Chapter 2 Radical Functions

Solve for x : $\sqrt{x+8} = x+6$ Square both sides

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2 + 6x + 6x + 36$$

$$x+8 = x^2 + 12x + 36$$ Bring all terms to same side

$$0 = x^2 + 12x + 36 - x - 8$$

$$0 = x^2 + 11x + 28$$ $4 \times 7 = 28$
$4 + 7 = 11$

$$0 = (x+4)(x+7)$$

$$x+4 = 0 \quad | \quad x+7 = 0$$ Set both factors = 0
$$x = -4 \quad | \quad x = -7$$

Test $x = -4$ is a solution   Test $x = -7$ is extraneous

$\sqrt{x+8} = x+6$ $\qquad\qquad$ $\sqrt{x+8} = x+6$

$\sqrt{-4+8}$ | $-4 + 6$ $\qquad$ $\sqrt{-7+8}$ || $-7 + 6$

$\sqrt{4}$ $\qquad$ 2 $\qquad\qquad$ $\sqrt{1}$ $\qquad$ -1
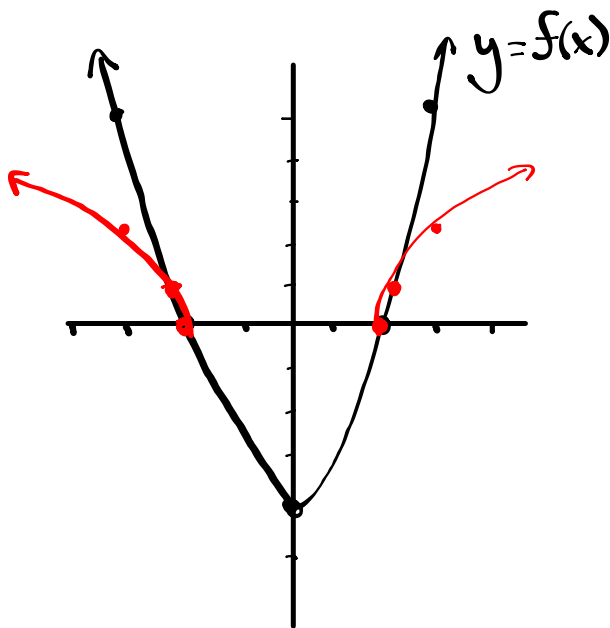
2 $\qquad\qquad\qquad$ 1

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. state the domain and range of each.

$y = f(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in R\}$ or $[-4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in R\}$

or $(-\infty, -2] + [2, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} \boxed{+7} = 71$

$2^{2x+2} = 64$

$2^{2x+2} = 2^6$

$2x+2 = 6$

$2x = 4$
$\frac{2x}{2} = \frac{4}{2}$

$\boxed{X = 2}$ is a solution

Test

$2^{2x+2} + 7 = 71$

$2^{2(2)+2} + 7$

$2^6 + 7$

$64 + 7$

71

b) $9^{2x+1} = 81(27^x)$

$(3^2)^{2x+1} = (3^4)(3^3)^x$

$3^{4x+2} = (3^4) \cdot (3^{3x})$

$3^{4x+2} = 3^{3x+4}$

$4x+2 = 3x+4$

$\boxed{X = 2}$ is a solution

Test.

$9^{2x+1} = 81(27^x)$

$9^{2(2)+1}$ $81(27^2)$

9^5 $81(729)$

59049 59049

$$\text{Chapter } 8 \rightarrow \text{Logarithms}$$

4. **Rewrite each expression as a single logarithm.**

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5(x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left(\frac{x^3 \cdot (x-1)^{1/2}}{(x^2+1)}\right)$$

$$\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}$$

ex. $3\log_2 x - 4\log_2 x^2 + \log_2 x - 2\log_2 x^5$

$$\log_2 x^3 - \log_2 x^8 + \log_2 x - \log_2 x^{10}$$

$$\log_2\left(\frac{x^3 \cdot x}{x^8 \cdot x^{10}}\right)$$

$$\log_2\left(\frac{x^4}{x^{18}}\right)$$

$$\log_2 x^{-14}$$

$$-14\log_2 x$$

Chapter 8 → Logarithms

7. **Solve the following equation (be sure to test your answers).**

$$\log_{10}(x+2) + \log_{10}(x-1) = 1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}(x^2 - x + 2x - 2) = 1$$

