# Chapter 2 Radical Functions

Solve for x: $\sqrt{x+8} = x+6$ Square both sides

$x+8 = (x+6)(x+6)$

$x+8 = x^2+6x+6x+36$

$x+8 = x^2+12x+36$ Bring all terms to same side

$0 = x^2+12x+36-x-8$

$0 = x^2+11x+28$ $\underline{4} \times \underline{7} = 28$, $\underline{4} + \underline{7} = 11$

$0 = (x+4)(x+7)$

| $x+4=0$ | $x+7=0$ |
|---|---|
| $x=-4$ | $x=-7$ |

set both factors = 0

Test $x=-4$ is a solution

$\sqrt{x+8} = x+6$

| | |
|---|---|
| $\sqrt{-4+8}$ | $-4+6$ |
| $\sqrt{4}$ | $2$ |
| $2$ | |

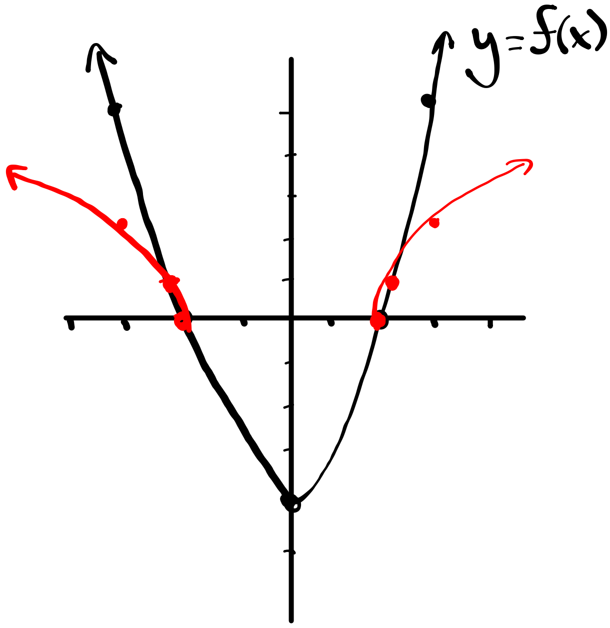
Test $x=-7$ is extraneous

$\sqrt{x+8} = x+6$

| | |
|---|---|
| $\sqrt{-7+8}$ | $-7+6$ |
| $\sqrt{1}$ | $-1$ |
| $1$ | |

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. State the domain and range of each.




$y = f(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in \mathbb{R}\}$ or $[-4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in \mathbb{R}\}$

or $(-\infty, -2] + [2, \infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

# Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2}+7=71$

(b) $9^{2x+1}=81(27^x)$

a) $2^{2x+2}+7=71$

$2^{2x+2}=64$

$\cancel{2}^{2x+2}=\cancel{2}^{6}$

$2x+2=6$

$\frac{2x}{2}=\frac{4}{2}$

$x=2$ is a solution

Test

$2^{2x+2}+7=71$

| | |
|---|---|
| $2^{2(2)+2}+7$ | 71 |
| $2^6+7$ | |
| $64+7$ | |
| $71$ | |

b) $9^{2x+1}=81(27^x)$

$(3^2)^{2x+1}=(3^4)((3^3)^x)$

$3^{4x+2}=(3^4)\cdot(3^{3x})$

$\cancel{3}^{4x+2}=\cancel{3}^{3x+4}$

$4x+2=3x+4$

$x=2$ is a solution

Test:

$9^{2x+1}=81(27^x)$

| | |
|---|---|
| $9^{2(2)+1}$ | $81(27^2)$ |
| $9^5$ | $81(729)$ |
| $59049$ | $59049$ |

# Chapter 8 → Logarithms

**4. Rewrite each expression as a single logarithm.**

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left(\frac{x^3 \cdot (x-1)^{1/2}}{(x^2+1)}\right)$$

$$\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}$$

ex: $3\log_2 x - 4\log_2 x^2 + \log_2 x - 2\log_2 x^5$

$$\log_2 x^3 - \log_2 x^8 + \log_2 x - \log_2 x^{10}$$

$$\log_2\left(\frac{x^3 \cdot x}{x^8 \cdot x^{10}}\right)$$

$$\log_2\left(\frac{x^4}{x^{18}}\right)$$

$$\log_2 x^{-14}$$

$$-14\log_2 x$$

# Chapter 8 → Logarithms

7. **Solve the following equation (be sure to test your answers).**

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}(x^2-x+2x-2)=1$$

$$\log_{\underset{\text{Base}}{10}}\underbrace{(x^2+x-2)}_{\text{ans}} = \underset{\text{exp}}{1} \quad \text{(log form)}$$

