

## Developing Trigonometric Functions from Properties...

Develop a trigonometric function that fits the following description...

- Models a sine function
- Period is  $120^\circ$
- Graph is reflected in  $x$ -axis
- Wave has a range of  $-8 \leq y \leq 2$
- Graph has a phase shift of  $60^\circ$  right
- Graph has a vertical translation of 3 units down

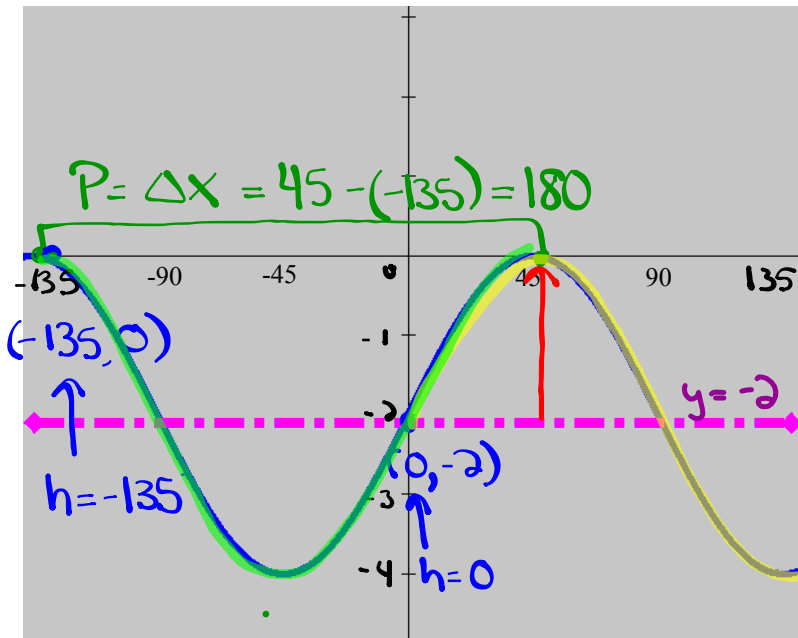
...Now we must learn how to identify all of the above information from a graph.

## Developing the Equation of a Sinusoidal Function

STEPS:  $y = a \sin[b(x-h)] + k$

- 1) Identify & label the **sinusoidal axis**.
- 2) Determine the **amplitude**, **period** & vertical translation.
- 3) Pick a trig function & determine the corresponding **phase shift**.

- the choices are: **positive sine**, **positive cosine**, negative sine, negative cosine



