

$$\textcircled{25} \quad \frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$$

$$(1+\cos\theta)(1-\cos\theta) = \sin^2\theta$$

$$1 - \cos\theta + \cos\theta - \cos^2\theta$$

$$\boxed{1 - \cos^2\theta}$$

$$\sin^2\theta$$

$$\textcircled{26} \quad \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2 \sec^2\theta$$

$$\frac{1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)} + \frac{1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = 2 \left(\frac{1}{\cos^2\theta} \right)$$

$$\frac{\cancel{1-\sin\theta} + \cancel{1+\sin\theta}}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{2}{\cos^2\theta}$$

$$\boxed{\frac{2}{1-\sin^2\theta}}$$

$$\frac{2}{\cos^2\theta}$$

$$\textcircled{26} \quad \cos^4 D = 1 - 2\sin^2 D + \sin^4 D$$

$$(\cos^2 D)(\cos^2 D)$$

$$(1 - \sin^2 D)(1 - \sin^2 D)$$

$$1 - \sin^2 D - \sin^2 D + \sin^4 D$$

$$1 - 2\sin^2 D + \sin^4 D$$

$$\textcircled{27} \quad \cot^4 B + \cot^2 B = \frac{\cos^2 B}{\sin^4 B}$$

$$\cot^2 B (\cot^2 B + 1)$$

$$\cot^2 B (\csc^2 B)$$

$$\left(\frac{\cos^2 B}{\sin^2 B}\right) \left(\frac{1}{\sin^2 B}\right)$$

$$\frac{\cos^2 B}{\sin^4 B}$$

Prove the identity:

$$\frac{\csc \theta}{1 + \csc \theta} + \frac{\csc \theta}{1 - \csc \theta} = -\frac{2 \sin \theta}{\cos^2 \theta}$$

$$\frac{\csc \theta (1 - \csc \theta)}{(1 + \csc \theta)(1 - \csc \theta)} + \frac{\csc \theta (1 + \csc \theta)}{(1 + \csc \theta)(1 - \csc \theta)}$$

$$\frac{\csc \theta - \cancel{\csc^2 \theta} + \csc \theta + \cancel{\csc^2 \theta}}{(1 + \csc \theta)(1 - \csc \theta)}$$

$$\frac{2 \csc \theta}{1 - \csc^2 \theta}$$

$$\frac{2 \csc \theta}{-\cot^2 \theta}$$

$$2 \left(\frac{1}{\sin \theta} \right) \div \left(-\frac{\cos^2 \theta}{\sin^2 \theta} \right)$$

$$\left(\frac{2}{\sin \theta} \right) \left(-\frac{\sin^2 \theta}{\cos^2 \theta} \right)$$

$$\frac{-\cancel{2} \sin^{\cancel{2}} \theta}{\cancel{\sin} \theta \cos^2 \theta}$$

$$\frac{-2 \sin \theta}{\cos^2 \theta}$$

$$-\frac{2 \sin \theta}{\cos^2 \theta}$$

Prove the identity:

$$\frac{1}{\sec^2 \theta \cot \theta} = \frac{\sin \theta - \sin^3 \theta}{\cos \theta}$$

$$\frac{1}{\sec^2 \theta} \cdot \frac{1}{\cot \theta}$$

$$\cos^2 \theta \cdot \frac{\sin \theta}{\cancel{\cos \theta}}$$

$$\sin \theta \cos \theta$$

$$\frac{\sin \theta (1 - \sin^2 \theta)}{\cos \theta}$$

$$\frac{\sin \theta \cancel{\cos^2 \theta}}{\cancel{\cos \theta}}$$

$$\sin \theta \cos \theta$$

$$\textcircled{5} \sin(x+y) \sin(x-y)$$

$$(\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$$

$$\sin^2 x \cos^2 y - \cos^2 x \sin^2 y$$

$$(1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y)$$

$$\cos^2 y - \cancel{\cos^2 x \cos^2 y} - \cos^2 x + \cancel{\cos^2 x \cos^2 y}$$

