

Chapter 2 Radical Functions

Solve for x: $\sqrt{x+8} = x+6$ *Square both sides*

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2 + 6x + 6x + 36$$

$$x+8 = x^2 + 12x + 36$$
 Bring all terms to same side

$$0 = x^2 + 12x + 36 - x - 8$$

$$0 = x^2 + 11x + 28$$
 $\frac{4}{4} \times \frac{7}{7} = 28$
 $\frac{4}{4} + \frac{7}{7} = 11$

$$0 = (x+4)(x+7)$$

$$x+4 = 0 \quad | \quad x+7 = 0$$

$$x = -4 \quad | \quad x = -7$$

*Set both factors = 0*Test $x = -4$ *is a solution*

$$\sqrt{x+8} = x+6$$

$$\sqrt{-4+8} \quad | \quad -4+6$$

$$\sqrt{4} \quad | \quad 2$$

$$2$$

Test $x = -7$ *is extraneous*

$$\sqrt{x+8} = x+6$$

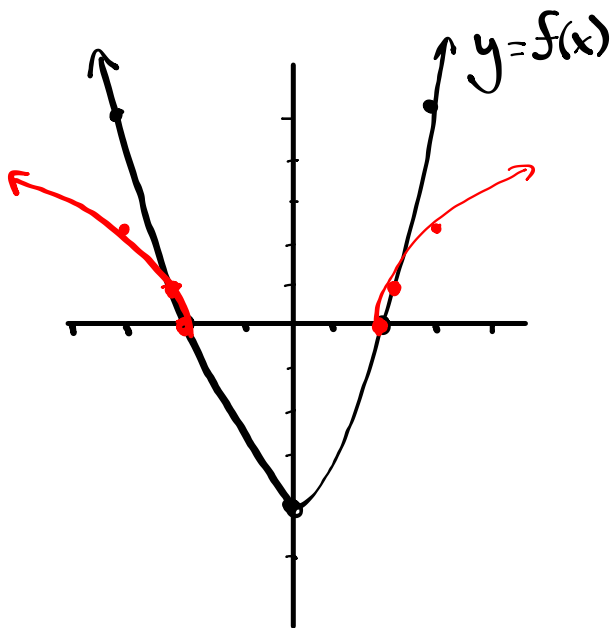
$$\sqrt{-7+8} \quad | \quad -7+6$$

$$\sqrt{1} \quad | \quad -1$$

$$1$$

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. state the domain and range of each.



$$y = f(x)$$

$$D: \{x | x \in \mathbb{R}\} \text{ or } (-\infty, \infty)$$

$$R: \{y | y \geq -4, y \in \mathbb{R}\} \text{ or } [-4, \infty)$$

$$y = \sqrt{f(x)}$$

$$D: \{x | x \leq -2, x \geq 2, x \in \mathbb{R}\}$$

$$\text{or } (-\infty, -2] + [2, \infty)$$

$$R: \{y | y \geq 0, y \in \mathbb{R}\} \text{ or } [0, \infty)$$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

~~$2^{2x+2} = 6$~~

$2x+2 = 6$

$2x = \frac{4}{2}$

 $x = 2$ is a solution

Test

$2^{2x+2} + 7 = 71$

$2^{2(2)+2} + 7$

$2^6 + 7$

$64 + 7$

71

b) $9^{2x+1} = 81(27^x)$

$(3^2)^{2x+1} = (3^4)(3^x)$

$3^{4x+2} = (3^4) \cdot (3^x)$

~~$3^{4x+2} = 3^{3x+4}$~~

$4x+2 = 3x+4$

 $x = 2$ is a solution

Test:

$9^{2x+1} = 81(27^x)$

$9^{2(2)+1}$

9^5

59049

$81(27^2)$

$81(729)$

59049

4. Rewrite each expression as a single logarithm.

$$3 \log_5 x + \frac{1}{2} \log_5 (x - 1) - \log_5 (x^2 + 1)$$

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x + 2) + \log_{10}(x - 1) = 1$$