$$\log_{10}(x^2 + x - 2) = 1 \quad (\text{log form})$$

↑ Base   ↑ ans   ↑ exp

$$10^1 = x^2 + x - 2 \quad (\text{exp. form})$$

$$10 = x^2 + x - 2 \quad \text{Bring everything to one side}$$

$$0 = x^2 + x - 12 \qquad \frac{-3 \times 4 = -12}{-3 + 4 = 1}$$

$$0 = (x-3)(x+4)$$

$$\begin{array}{c|c} x - 3 = 0 & x + 4 = 0 \\ x = 3 & x = -4 \end{array}$$

Test $x = 3$ is a solution

$$\log_{10}(x+2) + \log_{10}(x-1) = 1$$

$$\log_{10}(3+2) + \log_{10}(3-1) = 1$$

$$\log_{10} 5 + \log_{10} 2 = 1$$

$$\log_{10} 10 = 1$$

$$1 = 1$$

Test $x = -4$ is extraneous

$$\log_{10}(x+2) + \log_{10}(x-1) = 1$$

$$\log_{10}(-4+2) + \log_{10}(-4-1) = 1$$

$$\log_{10}(-2) + \log_{10}(-5) = 1$$

↑ undefined

Chapter 7 or 8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

$$y = (\text{Initial Amount})(\text{Base})^{\text{Time it takes}}$$

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

Given: I.A. = 60 mg   time = 5.3 years $y = 60\left(\frac{1}{2}\right)^{x/5.3}$

Base = ½ or (0.5) $m = 60\left(\frac{1}{2}\right)^{t/5.3}$

b) What amount will be present in 10.6 years? $x = 10.6$ [2]

$$y = 60\left(\frac{1}{2}\right)^{\frac{10.6}{5.3}} \rightarrow \quad y = 60\left(\frac{1}{4}\right)$$

$$y = 60\left(\frac{1}{2}\right)^{2} \qquad \boxed{y = 15\,mg}$$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

$60 \times 0.125 = 7.5$ $\dfrac{7.5}{60} = \dfrac{60(0.5)^{t/5.3}}{60}$

$y = 7.5$

$$0.125 = (0.5)^{t/5.3}$$

$$\dfrac{\log(0.125)}{\log(0.5)} \qquad (0.5)^{3} = (0.5)^{t/5.3}$$

$$5.3 \cdot 3 = \dfrac{t}{5.3} \cdot 5.3$$

$$15.9 \text{ years} = t$$

Using logs: $\dfrac{7.5}{60} = \dfrac{60(0.5)^{t/5.3}}{60}$

$$0.125 = (0.5)^{t/5.3}$$

$$\log(0.125) = \log(0.5)^{t/5.3}$$

$$5.3 \cdot \log(0.125) = \dfrac{t}{5.3}\log(0.5) \cdot 5.3$$

$$\dfrac{5.3\log(0.125)}{\log(0.5)} = \dfrac{t\,\log(0.5)}{\log(0.5)}$$

$$15.9 \text{ years} = t$$

Chapter 4 → Trigonometry/Radian Measure

2. Solve for all values of θ in the specified domain.

$\tan^2 \theta + \tan \theta = 0$, $0 \le \theta \le 2\pi$ (Radians)

$$\tan\theta \left(\tan\theta + 1 \right) = 0$$

$$\boxed{\bar{\theta} = 0, \pi, 2\pi, \frac{3\pi}{4}, \frac{7\pi}{4}}$$

$\tan\theta = 0$ (Unit Circle) $\theta = 0, \pi, 2\pi$	$\tan\theta + 1 = 0$ $\tan\theta = -1$ (Special triangle) $\bar{\theta} = \frac{\pi}{4}$	where is $\tan\theta < 0$

where is $\tan\theta < 0$

Q2	Q4
$\theta = \pi - \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

e. $\cos^2 \theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$ (Degrees)

$$\cos\theta \left(\cos\theta + \frac{1}{2} \right) = 0$$

$$\boxed{\theta = 90°, 120°, 240°, 270°}$$

$\cos\theta = 0$ (Unit Circle) $\theta = 90°, 270°$	$\cos\theta + \frac{1}{2} = 0$ $\cos\theta = -\frac{1}{2}$ (Special triangle) $\bar{\theta} = 60°$	where is $\cos\theta < 0$

where is $\cos\theta < 0$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

Chapter 4 → Trig/Radian Measure

$$\frac{5\tan^2\left(5\pi/4\right)}{6\sin\left(5\pi/6\right) + 4\sin\left(4\pi/3\right)}$$

$$\boxed{\frac{4\pi}{4}}, \frac{5\pi}{4}, \frac{6\pi}{4} \qquad \frac{4\pi}{6}, \frac{5\pi}{6}, \boxed{\frac{6\pi}{6}} \qquad \boxed{\frac{3\pi}{3}}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$1\pi \qquad\qquad 1\pi \qquad\qquad 1\pi$