Amp = 2      $a = \pm 2$

Period = 180

$b = \frac{360}{P} = \frac{360}{180} = 2$

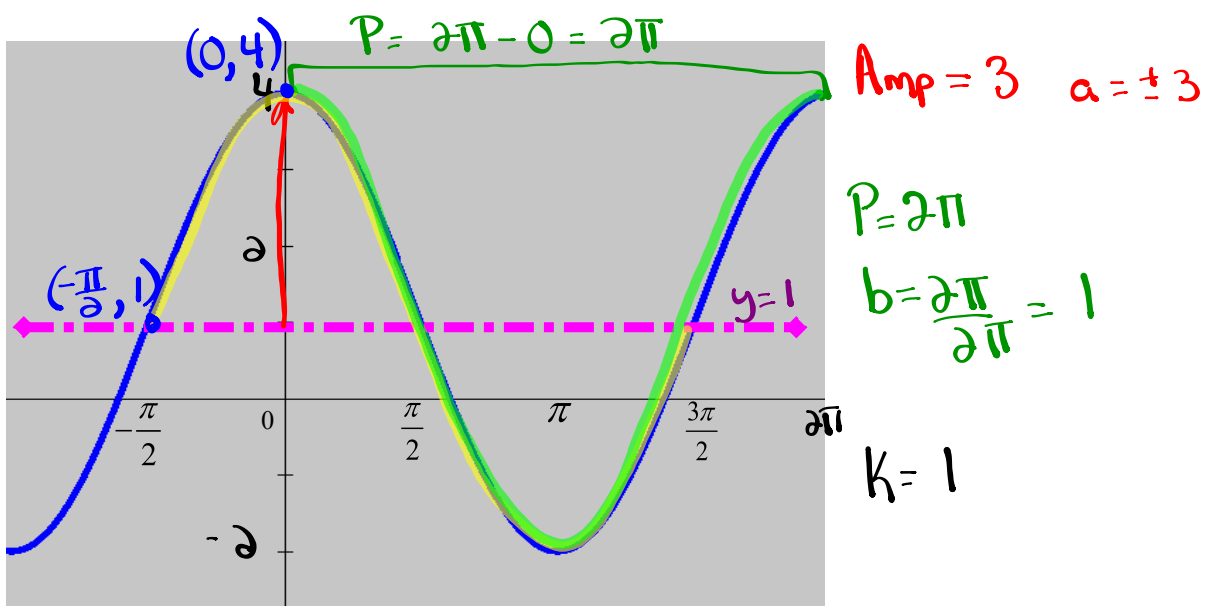
VT:  $k = -2$

$y = \sin x \quad (h=0)$   
 $y = 2 \sin[2(x-0)] - 2$   
 $y = 2 \sin[2(x)] - 2$

$y = \cos x \quad (h=-135)$   
 $y = 2 \cos[2(x+135)] - 2$

## Finding an Equation from a Graph:

Determine a sine and a cosine equation for this graph



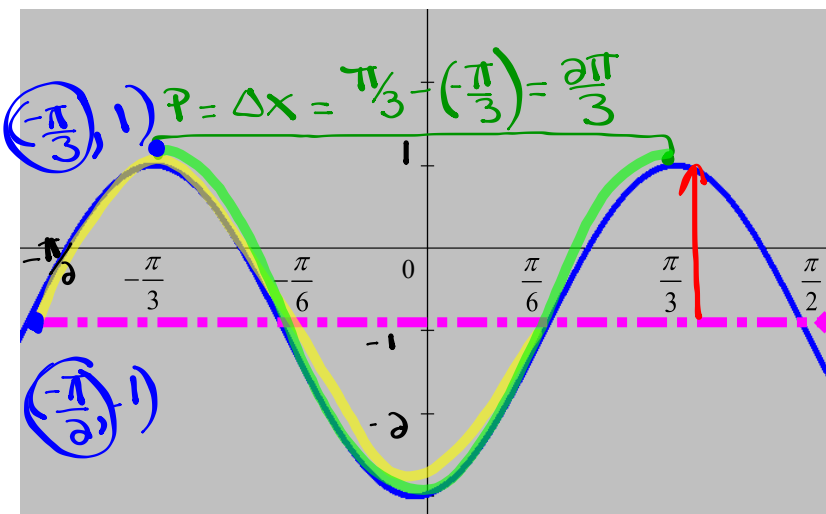
$$y = \sin x \quad (h = -\frac{\pi}{2})$$

$$y = 3 \sin \left[ 1 \left( x + \frac{\pi}{2} \right) \right] + 1$$

$$y = \cos x \quad (h = 0)$$

$$y = 3 \cos \left[ 1 (x - 0) \right] + 1$$

Determine a sine and a cosine equation for this graph



Amp = 2    a = ±2

Period =  $\frac{2\pi}{3}$

$b = \frac{2\pi}{P} = 2\pi \div \frac{2\pi}{3}$   
 $= \frac{2\pi}{1} \times \frac{3}{2\pi} = 3$