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

b) What amount will be present in 10.6 years? [2]

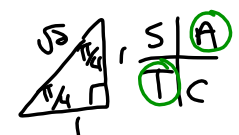
c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

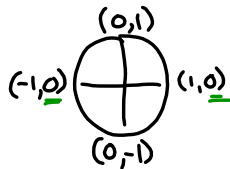
2. Solve for all values of θ in the specified domain.

$$\tan^2 \theta + \tan \theta = 0, \quad 0 \leq \theta \leq 2\pi$$

② a) $\sin \theta = \sin \theta \tan \theta$ $0 \leq \theta \leq 2\pi$
 $0 = \sin \theta \tan \theta - \sin \theta$
 $0 = (\sin \theta)(\tan \theta - 1)$

$\sin \theta = 0$ | $\tan \theta - 1 = 0$
 $\theta = 0, \pi, 2\pi$ | $\tan \theta = 1$
 $\theta_R = \frac{\pi}{4}$





Where is $\tan \theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

③ b) $3 \sin^2 \theta - 2 \sin \theta - 1 = 0$ $0 \leq \theta \leq 360^\circ$
 $(3 \sin^2 \theta - 3 \sin \theta + \sin \theta - 1) = 0$ $\frac{-3}{-3} \times \frac{1}{-1} = -3$
 $3 \sin \theta (\sin \theta - 1) + 1 (\sin \theta - 1) = 0$ $\frac{-3}{-3} + \frac{1}{-1} = -2$
 $(3 \sin \theta + 1)(\sin \theta - 1) = 0$

$3 \sin \theta + 1 = 0$ | $\sin \theta - 1 = 0$
 $\sin \theta = -\frac{1}{3}$ | $\sin \theta = 1$
 $\theta_R = \sin^{-1}(\frac{1}{3})$ → Unit Circle
 $\theta_R = 19$
 $\theta = 90^\circ$

Where is sine negative: 

Q3	Q4
$\theta = 180^\circ + \theta_R$	$\theta = 360^\circ - \theta_R$
$\theta = 180^\circ + 19$	$\theta = 360^\circ - 19$
$\theta = 199$	$\theta = 340$

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

$$y = \cos x$$

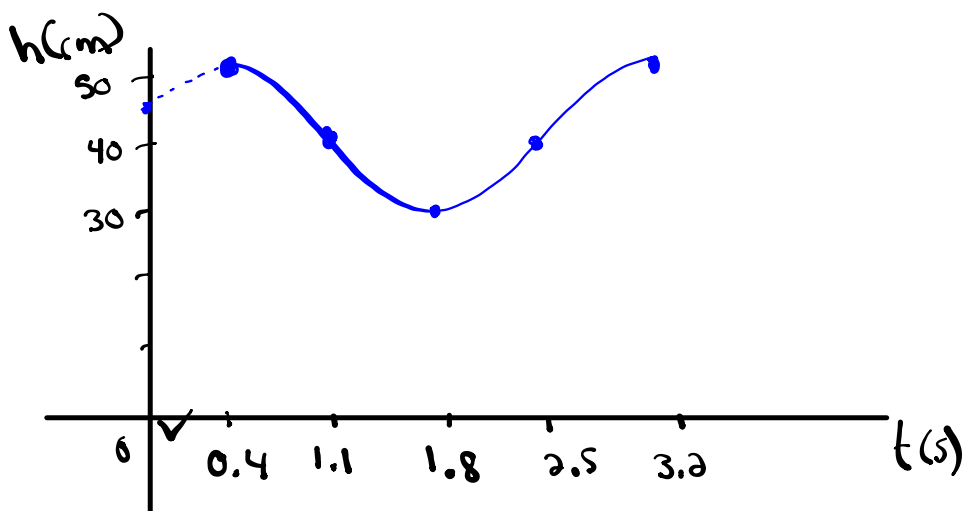
(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

$$\begin{aligned} \text{max} &= 50 \text{ cm} & \text{Amp} &= 10 & P &= 2(1.8 - 0.4) = 2.8 & h &= 0.4 \\ \text{min} &= 30 \text{ cm} & a &= \pm 10 & b &= \frac{360}{2.8} = 128.57 \\ K &= \frac{50 + 30}{2} = 40 & y &= 10 \cos[128.57(17.2 - 0.4)] + 40 \\ & & y &= 49.9 \text{ cm} \end{aligned}$$