$$\boxed{\tan\frac{5\pi}{4} = \frac{-1}{-1} = 1} \qquad \boxed{\sin\frac{5\pi}{6} = \frac{1}{2}} \qquad \boxed{\sin\frac{4\pi}{3} = \frac{-\sqrt{3}}{2}}$$
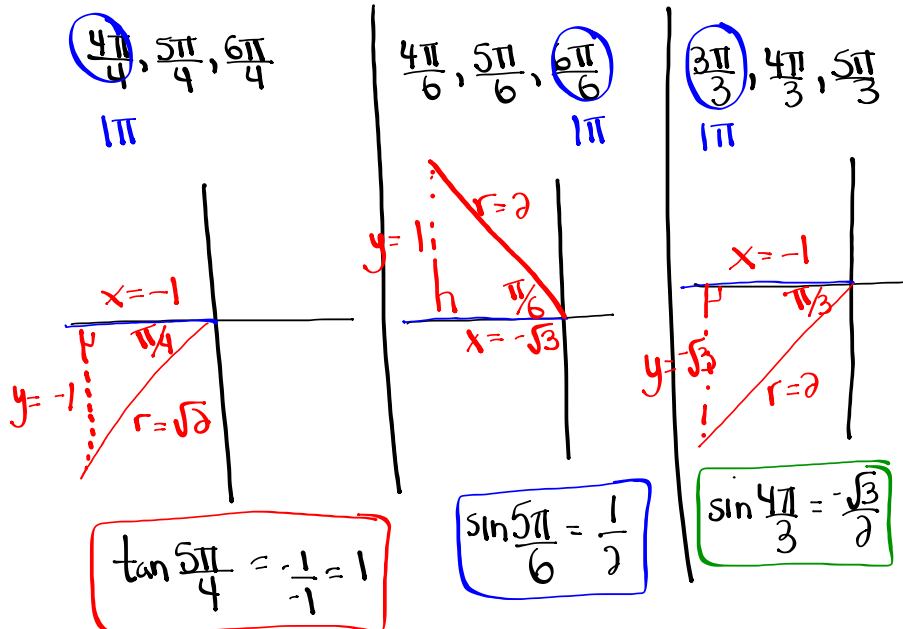
$$\frac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right) + {}^2 4\left(\frac{-\sqrt{3}}{2}\right)}$$

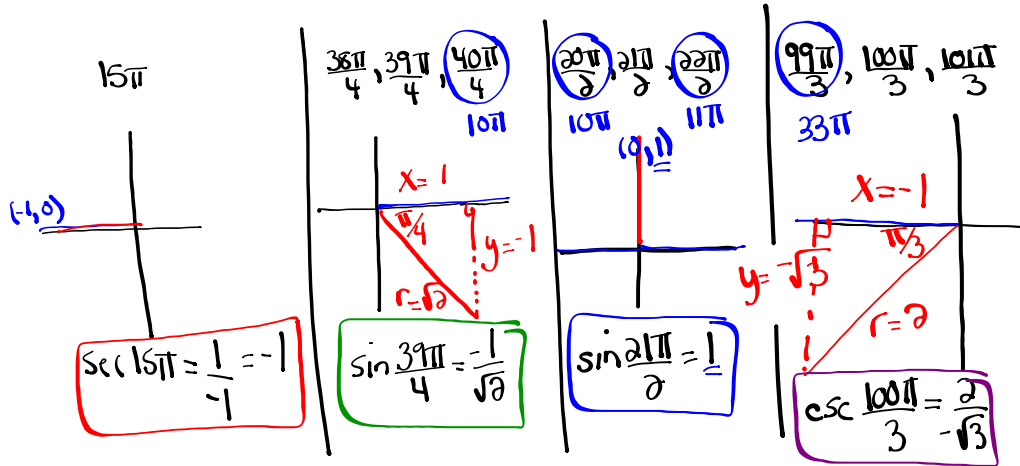$$\frac{5}{(3 - 2\sqrt{3})} \cdot \frac{(3 + 2\sqrt{3})}{(3 + 2\sqrt{3})}$$

$$\frac{15 + 10\sqrt{3}}{9 - 4(3)}$$

$$\boxed{\frac{15 + 10\sqrt{3}}{-3}} \quad \text{or} \quad -\frac{15 + 10\sqrt{3}}{3}$$