$$\cos^2 y - \cos^2 x$$

$$= \cos^2 y - \cos^2 x$$

Trig Identities # 4

$$\textcircled{4} \quad \underline{\tan^2 \theta} - \cot^2 \theta = \sec^2 \theta - \csc^2 \theta$$

$$\begin{array}{l} \sec^2 \theta - 1 - (\csc^2 \theta - 1) \\ \sec^2 \theta - 1 - \csc^2 \theta + 1 \\ \sec^2 \theta - \csc^2 \theta \end{array}$$

$$\textcircled{5} \quad \underline{\sec^2 \theta} - \underline{\sin^2 \theta} = \cos^2 \theta + \tan^2 \theta$$

$$\begin{array}{l} \tan^2 \theta + 1 - (1 - \cos^2 \theta) \\ \tan^2 \theta + 1 - 1 + \cos^2 \theta \\ \cos^2 \theta + \tan^2 \theta \end{array}$$

$$\textcircled{6} \quad \frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta \cos^2 \theta - \cos^4 \theta} = \frac{\csc^2 \theta}{\cot^2 \theta}$$

$$\begin{array}{l} \frac{(\cancel{\sin^2 \theta - \cos^2 \theta})(\sin^2 \theta + \cos^2 \theta)}{\cos^2 \theta (\cancel{\sin^2 \theta - \cos^2 \theta})} \\ \frac{1}{\cos^2 \theta} \end{array}$$

$$\begin{array}{l} \frac{1}{\sin^2 \theta} \div \frac{\cos^2 \theta}{\sin^2 \theta} \\ \frac{1}{\sin^2 \theta} \times \frac{\cancel{\sin^2 \theta}}{\cos^2 \theta} \\ \frac{1}{\cos^2 \theta} \end{array}$$

Review

$$\textcircled{32} \quad \frac{\cos y}{(1+\sin y)} + \frac{1+\sin y}{\cos y} = 2 \boxed{\sec y}$$

$$\frac{\cos^2 y + \cancel{(1+\sin y)^2}}{\cos y(1+\sin y)}$$

$$\frac{\underline{\cos^2 y} + \underline{1} + \underline{2\sin y} + \underline{\sin^2 y}}{\cos y(1+\sin y)}$$

Factor \rightarrow $\frac{2 + 2\sin y}{\cos y(1+\sin y)}$

$$\frac{\cancel{2(1+\sin y)}}{\cos y \cancel{(1+\sin y)}}$$

$$\boxed{\frac{2}{\cos y}}$$

$$2 \left(\frac{1}{\cos y} \right)$$

$$\boxed{\frac{2}{\cos y}}$$

$$\textcircled{33} \quad \boxed{\sin(x+y)} + \boxed{\sin(x-y)} = 2\sin x \cos y$$

$$\sin x \cos y + \cancel{\cos x \sin y} + \sin x \cos y - \cancel{\cos x \sin y}$$

$$\boxed{2\sin x \cos y}$$

$$\boxed{2\sin x \cos y}$$

$$\textcircled{28} \quad \boxed{\sin 2\theta} (1 - \boxed{\cos 2\theta}) = 4\sin^3 \theta \cos \theta$$

$$2\sin \theta \cos \theta (1 - (\cos^2 \theta - \sin^2 \theta))$$

$$2\sin \theta \cos \theta (1 - \underline{\cos^2 \theta} + \sin^2 \theta)$$

$$2\sin \theta \cos \theta (\sin^2 \theta + \sin^2 \theta)$$

$$(2\sin \theta \cos \theta)(2\sin^2 \theta)$$

$$\boxed{4\sin^3 \theta \cos \theta}$$

$$\boxed{4\sin^3 \theta \cos \theta}$$

$$\textcircled{15} \quad \cos^2 \theta - \boxed{\sin^2 \theta} = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$\boxed{2\cos^2 \theta - 1}$$