(b) How high was the weight above the floor when the stopwatch was initially started?

$$t = 0$$

$$\begin{aligned} y &= 10 \cos[128.57(0 - 0.4)] + 40 \\ y &= 46.23 \text{ cm} \end{aligned}$$



$$\text{count by } \frac{P}{4} = \frac{2.8}{4} = 0.7$$

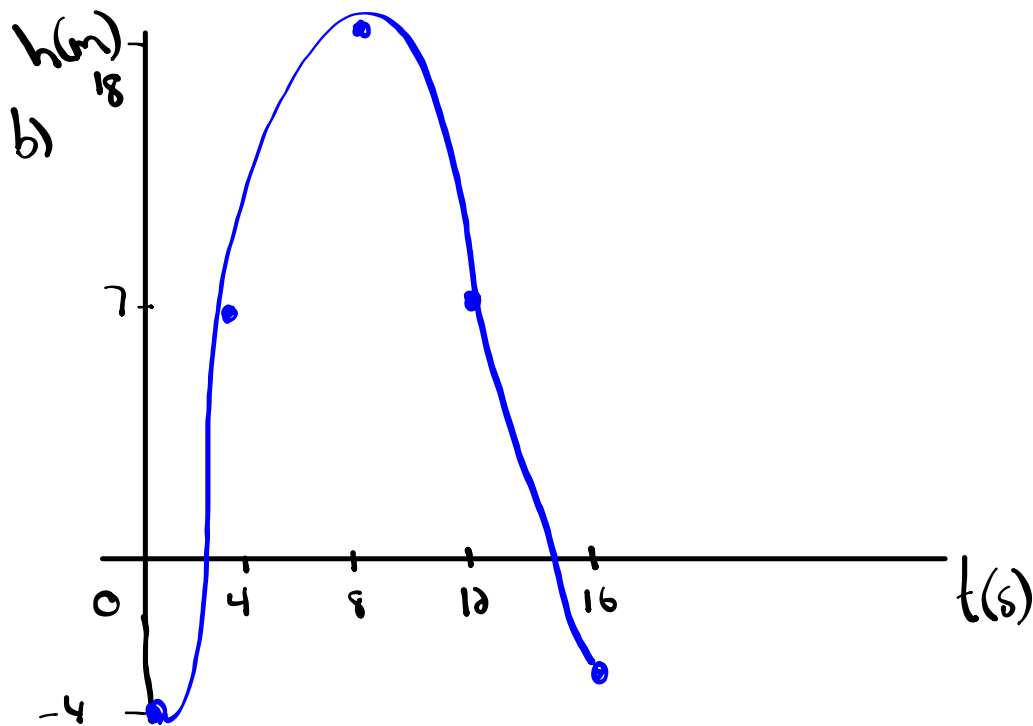
$$\textcircled{1} \text{ Amp} = 11 \quad P = 16 \quad \text{min} = -4$$

