{4})=-0.1$ $(0.25,-0.1)$ ← min

No max!

⑦ Intervals of concavity.

$f''(x)=12x^2-18x+6$

$f''(x)=6(2x^2-3x+1)$ ← Decomp.

$f''(x)=6[2x^2-2x-x+1]$

$f''(x)=6[2x(x-1)-1(x-1)]$

$f''(x)=6(2x-1)(x-1)$

CV: $x=1, \frac{1}{2}$

+ − +

(0) $\frac{1}{2}$ (0.75) 1 (2)

CU on $(-\infty,\frac{1}{2})$ & $(1,\infty)$

CD on $(\frac{1}{2},1)$

⑧ Inflection Points.

$f(\frac{1}{2})=(\frac{1}{2})^4-3(\frac{1}{2})^3+3(\frac{1}{2})^2-(\frac{1}{2})=-0.0625$ $(0.5,-0.0625)$

$f(1)=(1)^4-3(1)^3+3(1)-(1)=0$ $(1,0)$

$f(x) = \frac{8(x-2)}{x^3}$ $f'(x) = \frac{-8(x-4)}{x^3}$ $f''(x) = \frac{16(x-6)}{x^4}$

CV: $x = 0, 4$ CV: $x = 0, 6$

① Intercepts:

x int: $(y=0)$

$8(x-2)=0$

$x = 2$

$(2,0)$

y int: $(x=0)$

$y = \frac{8(0-2)}{(0)^3}$

$y = \frac{-16}{0}$

No y-intercept

② Asymptotes:

VA: (zeros of the denom)

$f(x) = \frac{8(x-2)}{x^3}$

$x=0$

$\lim_{x\to 0^-} f(x) = \frac{(-)}{(+)} = -\infty$

$\lim_{x\to 0^+} f(x) = \frac{(-)}{(+)} = -\infty$

HA: (compare degree)

$f(x) = \frac{8(x-2)}{x^3} = \frac{8x-16}{x^3}$

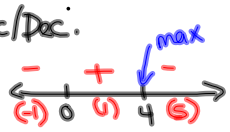
$\lim_{x\to\pm\infty} \frac{8x-16}{x^3} = 0$

$y=0$

③ Intervals of Inc/Dec:

$f'(x) = \frac{-8(x-4)}{x^3}$

CV: $x = 0, 4$



Decreasing on $(-\infty, 0) + (4, \infty)$

Increasing on $(0, 4)$

④ Local Max/min:

$f(0) = \frac{8(0-2)}{(0)^3} = \frac{-16}{0}$ = undefined (no min)

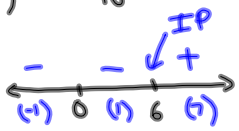
$x=0$ is VA

$f(4) = \frac{8(4-2)}{(4)^3} = \frac{16}{16} = 1$ $(4,1)$ max

⑤ Concavity:

$f''(x) = \frac{16(x-6)}{x^4}$

CV: $x = 0, 6$

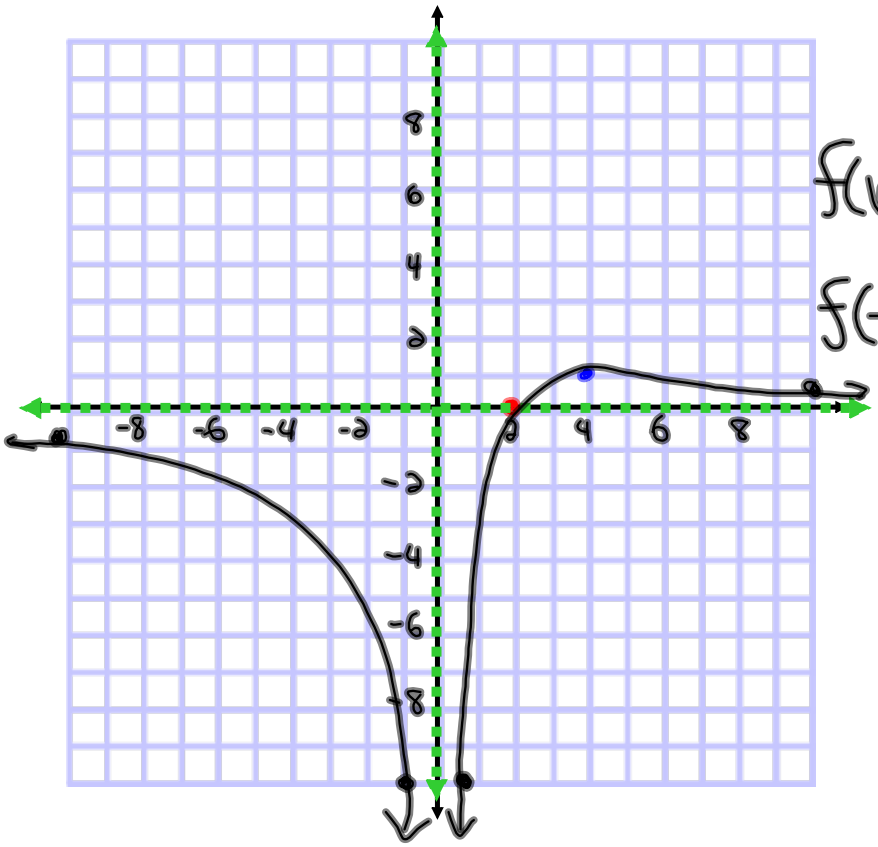


CU on $(6, \infty)$

CD on $(-\infty, 6)$

⑥ Inflection Point

$f(6) = \frac{8(6-2)}{(6)^3} = \frac{32}{36} = \frac{8}{9} = 0.\overline{8}$ $\left(6, 0.\overline{8}\right)$



$f(10) = 0.64$

$f(-10) = -0.64$