Trigonometric Identities

Prerequisite Skills...

Simple trinomial

Diff. of squares

Factor:

$\cos^2\theta + 10\cos\theta - 24$

$12 \times -2 = -24$
$12 + -2 = 10$

$(\cos\theta - 2)(\cos\theta + 12)$

$\sin^2\theta - \cos^2\theta$

$(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

trinomial decomposition

$9\tan^4 x - 6\tan^2 x + 1$

$-3 \times -3 = 9$
$-3 + -3 = -6$

$\left(\tan^2 x - \frac{3}{9}\right)\left(\tan^2 x - \frac{3}{9}\right)$

$\left(\tan^2 x - \frac{1}{3}\right)\left(\tan^2 x - \frac{1}{3}\right)$ (Reduce)

$(3\tan^2 x - 1)(3\tan^2 x - 1)$

Diff of squares

Simplify the following expression: $\dfrac{\tan^2\theta(\cos^2\theta - 1)}{\tan\theta\cos\theta + \tan\theta}$

Common factor

$\dfrac{\cancel{\tan\theta}\cos\theta}{\cancel{\tan\theta}} + \dfrac{\cancel{\tan\theta}}{\cancel{\tan\theta}}$

$\dfrac{\tan^{\cancel{2}}\theta\,\cancel{(\cos\theta + 1)}(\cos\theta - 1)}{\cancel{\tan\theta}\,\cancel{(\cos\theta + 1)}}$

$\boxed{\tan\theta(\cos\theta - 1)}$

Find a common denominator for each of the following:

$$\frac{3}{5a} - \frac{5}{4b}$$

$$\frac{3(4b)}{(5a)(4b)} - \frac{5(5a)}{(5a)(4b)}$$

$$\frac{12b}{20ab} - \frac{25a}{20ab}$$

$$\frac{12b - 25a}{20ab}$$

$$\frac{2}{x+3} + \frac{1}{x-6}$$

$$\frac{2(x-6)}{(x+3)(x-6)} + \frac{1(x+3)}{(x+3)(x-6)}$$

$$\frac{2x-12}{(x+3)(x-6)} + \frac{x+3}{(x+3)(x-6)}$$

$$\frac{3x-9}{(x+3)(x-6)} \text{ or } \frac{3x-9}{x^2-3x-18}$$

$$\frac{tanx}{1 - cosx} + \frac{sinx}{1 + cosx}$$

$$\frac{\tan x(1+\cos x)}{(1-\cos x)(1+\cos x)} + \frac{\sin x(1-\cos x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{\tan x + \tan x\cos x}{(1-\cos x)(1+\cos x)} + \frac{\sin x - \sin x\cos x}{(1-\cos x)(1+\cos x)}$$

$$\frac{\tan x + \tan x\cos x + \sin x - \sin x\cos x}{(1-\cos x)(1+\cos x)}$$

or

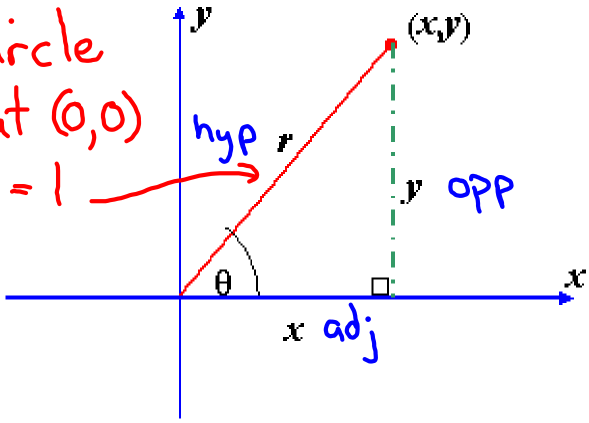
$$\frac{\tan x + \tan x\cos x + \sin x - \sin x\cos x}{1 - \cos^2 x}$$