VT: k = -1

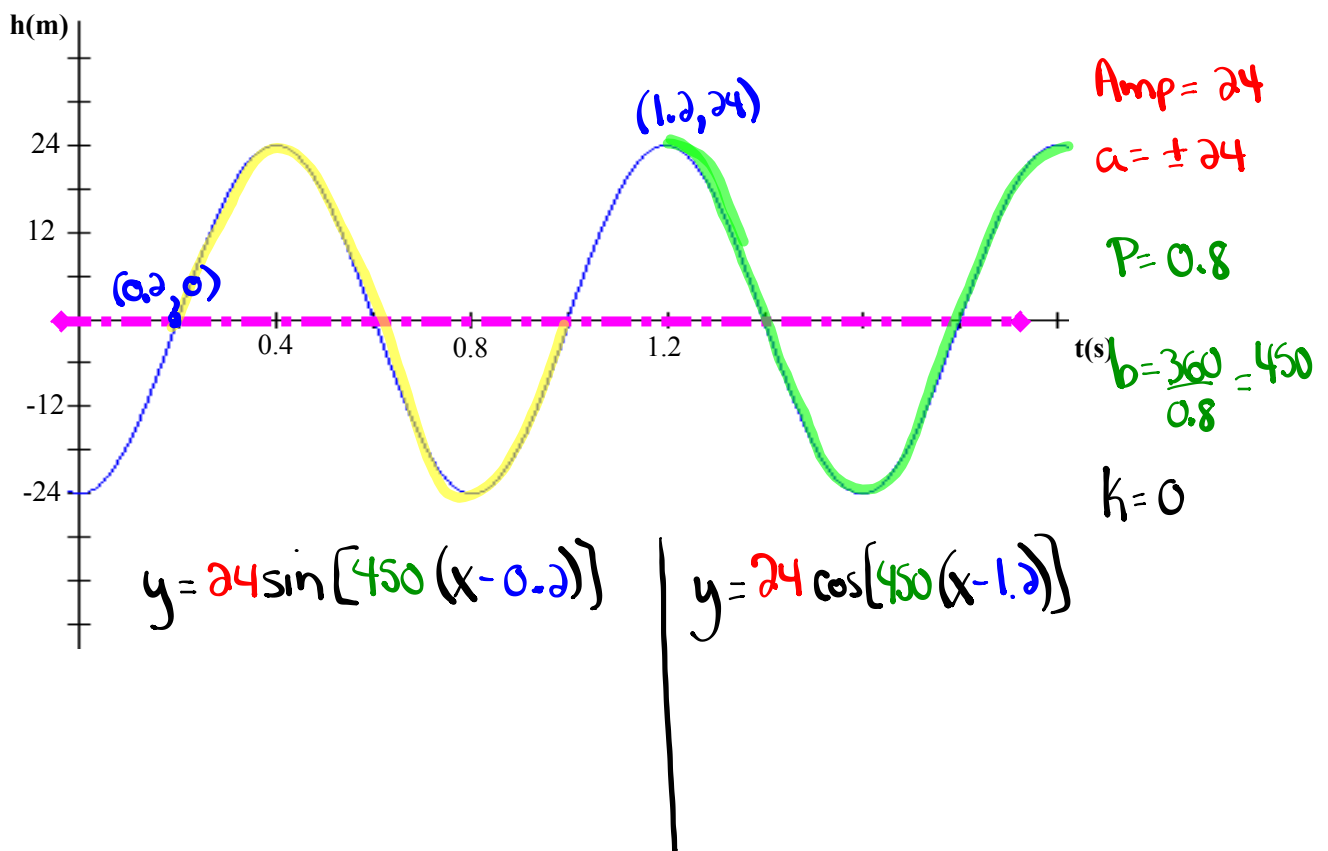
$y = \sin x \quad (h = -\frac{\pi}{2})$

$y = \cos x \quad (h = -\frac{\pi}{3})$

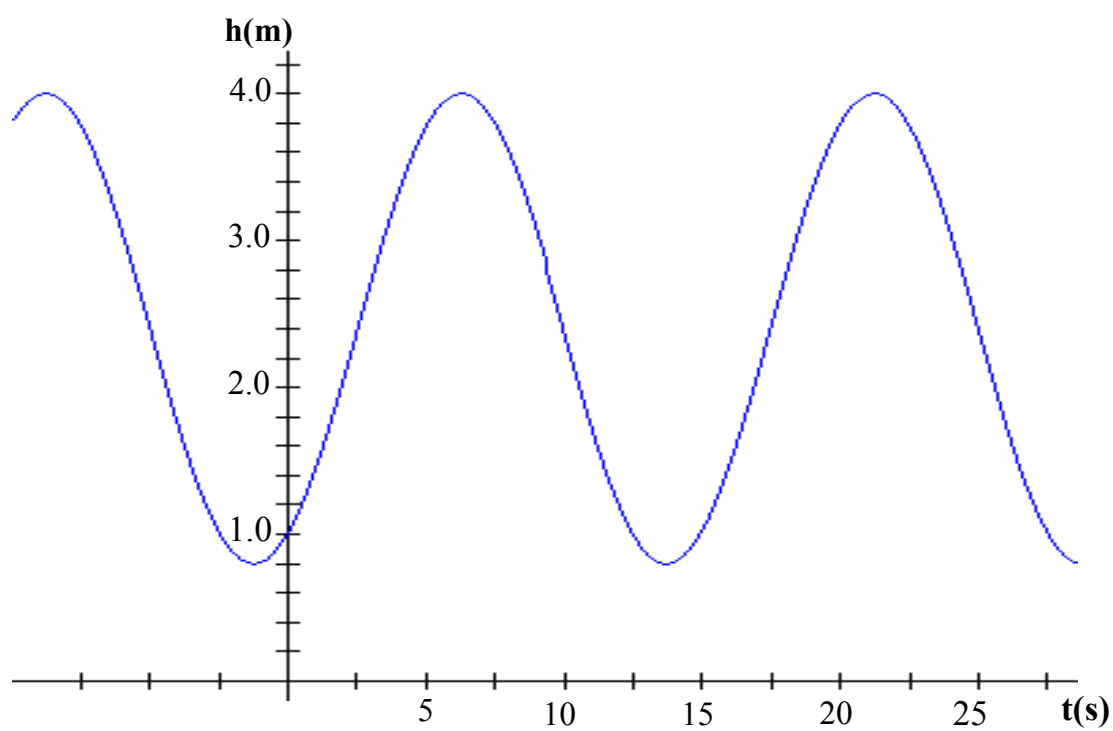
$y = 2 \sin [3(x + \frac{\pi}{2})] - 1$

$y = 2 \cos [3(x + \frac{\pi}{3})] - 1$

Determine a sine and a cosine equation for this graph



Find 4 equations to describe the graph.



## EXTRA PRACTICE...

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Worksheet: #28 a) - f)

## Applications of Sinusoidal Relations

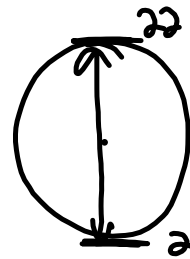
- Strategy: (1) Translate ALL key pieces of information from the problem.  
 (2) Draw a sketch with ALL key points identified.  
 (3) Develop an equation that models the problem.  
 (4) Answer the question(s) being asked.

CHECK??? Do the numbers make sense?

\* Radius = Amp.

\* Count by  $\frac{P}{4}$  on x-axis

\* min + diameter = max  
 min + radius = sin axis



$r=10$   
 min = 2  
 max = ?

\* From max to min or min to max is half the period

Ex: max @ 10s       $P=10s$   
 min @ 15s  
 max @ 20s



## Applications of Sinusoidal Functions

A carnival Ferris wheel with a radius of 14 m makes one complete revolution every 16 seconds. The bottom of the wheel is 1.5 m above the ground. If a person is at the top of the wheel when a stop watch is started, determine how high above the ground that person will be after 1 minute and 7 seconds? Sketch one period of this function.