$$a = \pm 11 \quad b = \frac{360}{16} = 22.5 \quad \text{max} = -4 + 22 = 18$$

$$K = -4 + 11 = 7$$

$$h = 0$$

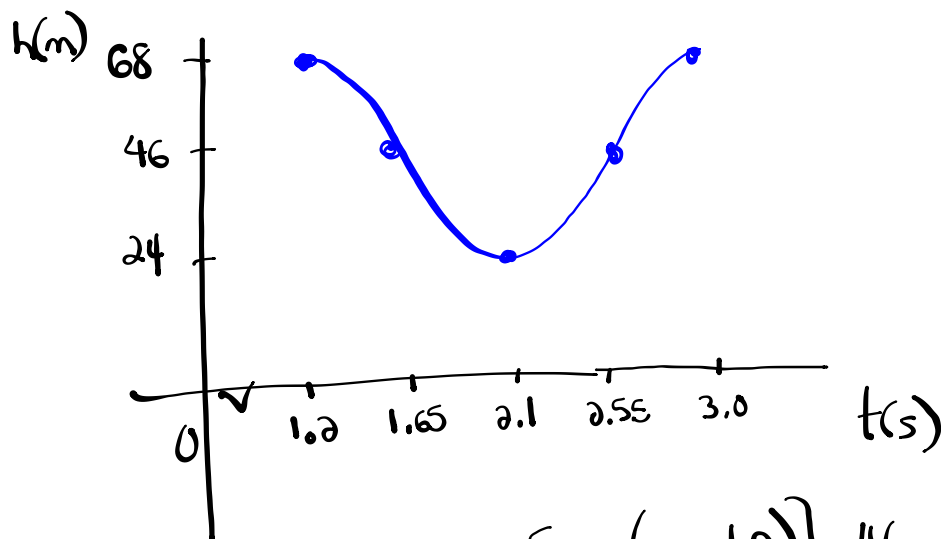
a) equation: $y = -11\cos[22.5(x)] + 7$



$$\textcircled{4} \quad \begin{array}{ll} \max = 68 & \text{Amp} = 68 - 46 = 22 \\ \min = 24 & a = \pm 22 \\ k = \frac{68 + 24}{2} = 46 & P = 2(2.1 - 1.2) \\ & P = 1.8 \\ & b = \frac{360}{1.8} = 200 \end{array}$$

$$h = \underline{1.2}$$

$$\frac{P}{4} = \frac{1.8}{4} = 0.45$$



$$y = 22 \cos[200(x - 1.2)] + 46$$

$$\textcircled{5} \text{ c) } y = \frac{1}{2} \cos(\theta + \underline{\pi}) - \underline{4}$$

$$a = \frac{1}{2}$$

$$(x, y) \rightarrow \left(\frac{1}{2}x - \pi, \frac{1}{2}y - 4 \right)$$

$$b = 1$$

$$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

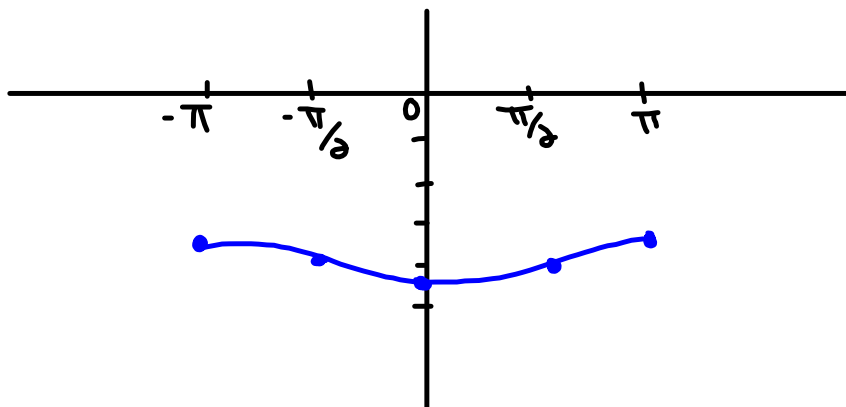
$$c = -\pi$$

$$d = -4$$

$$y = \cos \theta$$

x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

x	y
$-\pi$	$-\frac{7}{2}$ -3.5
$-\frac{\pi}{2}$	-4 -4
0	$-\frac{9}{2}$ -4.5
$\frac{\pi}{2}$	-4 -4
π	$-\frac{7}{2}$ -3.5



$$\frac{1}{\sec^2 \theta \cot \theta} = \frac{\sin \theta - \sin^3 \theta}{\cos \theta}$$