Chapter 8

Financial Mathematics: Investing Money

LEARNING GOALS

You will be able to develop your number sense in financial applications by

- Understanding and comparing the effects of simple interest and compound interest
- Determining how changes in the variables of an investment affect the return
- Being aware of a variety of different investment instruments
- Comparing different investment strategies

8.1 Simple Interest

term
The contracted dur... investment or loan.

interest
The amount of money earned on an investment or paid on a loan.

fixed interest rate
An interest rate that is guaranteed not to change during the term of an investment or loan.

principal
The original amount of money invested or loaned.

maturity
The contracted end date of an investment or loan, at the end of the term.

future value
The amount, A, that an investment will be worth after a specified period of time.

GOAL
Solve problems that involve simple interest

simple interest
The amount of interest earned on an investment or paid on a loan based on the original amount (the principal) and the simple interest rate.

Communication *Tip*
Interest rates are communicated as a percent for a time period. Since most often the time period is per year or **per annum** (abbreviated as **/a**), a given percent is assumed to be annual unless otherwise stated. For example, an interest rate of 4% means 4%/a or 4% interest per year.

SIMPLE Interest	Based on the **principal** (original amount) that is invested/borrowed. Interest is a certain percentage per **annum** (year). Often used for personal loans and short-term investments. The length of time for the investment/loan is called the **term**.

Interest = Principal x rate x time

$\boxed{I = Prt}$

&

$\boxed{A = P + I}$

OR

$A = P + Prt$

$A = P(1 + rt)$

- I - interest earned
- P - principal (original investment/loan)
- r - interest rate as a percent **(change to a decimal)**
- t - is ALWAYS time in **years**

(how long the money is invested/borrowed)

- **A - amount of money including interest**

$= 2500(1 + (0.025)(10))$

$= 2500(1 + 0.25)$

$= 2500(1.25)$

$= 3125$

APPLY the Math p. 446

EXAMPLE 1 Solving a simple interest problem

Marty invested in a $2500 guaranteed investment certificate (GIC) at 2.5% simple interest paid annually, with a term of 10 years.

a) How much interest will accumulate over the term of Marty's investment?

b) What is the **future value** of his investment at maturity?

NOTE:
Means that interest is paid only in yearly increments.

Given:

$P = 2500$

$r = 2.5\% = 0.025$

$t = 10$

a) $I = Prt$

$I = (2500)(0.025)(10)$

$I = \$625$

b) $A = P + I$

$A = 2500 + 625$

$A = \$3125$

EXAMPLE #2:
Betty-Ann's bank offers a simple interst rate of 4% per annum. How much interest would Betty-Ann earn on her investment of $4000 after 8 months.

Given:

$r = 4\% = 0.04$

$P = 4000$

$t = 8 \text{ months}$

$t = \frac{8}{12} = \frac{2}{3}$

$I = Prt$

$I = 4000\,(0.04)\,\underset{(0.67)}{(8/12)}$

$I = \$106.67$

Time in years!!

rate of return
The ratio of money earned (or lost) on an investment relative to the amount of money invested, usually expressed as a decimal or a percent.

$$ROR = \frac{\text{earn / lost}}{\text{invested}}$$

Principal

EXAMPLE 3 p. 448 — Determining the duration of a simple interest investment

Ingrid invested her summer earnings of \$5000 at 8% simple interest, paid annually. She intends to use the money in a few years to take a holiday with a girlfriend.

Solve for t

a) How long will it take for the future value of the investment to grow to \$8000? ← A = 8000

b) What is Ingrid's **rate of return**?

Given: P = 5000
r = 8% = 0.08

Ingrid's Solution

a) $A = P + Prt$

P is \$5000.
r is 8%, or 0.08.
A is \$8000.

$$8000 = 5000 + (5000)(0.08)t$$
$$3000 = 400t$$
$$7.5 = t$$

I knew P, r, and A. I determined t by substituting these known values into the formula $A = P + Prt$ and solving for t.

Because I needed to isolate t, I knew that the $A = P + Prt$ form of the equation would have fewer solution steps than the $A = P(1 + rt)$ form would.

It will take 8 years for the future value of the investment to be at least \$8000.