$$\text{Amp} = 14$$

$$P = 16$$

$$\text{min} = 1.5$$

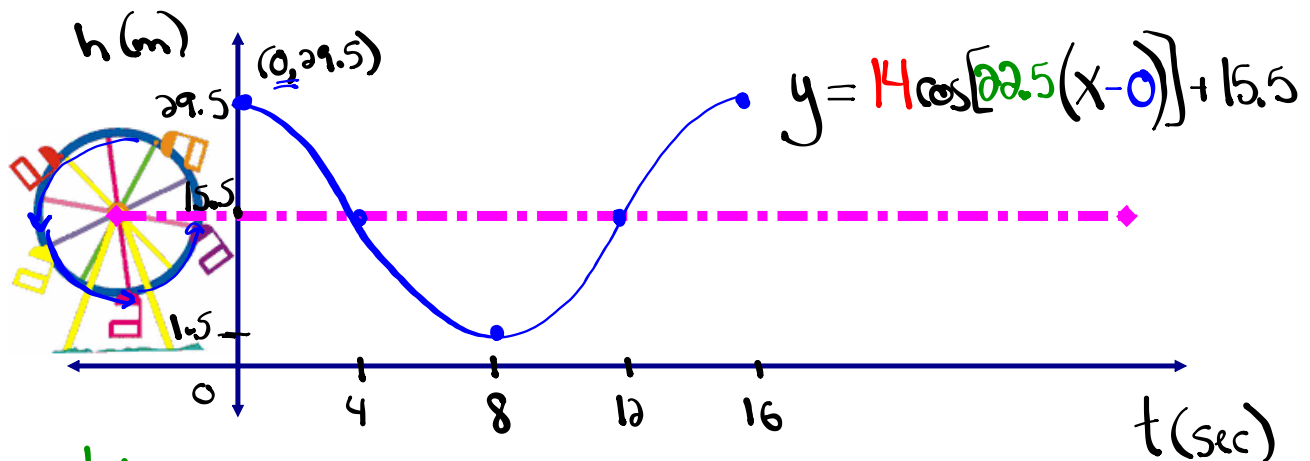
$$y = \cos x$$

$$a = \pm 14$$

$$b = \frac{360}{16} = 22.5$$

$$\text{max} = 29.5 \quad (\text{min} + \text{diameter})$$

$$k = 15.5 \quad (\text{min} + \text{radius})$$



$$\text{count by } \frac{16}{4} = 4$$

$$1 \text{ min and } 7 \text{ seconds} = \underline{\underline{67}} \text{ sec}$$

$$y = 14 \cos[22.5(x)] + 15.5$$

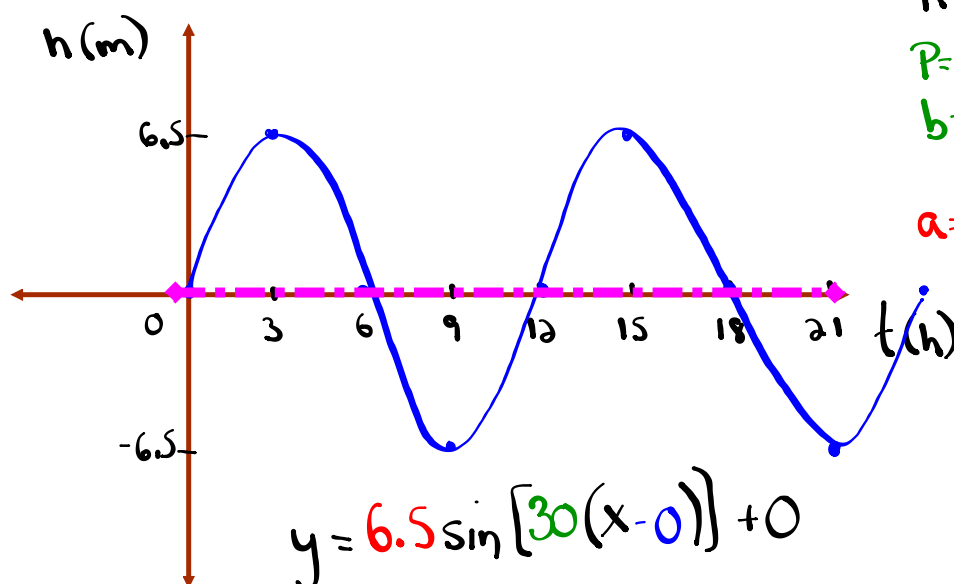
$$y = 14 \cos[22.5(67)] + 15.5$$

$$y = 20.86 \text{ m}$$

## Ocean Tides

The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 12 h. Assuming the height of water with respect to mean sea-level to be modelled by a sine function,

- (a) draw the graph for a the motion of the tides for one complete day;  
 (b) find an equation for the graph in (a).