$$10^1 = x^2+x-2 \quad \text{(exp. form)}$$

$$10 = x^2+x-2 \quad \text{Bring everything to one side}$$

$$0 = x^2+x-12 \qquad -3 \times 4 = -12$$

$$-3 + 4 = 1$$

$$0=(x-3)(x+4)$$

| | |
|---|---|
| $x-3=0$ | $x+4=0$ |
| $x=3$ | $x=-4$ |

| Test $x=3$ is a solution | Test $x=-4$ is extraneous |
|---|---|
| $\log_{10}(x+2)+\log_{10}(x-1)=1$ | $\log_{10}(x+2)+\log_{10}(x-1)=1$ |
| $\log_{10}(3+2)+\log_{10}(3-1)=1$ | $\log_{10}(-4+2)+\log_{10}(-4-1)=1$ |
| $\log_{10}5+\log_{10}2=1$ | $\cancel{\log_{10}(-2)}+\cancel{\log_{10}(-5)}=1$ |
| $\log_{10}10=1$ | undefined |
| $1=1$ | |

Chapter 7 or 8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

$y = (\text{Initial Amount})(\text{Base})^{\frac{\text{Time}}{\text{it takes}}}$

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

Given: I.A. = 60 mg   time = 5.3 years   $y = 60\left(\frac{1}{2}\right)^{x/5.3}$

Base = ½ or (0.5)   $m = 60\left(\frac{1}{2}\right)^{t/5.3}$

b) What amount will be present in 10.6 years? [2]  $x = 10.6$

$y = 60\left(\frac{1}{2}\right)^{\frac{10.6}{5.3}}$

$y = 60\left(\frac{1}{2}\right)^{2}$

$y = 60\left(\frac{1}{4}\right)$

$\boxed{y = 15\text{mg}}$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

$60 \times 0.125 = 7.5$

$y = 7.5$

$\frac{7.5}{60} = \frac{60(0.5)^{t/5.3}}{60}$

$0.125 = (0.5)^{t/5.3}$

$\frac{\log(0.125)}{\log(0.5)}$

$\cancel{(0.5)}^{3} = \cancel{(0.5)}^{t/5.3}$

$5.3 \cdot 3 = \frac{t}{5.3} \cdot 5.3$

$15.9 \text{ years} = t$

Using logs: $\frac{7.5}{60} = \frac{60(0.5)^{t/5.3}}{60}$

$0.125 = (0.5)^{t/5.3}$

$\log(0.125) = {t/5.3}\log(0.5)$

$5.3 \cdot \log(0.125) = \frac{t}{5.3}\log(0.5) \cdot 5.3$

$\frac{5.3\log(0.125)}{\log(0.5)} = \frac{t\,\cancel{\log(0.5)}}{\cancel{\log(0.5)}}$

$15.9 \text{ years} = t$

2. Solve for all values of $\theta$ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$

$0 \le \theta \le 2\pi$

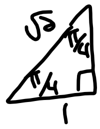
12 a) $\sin\theta = \sin\theta\tan\theta$

Common Factor

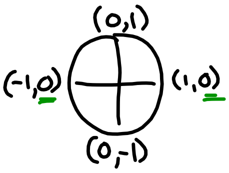
$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

| | |
|---|---|
| $\sin\theta = 0$ | $\tan\theta - 1 = 0$ |
| $\theta = 0, \pi, 2\pi$ | $\tan\theta = 1$ |
| | $\theta_R = \frac{\pi}{4}$ |




S | A
T | C




Where is $\tan\theta$ positive

| Q1 | Q3 |
|---|---|
| $\theta = \theta_R$ | $\theta = \pi + \theta_R$ |
| $\theta = \frac{\pi}{4}$ | $\theta = \pi + \frac{\pi}{4}$ |
| | $\theta = \frac{5\pi}{4}$ |

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

13 b) $3\sin^2\theta - 2\sin\theta - 1 = 0$ , $0 \le \theta \le 360°$

$\frac{-3}{} \times \frac{1}{} = -3$

$\frac{-3}{} + \frac{1}{} = -2$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

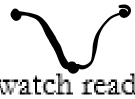
| | |
|---|---|
| $3\sin\theta + 1 = 0$ | $\sin\theta - 1 = 0$ |
| $\sin\theta = -\frac{1}{3}$ | $\sin\theta = 1$ |
| $\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$ | $\theta = 90°$ |
| $\theta_R = 19$ | |

Unit Circle

Where is sine negative: S | A / T | C

| Q3 | Q4 |
|---|---|
| $\theta = 180° + \theta_R$ | $\theta = 360° - \theta_R$ |
| $\theta = 180° + 19$ | $\theta = 360° - 19$ |
| $\theta = 199$ | $\theta = 340$ |

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 ~~cm above~~ the floor, occurs at 1.8 sec.