I knew 7.5 years would not work because the interest is paid annually. This meant that I had to round up to the next whole year. It also meant that, at 8 years, the future value would be more than \$8000.

b) After 8 years:
$A = P + Prt$
$A = 5000 + (5000)(0.08)(8)$
$A = 8200$

$I = Prt$
$I = 5000(0.08)(8)$
$I = \underline{3200}$

At 8 years, the future value will be \$8200.

Interest earned:
\$8200 − \$5000 = \$3200

I determined the interest earned by subtracting the principal from the future value.

$$\text{Rate of return} = \frac{3200}{5000}$$

$$\text{Rate of return} = 0.64$$

I compared the interest earned with the principal to determine the rate of return.

The rate of return is 64% over 8 years.

EXAMPLE 4 Determining the rate of interest on a simple interest investment p. 450

Grant invested $25 000 in a simple interest Canada Savings Bond (CSB) that paid interest annually.

a) If the future value of the CSB is $29 375 at the end of 5 years, what interest rate does the CSB earn? Solve for r

b) Grant cashed in the bond after 4.5 years because a house he had been admiring came up for sale and he needed a down payment. How much money did he have for the down payment?

Given:

$P = 25000$

$A = 29375$

$t = 5$

$A = P + Prt$

$29375 = 25000 + (25000)(r)(5)$

$\frac{4375}{125000} = \frac{\cancel{125000}r}{\cancel{125000}}$

$0.035 = r$

as a percentage: $r = 0.035 \times 100$
$= 3.5\%$

b) $A = P(1 + rt)$

$A = 25000(1 + (0.035)(4.5))$

$A = 25000(1 + 0.1575)$

$A = 25000(1.1575)$

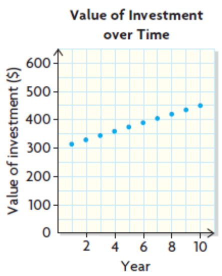
$A = \$28937.50$

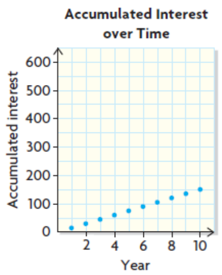
SIMPLE INTEREST...

In Summary p. 451

Key Ideas

- Simple interest is determined only on the principal of an investment.
- The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest earned over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. For example, the following graphs show principal of \$300 invested at 5% interest, paid annually, over a term of 10 years.





Need to Know

- The amount of simple interest earned on an investment can be determined using the formula

$$I = Prt$$

  where I is the interest, P is the principal, r is the annual interest rate expressed as a decimal, and t is the time in years.
- The future value or amount, A, of an investment that earns simple interest can be determined using the formula

$$A = P + Prt$$
$$\text{or } A = P(1 + rt)$$

  where P is the principal, r is the interest rate expressed as a decimal, and t is the time in years.
- Unless otherwise stated, an interest rate is assumed to be annual, or per annum.
- Even though interest rates are usually annual, interest can be paid out at different intervals, such as annually, semi-annually, monthly, weekly, and daily.

$$$ Questions...great website for answers!!!

HOMEWORK...

p. 452: 1, 2, 3a, 4, 5, 10

$$\boxed{I = Prt}$$

&

$$\boxed{A = P + I}$$

OR

$$\boxed{\begin{array}{l} A = P + Prt \\ A = P(1 + rt) \end{array}}$$

Solutions

1. Determine the future value of each investment if it earns simple interest.

a) 8-year term on a principal of \$30 000 at 3.5 %.

$A = P + I$

$A = P + Prt$

or $A = P(1 + rt)$

$= \$30\,000\,[1 + (0.035)(8)]$

$= \$30\,000\,(1 + 0.28)$

$= \$30\,000\,(1.28)$

$= \$38\,400$

b) 1.25% interest paid quarterly for 4 years on \$10 000.