$$y = 6.5 \sin [30(x-0)] + 0$$

$$y = 6.5 \sin [30x]$$

# Solutions to Homework

**Applications of Sinusoidal Functions**

1. A Ferris wheel has a radius of 12m and makes one complete revolution every 12 seconds. The bottom of the wheel is 2m above the ground. If a person gets on at the bottom and goes up, determine the following:

a) Amplitude: = 12m    b) Period: 12s    c) b: 30    d) d: = 14m  
 e) Minimum Height: = 2m    f) Maximum Height: = 26m  $\rightarrow (2+24)$   
min + diameter  
 g) Equation:  $y = -12\cos(30x) + 14$   
 h) Sketch the graph for two revolutions (periods):

i) How high is the person at these times?  
 i) 12 seconds  $\rightarrow y = -12\cos(30(12)) + 14 = \boxed{2m}$   
 ii) 5 minutes  $\rightarrow y = -12\cos(30(300)) + 14 = \boxed{2m}$

$y = -\cos x$

2. The tide on the Miramichi River has a range of 8m. At time 0 the tide is midway and at 4 hours the tide is low. The tide completes one full cycle in 16 hours.

a) Draw the graph for a 32 hour period.

b) Write the equation for this graph.

$a = 4$   
 $P = 16$   
 $b = \frac{360}{16} = 22.5$   
 $c = 0$   
 $d = 0$

$y = -4\sin(22.5x)$

If range is 5m  
 $\rightarrow \max = 2.5$   
 $\rightarrow \min = -2.5$

# Solutions to Homework

under water  $\min = -3$

3. A water wheel has a radius of 10m. 3m of the wheel is submerged under water. If the wheel makes one revolution in 360 degrees and the bucket starts at the center and goes up, find:

a) Amplitude: = 10m    b) Period: = 360°    c) b: = 1    d) d: = 7  
 e) Min Height: = -3m    f) Max Height: = 17m  $\rightarrow (-3 + 20)$   
 g) Equation of Graph:  $y = 10\sin(x) + 7$  min + diameter  
 h) Sketch the graph for 2 revolutions:

i) How high will the bucket be after?

(i)  $40^\circ \rightarrow y = 10\sin(40^\circ) + 7 = 13.43m$   
 (ii)  $110^\circ \rightarrow y = 10\sin(110^\circ) + 7 = 16.4m$   
 (iii)  $200^\circ \rightarrow y = 10\sin(200^\circ) + 7 = 3.58m$

j) After how many degrees would the height be equal to 11? (Hint sub 11 in for y)

$11 = 10\sin(x) + 7$   
 $4 = 10\sin(x)$   
 $0.4 = \sin(x)$   
 $x = 23.6^\circ$      $\sin^{-1}(0.4) = x$

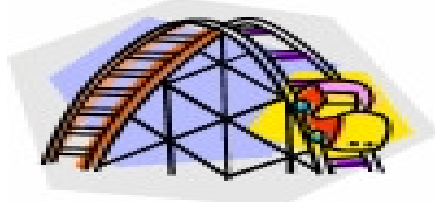
4. A water wheel is defined by the equation  $y = 7\cos[18(x)] + 4$     a = 7    b = 18    c = 0    d = 4  
 Find:

a) Amplitude = 7    b) Period =  $\frac{360}{18} = 20s$   
 c) Sketch the graph if the bucket starts at the top and goes down. Assume this function models the height of the bucket in meters over time in seconds.

d) How much of the wheel is submerged? 3m (min = -3)  
 e) What is the Radius? = 7m    f) When is the bucket 5m high?  
 $5 = 7\cos(18x) + 4$   
 $1 = 7\cos(18x)$   
 $0.1428 = \cos(18x)$   
 $81.79 = 18x$   
 $4.54s = x$      $\cos^{-1}(0.1428) = 18x$