$y = \cos x$

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

max = 50cm

min = 30cm

$K = \frac{50+30}{2} = 40$

Amp = 10 $\quad P = 2(1.8-0.4) = 2.8 \quad h = 0.4$

$a = \pm 10 \quad b = \frac{360}{2.8} = 128.57$

$y = 10\cos[128.57(17.2-0.4)]+40$

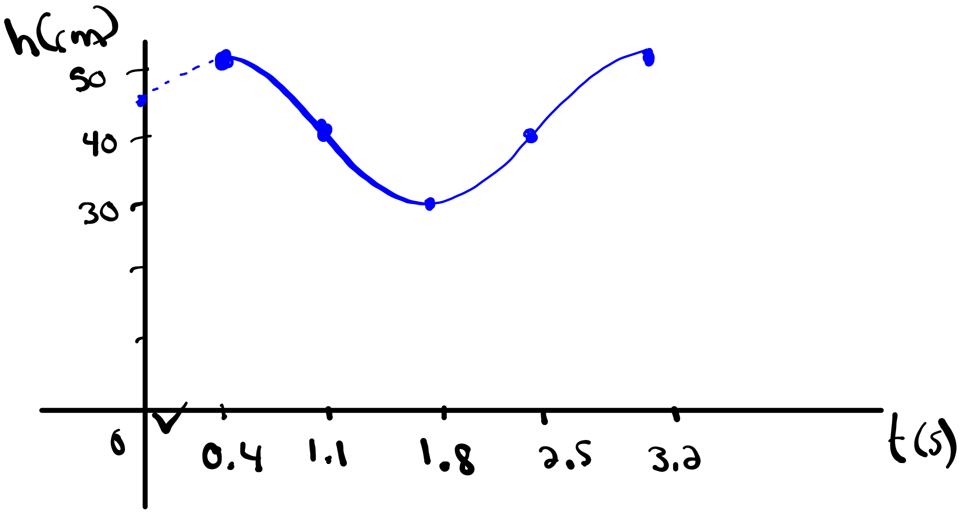
$y = 49.9$ cm

(b) How high was the weight above the floor when the stopwatch was initially started?

$t=0$

$y = 10\cos[128.57(0-0.4)]+40$

$y = 46.23$cm




Count by $\frac{P}{4} = \frac{2.8}{4} = 0.7$

① Amp = 11 P= 16 min = -4

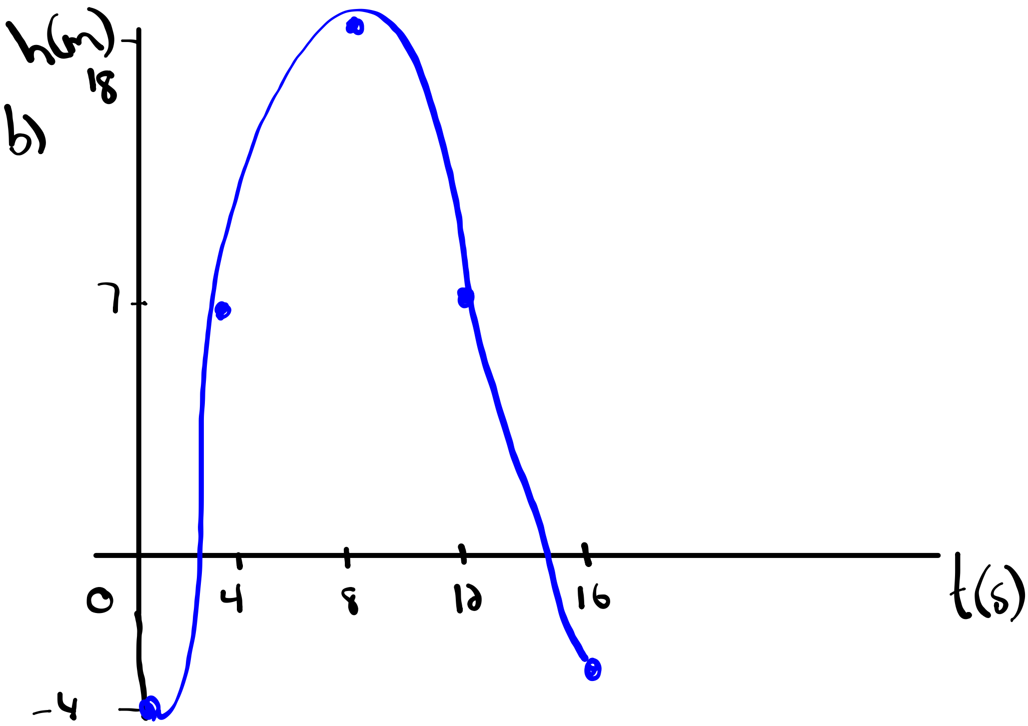
$a = \pm 11$ $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)