Chapter 4 → Trig/Radian Measure

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

15π	$\frac{38\pi}{4},\frac{39\pi}{4},\boxed{\frac{40\pi}{4}}$	$\boxed{\frac{20\pi}{2}},\frac{21\pi}{2},\boxed{\frac{22\pi}{2}}$	$\boxed{\frac{99\pi}{3}},\frac{100\pi}{3},\frac{101\pi}{3}$
$(-1,0)$	10π	$10\pi \quad (0,1) \quad 11\pi$	33π

$x=1$, $\frac{\pi}{4}$, $y=-1$, $r=\sqrt{2}$

$y=-\sqrt{3}$, $x=-1$, $\frac{\pi}{3}$, $r=2$

$$\boxed{\sec 15\pi = \frac{1}{-1} = -1}$$

$$\boxed{\sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}}$$

$$\boxed{\sin\frac{21\pi}{2} = 1}$$

$$\boxed{\csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}}$$

$$\underline{\sec 15\pi} + \sqrt{2}\,\underline{\sin\frac{39\pi}{4}}\,\underline{\sin\frac{21\pi}{2}} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{-2}{\sqrt{3}}\right)^2$$

$$-1 \quad -\frac{\sqrt{2}}{\sqrt{2}} \quad -\frac{4}{3}$$
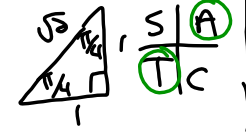
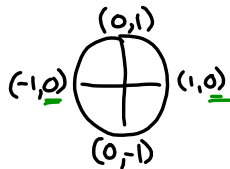$$-1 -1 -\frac{4}{3}$$

$$-2 - \frac{4}{3}$$

$$\frac{-6}{3} - \frac{4}{3}$$

$$\boxed{\frac{-10}{3}}$$

⑫ a) $\sin\theta = \sin\theta\tan\theta$ $0 \le \theta \le 2\pi$
                                                  Common Factor

$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$ | $\tan\theta - 1 = 0$

$\theta = 0, \pi, 2\pi$ | $\tan\theta = 1$ $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

$\theta_R = \dfrac{\pi}{4}$

(0,1)
(-1,0) (1,0)
(0,-1)

Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \dfrac{\pi}{4}$	$\theta = \pi + \dfrac{\pi}{4}$
	$\theta = \dfrac{5\pi}{4}$

Solutions are: $0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}, 2\pi$

⑬ b) $3\sin^2\theta - 2\sin\theta - 1 = 0$ $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$ $\dfrac{-3 \times 1}{-3} = -3$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$ $\dfrac{-3 + 1}{} = -2$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$ | $\sin\theta - 1 = 0$

$\sin\theta = -\dfrac{1}{3}$ | $\sin\theta = 1$ → Unit Circle

$\theta = 90°$

$\theta_R = \sin^{-1}\left(\dfrac{1}{3}\right)$

$\theta_R = 19$

Where is sine negative: $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

Chapter 5 → Trig Functions

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 ~~cm above~~ above the floor, occurs at 1.8 sec.
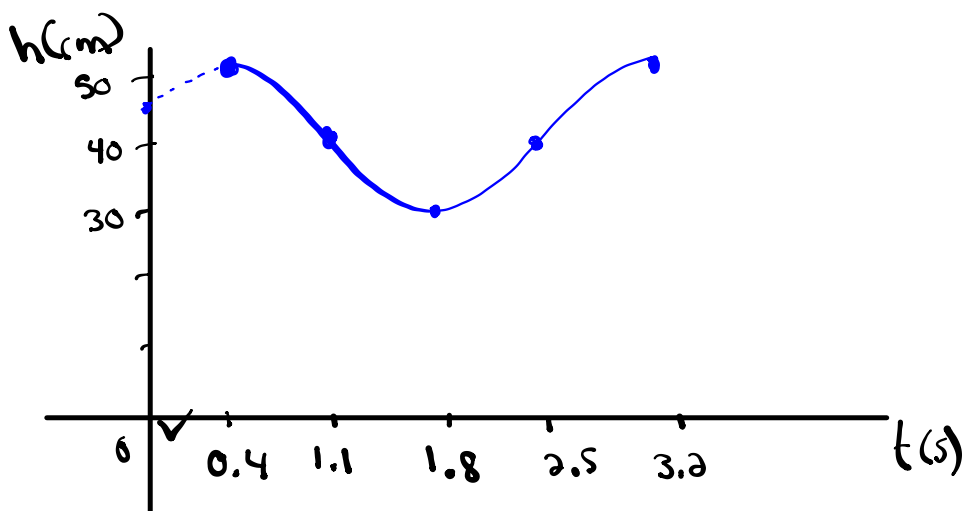
$y = \cos x$

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

$$max = 50 cm$$
$$min = 30 cm$$
$$K = \frac{50 + 30}{2} = 40$$

$$Amp = 10 \quad P = 2(1.8 - 0.4) = 2.8 \quad h = 0.4$$
$$a = \pm 10 \quad b = \frac{360}{2.8} = 128.57$$

$$y = 10\cos\left[128.57(17.2 - 0.4)\right] + 40$$
$$y = 49.9 \, cm$$

(b) How high was the weight above the floor when the stopwatch was initially started?