Trig Identities

Reciprocal Identities

Unit Circle
- Center at (0,0)
- Radius = 1

r = 1



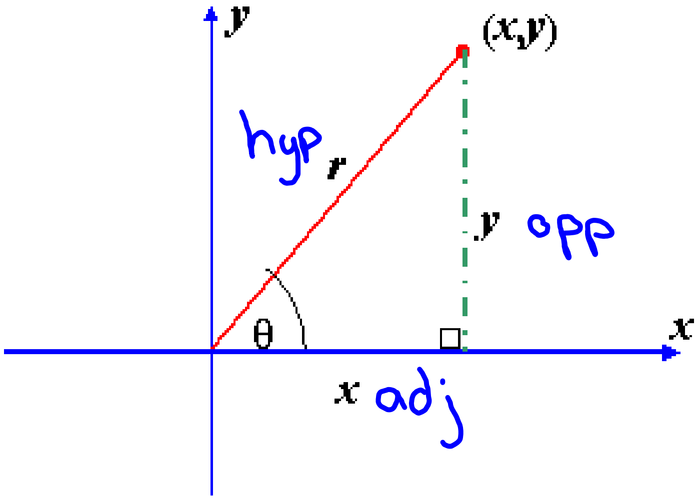
$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$	$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$	$\tan\theta = \frac{y}{x}$
$\csc\theta = \frac{r}{y} = \frac{1}{y}$	$\sec\theta = \frac{r}{x} = \frac{1}{x}$	$\cot\theta = \frac{x}{y}$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta = \frac{1}{\csc^2\theta}$	$\cos^2\theta = \frac{1}{\sec^2\theta}$	$\tan^2\theta = \frac{1}{\cot^2\theta}$
$\csc^2\theta = \frac{1}{\sin^2\theta}$	$\sec^2\theta = \frac{1}{\cos^2\theta}$	$\cot^2\theta = \frac{1}{\tan^2\theta}$

Quotient Identities



$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

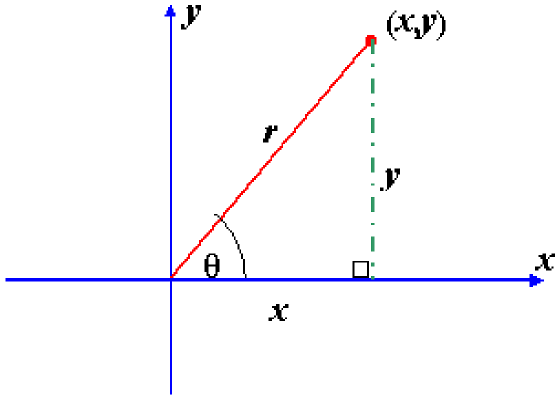
$$\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

Quotient Identities

$$\boxed{\begin{gathered}\tan\theta = \frac{\sin\theta}{\cos\theta} \\ \cot\theta = \frac{\cos\theta}{\sin\theta}\end{gathered}}$$

$$\boxed{\begin{gathered}\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \\ \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}\end{gathered}}$$

Pythagorean Identities



$\boxed{r=1}$

$\div r^2$

$$\frac{x^2}{r^2}+\frac{y^2}{r^2}=\frac{r^2}{r^2}$$

$$x^2+y^2=1$$

$$\cos^2\theta+\sin^2\theta=1$$

$$\boxed{\sin^2\theta+\cos^2\theta=1}$$

$$\sin^2\theta=1-\cos^2\theta$$

$$\cos^2\theta=1-\sin^2\theta$$

$\div x^2$

$$\frac{x^2}{x^2}+\frac{y^2}{x^2}=\frac{r^2}{x^2}$$

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$1+\tan^2\theta=\sec^2\theta$$

$$\boxed{\tan^2\theta+1=\sec^2\theta}$$

$$\tan^2\theta=\sec^2\theta-1$$

$$1=\sec^2\theta-\tan^2\theta$$

$\div y^2$

$$\frac{x^2}{y^2}+\frac{y^2}{y^2}=\frac{r^2}{y^2}$$

$$\frac{\cos^2\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\cot^2\theta+1=\csc^2\theta$$

$$\boxed{\cot^2\theta+1=\csc^2\theta}$$

$$\cot^2\theta=\csc^2\theta-1$$

$$1=\csc^2\theta-\cot^2\theta$$

Trigonometric Identities

You must know these!