$$\boxed{2\cos^2 \theta - 1}$$

$$\textcircled{16} \quad \overset{\text{Diff. of Squares}}{\cos^4 \theta - \sin^4 \theta} = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\boxed{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}$$

$$(1)(\cos^2 \theta - \sin^2 \theta)$$

$$\boxed{(\cos^2 \theta - \sin^2 \theta)}$$

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$$\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)}$$

$$= 2 \sec^2 \theta$$

$$\frac{\cancel{1-\sin\theta} + \cancel{1+\sin\theta}}{(1+\sin\theta)(1-\sin\theta)}$$

$$2 \left(\frac{1}{\cos^2 \theta} \right)$$

$$\frac{2}{(1-\sin^2\theta)}$$

$$\frac{2}{\cos^2 \theta}$$

$$\frac{2}{\cos^2 \theta}$$

$$\textcircled{2} \quad \boxed{\tan\theta} + \frac{1}{\boxed{\tan\theta}} = \boxed{\csc\theta} \boxed{\sec\theta}$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\left(\frac{1}{\sin\theta}\right) \left(\frac{1}{\cos\theta}\right)$$

$$\frac{\boxed{\sin^2\theta + \cos^2\theta}}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta}$$

$$\textcircled{29} \quad \underline{\cos^4 \theta} = 1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\underline{(\cos^2 \theta)(\cos^2 \theta)}$$

$$(1 - \sin^2 \theta)(1 - \sin^2 \theta)$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$(17) \frac{1 - \sin^2 \theta}{\csc^2 \theta - 1} = \sin^2 \theta$$

$$\frac{\cos^2 \theta}{\cot^2 \theta}$$

$$\frac{\cos^2 \theta}{\frac{\cos^2 \theta}{\sin^2 \theta}}$$

$$\cancel{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cancel{\cos^2 \theta}}$$

$$\sin^2 \theta$$

Review Period 2

$$\textcircled{15} \quad \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$2\cos^2 \theta - 1$$

$$\textcircled{6} \quad \cos \theta \sin \theta \cot \theta = \cos^2 \theta$$

$$(\cos \theta) \cancel{(\sin \theta)} \left(\frac{\cos \theta}{\cancel{\sin \theta}} \right)$$

$$\cos^2 \theta$$

$$\textcircled{16} \quad \cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

D. FF of Squares

$$(\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta)$$

$$(\cos^2 \theta - \sin^2 \theta) (1)$$

$$\cos^2 \theta - \sin^2 \theta$$

$$\begin{aligned}
 \textcircled{20} \quad & \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2 \sec^2\theta \\
 & \frac{\cancel{1-\sin\theta} + \cancel{1+\sin\theta}}{(1+\sin\theta)(1-\sin\theta)} = 2 \left(\frac{1}{\cos^2\theta} \right) \\
 & \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} \\
 & \frac{2}{\cos^2\theta}
 \end{aligned}$$

$$\textcircled{29} \quad \cos^4 \theta =$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\underline{\cos^2 \theta} \underline{\cos^2 \theta}$$

$$(1 - \sin^2 \theta)(1 - \sin^2 \theta)$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\textcircled{24} \quad \frac{\sec\theta - 1}{\sec\theta + 1} \stackrel{=}{\neq} \frac{1 - \cos\theta}{1 + \cos\theta}$$