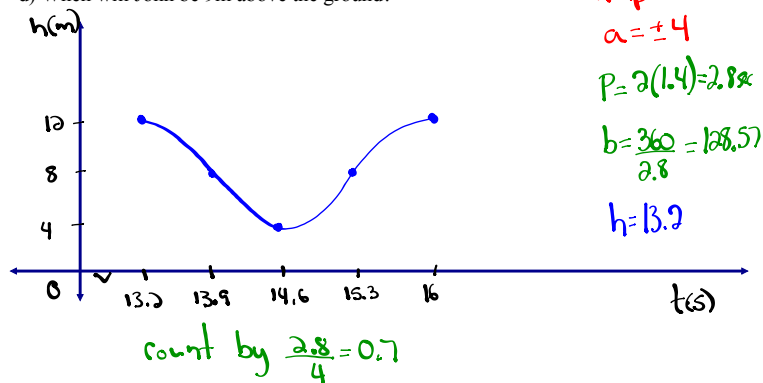
$y = b\sin(x) + 7$   
 $11 = 10\sin(x) + 7$   
 $4 = 10\sin(x)$   
 $0.4 = \sin(x)$   
 $\sin^{-1}(0.4) = x$   
 $23.6^\circ = x$

## Roller Coaster



John rides on a roller coaster at Six Flags Amusement Park. An observer starts a stopwatch and observes that John is at a maximum height of 12 m at  $t = 13.2$  s. At  $t = 14.6$  s, John reaches a minimum height of 4 m.

- a) Sketch a graph of the function. from max to min took  $14.6 - 13.2 = 1.4$  sec. max = 12
- b) Find an equation that expresses John's height in terms of time. min = 4
- c) How high is John above the ground at  $t = 20.8$  s? k = 8
- d) When will John be 9m above the ground? Amp = 4



c)  $x = 20.8$  s

$$y = 4 \cos[128.57(20.8 - 13.2)] + 8$$

$$\boxed{y = 7.1 \text{ m}}$$

d)  $y = 9$  m

$$9 = 4 \cos[128.57(x - 13.2)] + 8$$

$$\frac{1}{4} = \frac{4 \cos[128.57(x - 13.2)]}{4}$$

$$0.25 = \cos[128.57(x - 13.2)]$$

$$\cos^{-1}(0.25) = 128.57(x - 13.2)$$

$$\frac{75.52}{128.57} = \frac{128.57(x - 13.2)}{128.57}$$

$$0.59 = x - 13.2$$

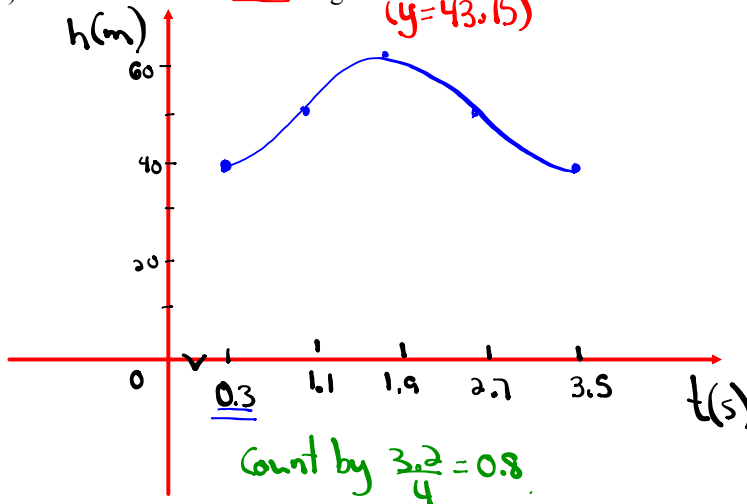
$$\boxed{13.79 \text{ s} = x}$$

# Spring Problem

A weight attached to a long spring is being bounced up and down by an electric motor. As it bounces, its distance from the floor varies periodically with time. You start a stopwatch. When the stopwatch reads 0.3 seconds the weight reaches its first low point 40 cm above the ground. The next high point, 60 cm above the ground, occurs at 1.9 seconds.