$t = 0$

$$y = 10\cos\left[128.57(0 - 0.4)\right] + 40$$
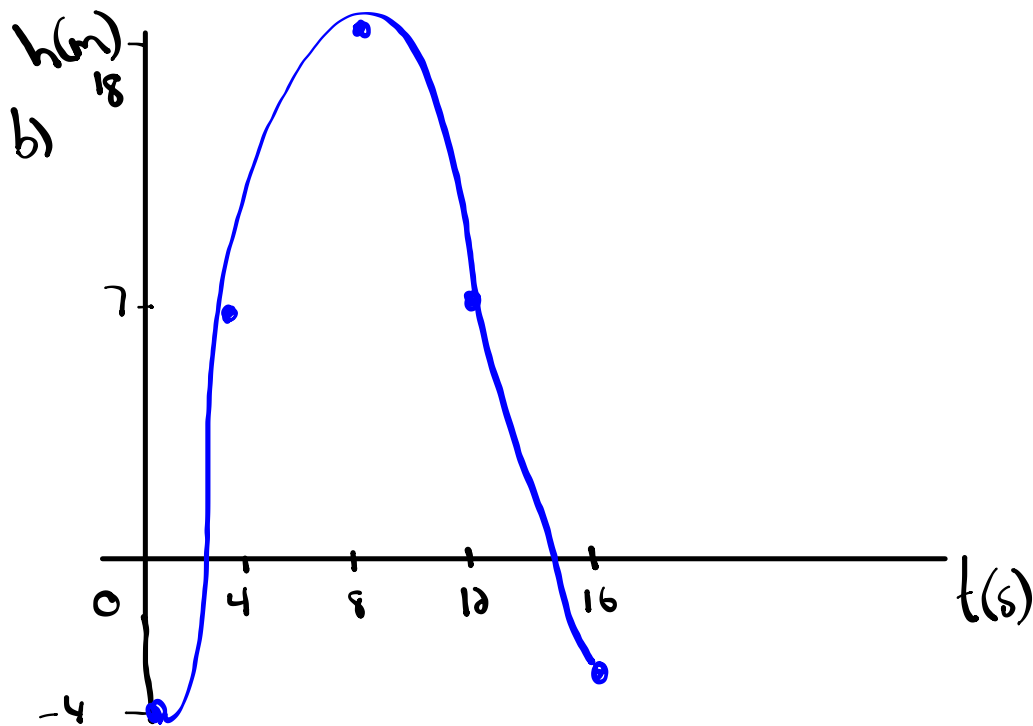$$y = 46.23 \, cm$$

Count by $\frac{P}{4} = \frac{2.8}{4} = 0.7$

1

① Amp = 11   P = 16   min = -4

a = ±11 $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

④   max $= 68$ Amp $= 68 - 46 = 22$ $P = 2(2.1 - 1.2)$

      min $= 24$ $a = \pm 22$ $P = 1.8$

      $k = \dfrac{68 + 24}{2} = 46$

      $b = \dfrac{360}{1.8} = 200$

      $h = \underline{\underline{1.2}}$

      $\dfrac{P}{4} = \dfrac{1.8}{4} = 0.45$

$$y = 22 \cos\left[200(x - 1.2)\right] + 46$$

(graph: h(n) vs t(s) with y-axis marked 68, 46, 24, and x-axis marked 1.2, 1.65, 2.1, 2.55, 3.0)

⑤ c) $y = \frac{1}{2}\cos(\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

$(x, y) \rightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\longrightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\pi/2$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5

$$\frac{1}{\sec^2\theta \cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$

$$\boxed{\frac{1}{\sec^2\theta}} \cdot \boxed{\frac{1}{\cot\theta}} \qquad \frac{\sin\theta\,\boxed{(1-\sin^2\theta)}}{\cos\theta}$$

$$\cos^2\theta \cdot \boxed{\tan\theta} \qquad \frac{\sin\theta \cos^2\theta}{\cos\theta}$$

$$\cos^2\theta \cdot \frac{\sin\theta}{\cos\theta} \qquad \sin\theta\cos\theta$$

$$\sin\theta\cos\theta$$

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