$(\sec\theta - 1)(1 + \cos\theta)$ $\sec\theta + \cos\theta \boxed{\sec\theta} - 1 - \cos\theta$ $\sec\theta + \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) - 1 - \cos\theta$ $\sec\theta + 1 - 1 - \cos\theta$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\sec\theta - \cos\theta$ </div>	$(\sec\theta + 1)(1 - \cos\theta)$ $\sec\theta - \cos\theta \boxed{\sec\theta} + 1 - \cos\theta$ $\sec\theta - \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) + 1 - \cos\theta$ $\sec\theta - 1 + 1 - \cos\theta$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\sec\theta - \cos\theta$ </div>
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$$\textcircled{16} \cos^4 \theta - \sin^4 \theta = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{(1)(\cos^2 \theta - \sin^2 \theta)} = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\textcircled{17} \frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = 2 \sec^2 \theta$$

$$\frac{1-\sin \theta + 1+\sin \theta}{(1+\sin \theta)(1-\sin \theta)} = 2 \left(\frac{1}{\cos^2 \theta} \right)$$

$$\frac{2}{1-\sin^2 \theta} = \frac{2}{\cos^2 \theta}$$

$$\boxed{\frac{2}{\cos^2 \theta}}$$

$$\textcircled{18} \frac{\cos y}{1+\sin y} + \frac{1+\sin y}{\cos y} = 2 \sec y$$

$$\frac{\cos y + (1+\sin y)(1+\sin y)}{\cos y(1+\sin y)} = 2 \left(\frac{1}{\cos y} \right)$$

$$\frac{\cos y + 1 + 2\sin y + \sin^2 y}{\cos y(1+\sin y)} = \frac{2}{\cos y}$$

Common Factor of 2

$$\frac{2 + 2\sin y}{\cos y(1+\sin y)} = \frac{2}{\cos y}$$

$$\frac{2(1+\sin y)}{\cos y(1+\sin y)} = \frac{2}{\cos y}$$

$$\boxed{\frac{2}{\cos y}}$$

$$\textcircled{19} \cos^4 \theta = 1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\cos^4 \theta - \sin^4 \theta = \boxed{1 - 2\sin^2 \theta}$$

$$\frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{(1)(\cos^2 \theta - \sin^2 \theta)} = \boxed{1 - 2\sin^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{1 - \sin^2 \theta - \sin^2 \theta} = \boxed{1 - 2\sin^2 \theta}$$

$$\textcircled{20} \tan \theta + \frac{1}{\tan \theta} = \csc \theta \sec \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{1}{\frac{\sin \theta}{\cos \theta}} = \left(\frac{1}{\sin \theta} \right) \left(\frac{1}{\cos \theta} \right)$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\boxed{\frac{1}{\sin \theta \cos \theta}}$$

$$\textcircled{a)} \quad \frac{\sec\theta - 1}{\sec\theta + 1} \quad \begin{array}{c} \swarrow \searrow \\ \nwarrow \nearrow \end{array} \quad \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$(\sec\theta - 1)(1 + \cos\theta)$$

$$\sec\theta + \boxed{\sec\theta} \cos\theta - 1 - \cos\theta$$

$$\sec\theta + \left(\frac{1}{\cancel{\cos\theta}}\right) (\cancel{\cos\theta}) - 1 - \cos\theta$$

$$\sec\theta + 1 - 1 - \cos\theta$$

$$\boxed{\sec\theta - \cos\theta}$$

$$= (1 - \cos\theta)(\sec\theta + 1)$$

$$\sec\theta + 1 - \cos\theta \boxed{\sec\theta} - \cos\theta$$

$$\sec\theta + 1 - \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) - \cos\theta$$

$$\sec\theta + 1 - 1 - \cos\theta$$

$$\boxed{\sec\theta - \cos\theta}$$

$$\textcircled{16} \cos^4 \theta - \sin^4 \theta = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\boxed{(\cos^2 \theta + \sin^2 \theta)} (\cos^2 \theta - \sin^2 \theta)$$

$$\textcircled{1} (\cos^2 \theta - \sin^2 \theta)$$

$$\boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\textcircled{15} \cos^2 \theta - \boxed{\sin^2 \theta} = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$2\cos^2 \theta - 1$$

$$\textcircled{15} \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$\boxed{1 - \sin^2 \theta} = 2\cos^2 \theta - \cos^2 \theta$$