$A = P(1 + rt)$

$= \$10\,000\,[1 + (0.0125)(4)]$

$= \$10\,000\,(1 + 0.05)$

$= \$10\,000\,(1.05)$

$= \$10\,500$

c) 0.5% interest paid weekly for 2 years on \$25 000.

$$A = P(1 + rt)$$
$$= \$25\,000\,[1 + (0.005)(2)]$$
$$= \$25\,000\,(1 + 0.01)$$
$$= \$25\,000\,(1.01)$$
$$= \$25\,250$$

d) 20-year term at 7.4% on a deposit of \$12 000.

$$A = P(1 + rt)$$
$$= \$12\,000\,[1 + (0.074)(20)]$$
$$= \$12\,000\,(1 + 1.48)$$
$$= \$12\,000\,(2.48)$$
$$= \$29\,760$$

2. Cam has \$5000 to invest. He wants his principal to grow to \$6500 in 5 years so that he can afford a new drum kit.

a) What simple interest rate will allow him to meet his goal?

$$A = P + I$$
$$\$6500 = \$5000 + I$$
$$\$6500 - \$5000 = I$$
$$\$1500 = I$$
$$\$1500 = Prt$$
$$\$1500 = \$5000(r)(5)$$
$$\$1500 = \$25000r$$
$$\frac{\$1500}{\$25000} = \frac{\cancel{\$25000}r}{\cancel{\$25000}}$$
$$0.06 = r$$

A 6% interest rate will allow him to meet his goal.

b) Suppose that interest is paid semi-annually and Cam withdraws all the money after 3.25 years. How much money will be have?

3

$$A = P(1 + rt)$$
$$= \$5000[1 + (0.06)(3)]$$
$$= \$5000(1 + 0.18)$$
$$= \$5000(1.18)$$
$$= \$5900$$

3a) Principal of \$1000 is invested at 5% simple interest, paid annually, for 5 years. What is the rate of return?

$$\begin{aligned} A &= P(1+rt) \\ &= \$1000[1+(0.05)(5)] \\ &= \$1000(1+0.25) \\ &= \$1000(1.25) \\ &= \$1250 \end{aligned}$$

$$\begin{aligned} ROR &= \frac{\text{earn/lost}}{\text{invested}} \\ &= \frac{\$250}{\$1000} \\ &= 0.25 \text{ or } 25\% \end{aligned}$$

4. Char invested \$4000 at a simple interest rate of 2.3%.

a) What is the value of the investment after 5 years?

$$A = P(1+rt)$$
$$= \$4000[1+(0.023)(5)]$$
$$= \$4000(1+0.115)$$
$$= \$4000(1.115)$$
$$= \$4460$$

b) What is its value after 10 years?

$$A = P(1+rt)$$
$$= \$4000[1+(0.023)(10)]$$
$$= \$4000(1+0.23)$$
$$= \$4000(1.23)$$
$$= \$4920$$

5. Both Brad and Chris purchased a $15000 GIC.

- Brad's GIC has a term of 6 years and a simple interest rate of 3.2 %.
- Chris's GIC has a term of 5 years at a simple interest rate of 3.3%.

Whose GIC will have the greatest future value at maturity? Explain.

BRAD

$$A = P(1+rt)$$
$$= \$15000[1 + (0.032)(6)]$$
$$= \$15000(1 + 0.192)$$
$$= \$15000(1.192)$$
$$= \$17\,880$$

CHRIS

$$A = P(1+rt)$$
$$= \$15000[1+(0.033)(5)]$$
$$= \$15\,000(1 + 0.165)$$
$$= \$15\,000(1.165)$$
$$= \$17\,475$$

★ Brad's GIC will have the greatest future value. Even though the interest rates are almost the same, Brad's GIC has a longer term.

10. Shaun has been looking at houses. He has $10 000 that he wants to invest, hoping that he can end up with $15 000 to make a down payment on a house. He has an opportunity to invest at 6.5% simple interest, paid annually. How long will it take before Shaun can make a down payment of $15 000?

$$A = P + I$$
$$\$15000 = \$10\,000 + I$$
$$\$15000 - \$10000 = I$$
$$\$5000 = I$$
$$\$5000 = Prt$$
$$\$5000 = \$10\,000(0.065)t$$
$$\$5000 = 650t$$
$$\frac{\$5000}{650} = \frac{\cancel{650}t}{\cancel{650}}$$
$$7.7 = t$$

It will take Shaun 8 years to make a down payment of $15 000.