Quotient

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$ $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$ $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$

Pythagorean

$$\sin^2\theta + \cos^2\theta = 1$$ $\sin^2\theta = 1-\cos^2\theta$ $\cos^2\theta = 1-\sin^2\theta$

$$\tan^2\theta + 1 = \sec^2\theta$$ $\tan^2\theta = \sec^2\theta - 1$ $\sec^2\theta - \tan^2\theta = 1$

$$1 + \cot^2\theta = \csc^2\theta$$ $\cot^2\theta = \csc^2\theta - 1$ $\csc^2\theta - \cot^2\theta = 1$

Reciprocal

$$\csc\theta = \frac{1}{\sin\theta}$$ $\csc^2\theta = \frac{1}{\sin^2\theta}$ $$\sin\theta = \frac{1}{\csc\theta}$$ $\sin^2\theta = \frac{1}{\csc^2\theta}$

$$\sec\theta = \frac{1}{\cos\theta}$$ $\sec^2\theta = \frac{1}{\cos^2\theta}$ $$\cos\theta = \frac{1}{\sec\theta}$$ $\cos^2\theta = \frac{1}{\sec^2\theta}$

$$\cot\theta = \frac{1}{\tan\theta}$$ $\cot^2\theta = \frac{1}{\tan^2\theta}$ $$\tan\theta = \frac{1}{\cot\theta}$$ $\tan^2\theta = \frac{1}{\cot^2\theta}$

Strategies for Proving Identities:

- Work on the most complex side and simplify so it has the same form as the simpler side LHS = RHS
- Methods used in simplifying: direct substitution, factoring, finding a common denominator, multiplying by the conjugate

* $\sin\theta$ and $\cos\theta$ are "good guys" → as much as possible, we will leave them alone

Prove the following:

$$\frac{\boxed{\tan x}}{\sin x} = \boxed{\sec x}$$

$\frac{\sin x}{\cos x} \div \sin x$ | $\frac{1}{\cos x}$

$\frac{\cancel{\sin x}}{\cos x} \times \frac{1}{\cancel{\sin x}}$

$\frac{1}{\cos x}$

$$\cos\theta \bullet \boxed{\sec\theta} = 1$$

$\cancel{\cos\theta} \bullet \frac{1}{\cancel{\cos\theta}}$ |

1

Prove the following:

$$\boxed{\cot\theta} \bullet \sin\theta = \cos\theta$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta}$$

$$\cos\theta$$

$$\frac{\cos x}{\boxed{\tan x}} = \frac{\boxed{1-\sin^2 x}}{\sin x}$$

Pythagorean

$$\cos x \div \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x}{\sin x}$$

$$\cos x \bullet \frac{\cos x}{\sin x}$$

$$\frac{\cos^2 x}{\sin x}$$

Ex. Prove that $\sin y + \sin y \cot^2 y = \boxed{\csc y}$

$\sin y \boxed{(1 + \cot^2 y)}$ | $\frac{1}{\sin y}$

$\sin y (\csc^2 y)$

$\cancel{\sin y} \left(\frac{1}{\sin^{\cancel{2}} y} \right)$

$\frac{1}{\sin y}$

Homework

1. $\tan\theta\cos\theta = \sin\theta$

2. $\cot\theta\sec\theta = \csc\theta$

3. $\dfrac{1+\cot^2\theta}{\csc^2\theta} = 1$

4. $\dfrac{\tan^2\theta}{1+\tan^2\theta} = \sin^2\theta$

5. $\dfrac{\tan^2\theta}{\sin^2\theta} = 1+\tan^2\theta$