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(4)

max = 68

min = 24

$K = \frac{68+24}{2} = 46$

Amp = 68 − 46 = 22

$a = \pm 22$

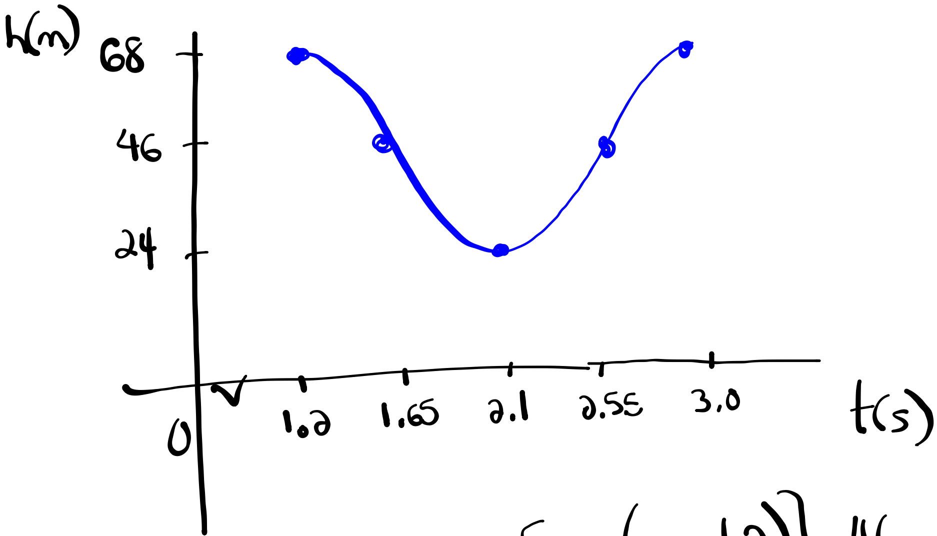
$P = 2(2.1 - 1.2)$

$P = 1.8$

$b = \frac{360}{1.8} = 200$

$h = 1.2$

$\frac{P}{4} = \frac{1.8}{4} = 0.45$




$y = 22\cos[200(x - 1.2)] + 46$

5 c) $y = \frac{1}{2}\cos(1\theta + \underline{\underline{\pi}}) - \underline{\underline{4}}$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

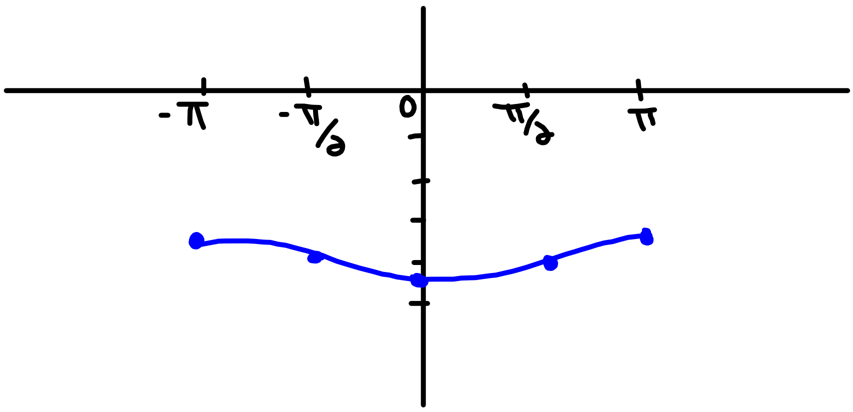
$(x, y) \longrightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

| x | y |
|---|---|
| 0 | 1 |
| $\pi/2$ | 0 |
| $\pi$ | -1 |
| $3\pi/2$ | 0 |
| $2\pi$ | 1 |

$\longrightarrow$

| x | y | |
|---|---|---|
| $-\pi$ | $-7/2$ | -3.5 |
| $-\frac{\pi}{2}$ | -4 | -4 |
| 0 | $-9/2$ | -4.5 |
| $\pi/2$ | -4 | -4 |
| $\pi$ | $-7/2$ | -3.5 |



$$\frac{1}{\sec^2\theta\cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$