- a) Sketch a graph of the function. *From max to min took 1.6 sec. (1.9-0.3)*
- b) Write an equation expressing the distance above the ground in terms of the numbers of seconds the stopwatch reads.  $y = 10 \cos [112.5(x - 0.3)] + 50$

- c) When is the mass 43.75m high?



$$\text{max} = 60 \text{ cm}$$

$$\text{min} = 40 \text{ cm}$$

$$k = 50$$

$$\text{Amp} = 10$$

$$a = \pm 10$$

$$P = 2(1.6) = 3.2 \text{ sec}$$

$$b = \frac{360}{3.2} = 112.5$$

$$h = 0.3$$

$$\begin{aligned} c) \quad y &= 10 \cos [112.5(x - 0.3)] + 50 \\ 43.75 &= -10 \cos [112.5(x - 0.3)] + 50 \\ -6.25 &= \frac{-10 \cos [112.5(x - 0.3)]}{-10} \end{aligned}$$

$$0.625 = \cos [112.5(x - 0.3)]$$

$$\cos^{-1}(0.625) = \cos^{-1}(\cos [112.5(x - 0.3)])$$


$$\frac{51.32}{112.5} = \frac{112.5(x - 0.3)}{112.5}$$

$$0.46 = x - 0.3$$

$$\boxed{0.76 \text{ s} = x}$$

## PRACTICE TIME...

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 Review - Practice Test for Sinusoidal Functions.doc

# Practice Test Solutions

**Part A: Multiple Choice**

- |       |                     |
|-------|---------------------|
| 1. A  | 11. A (second hand) |
| 2. D  | 12. C               |
| 3. A  | 13. A               |
| 4. C  | 14. C               |
| 5. B  | 15. D               |
| 6. D  | 16. D               |
| 7. A  | 17. B               |
| 8. D  | 18. D               |
| 9. B  | 19. A               |
| 10. A | 20. A               |

**Part B: Open Response**

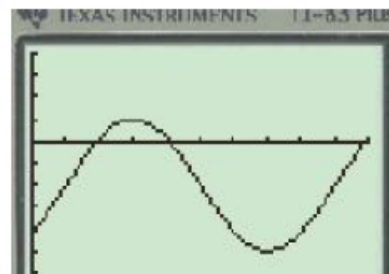
1.  $-\frac{5}{4}$

2. (i)  $y = 3 \sin \frac{3}{2}(x - 160^\circ) - 6$

$y = 3 \cos \frac{3}{2}(x + 20^\circ) - 6$

(ii)  $(x, y) \rightarrow \left( \frac{2}{3}x + 160^\circ, 3y - 6 \right)$

3.



X	Y1
15	-2
45	1
75	-2
105	1
135	-2
165	1
195	-2

X=195

4. 10.28 m



# MORE PRACTICE???

Review - Trigonometric Functions.doc

## SOLUTIONS

1. (a)  $39^\circ$

(b)  $53^\circ$

2. (a)  $-2$

(b)  $\frac{7-2\sqrt{3}}{4}$

3. (a) II

(b) II

4. (a)  $-1.2799$

c)  $1.2690$

(e)  $-5$

(b)  $-1.0864$

(d)  $39^\circ$

(f)  $25^\circ$

5.  $\sin \theta = \frac{-\sqrt{5}}{5}$

$\csc \theta = -\sqrt{5}$

$\cos \theta = \frac{-2\sqrt{5}}{5}$

$\sec \theta = \frac{\sqrt{5}}{2}$

$\tan \theta = \frac{1}{2}$

$\cot \theta = 2$

6.  $\frac{-\sqrt{10}}{2}$

8. Amp = 3

Period =  $180^\circ$

V.T. = Up 2

P.S. = none

Domain:  $0^\circ \leq \theta \leq 360^\circ$

(b) Amp = 2

Period =  $120^\circ$

V.T. = Down 2

P.S. =  $60^\circ$  left

Domain:  $\mathbb{R}$

(c) Amp = 2

Period =  $720^\circ$

V.T. = Up 5

P.S. = none

Domain:  $-90^\circ \leq \theta \leq 360^\circ$

Range:  $-3 \leq y \leq 7$

(d) Amp = 6

Period =  $360^\circ$

V.T. = None

P.S. =  $90^\circ$  right

Domain:  $\mathbb{R}$

Range:  $-6 \leq y \leq 6$

10. 11.9 m

11. 46.2 cm

## Attachments

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Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc

Sketching Sinusoidal Functions #2.pdf

Sketching Sinusoidal Functions #2.doc

Sketching Sinusoidal Functions #3 (Solutions).doc

worksheet-sketching in radian measure.doc