$$\boxed{\cos^2 \theta} \rightarrow \text{Pythagorean}$$

$$\boxed{1 - \sin^2 \theta}$$

$$\textcircled{15} \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - \boxed{1}$$

$$2\cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)$$

$$2\cos^2 \theta - \sin^2 \theta - \cos^2 \theta$$

$$\cos^2 \theta - \sin^2 \theta$$

Final Review

$$\textcircled{3} \quad \boxed{\sin(x+y)} \boxed{\sin(x-y)} = \cos^2 y - \cos^2 x$$

$$\boxed{(\sin x \cos y + \cos x \sin y)} \boxed{(\sin x \cos y - \cos x \sin y)} \quad \boxed{\cos^2 y - \cos^2 x}$$

$$\boxed{\sin^2 x} \cos^2 y - \cos^2 x \boxed{\sin^2 y}$$

$$\swarrow \quad \searrow \quad \swarrow \quad \searrow$$

$$\boxed{(1 - \cos^2 x)} \cos^2 y - \cos^2 x \boxed{(1 - \cos^2 y)}$$

$$\cos^2 y - \cancel{\cos^2 x \cos^2 y} - \cos^2 x + \cancel{\cos^2 x \cos^2 y}$$

$$\boxed{\cos^2 y - \cos^2 x}$$

Final Review

$$\begin{aligned}
 \textcircled{4} \quad \boxed{\cos^4 \theta} &= 1 - 2\sin^2 \theta + \sin^4 \theta \quad \leftarrow \text{Factor} \\
 &= (1 - \sin^2 \theta)(1 + \sin^2 \theta) \quad \leftarrow \text{Pythagorean Identity} \\
 &= (\cos^2 \theta)(\cos^2 \theta) \\
 &= \boxed{\cos^4 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad \boxed{\tan^4 \theta} &= \boxed{\sec^4 \theta} (1 - 2\cos^2 \theta + \cos^4 \theta) \quad \leftarrow \text{Factor} \\
 \boxed{\frac{\sin^4 \theta}{\cos^4 \theta}} & \left| \begin{aligned} & \frac{1}{\cos^4 \theta} \boxed{(1 - \cos^2 \theta)(1 - \cos^2 \theta)} \quad \leftarrow \text{Pythag.} \\ & \frac{1}{\cos^4 \theta} (\sin^2 \theta)(\sin^2 \theta) \\ & \boxed{\frac{\sin^4 \theta}{\cos^4 \theta}} \end{aligned} \right.
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad \boxed{\tan \theta} &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\
 \boxed{\frac{\sin \theta}{\cos \theta}} & \left| \begin{aligned} & \frac{\sin^2 \theta + \sin \theta}{\cos \theta (1 + \sin \theta)} \quad \leftarrow \text{Factor} \\ & \frac{\sin \theta (\cancel{\sin \theta + 1})}{\cos \theta (\cancel{1 + \sin \theta})} \end{aligned} \right.
 \end{aligned}$$

$$⑦ \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2 \boxed{\sec\theta}$$

$$\frac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)} \quad \boxed{\frac{1}{\cos\theta}}$$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} \quad \boxed{\frac{2}{\cos\theta}}$$

Pythag

$$\frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)} \quad \leftarrow \text{Factor}$$

$$\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$\boxed{\frac{2}{\cos\theta}}$$

$$⑩ \sin(x+y) - \sin(x-y) = 2\cos x \sin y$$

$$\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y) = 2\cos x \sin y$$

$\boxed{2\cos x \sin y}$

$$⑪ \frac{\sin^4\theta - \cos^4\theta}{\sin^3\theta \cos^3\theta - \cos^4\theta} = \frac{\boxed{\csc^2\theta}}{\boxed{\cot^2\theta}}$$

$$\frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{\cos^3\theta(\sin^2\theta - \cos^2\theta)}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\sin^2\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\textcircled{30} \quad \underline{\cos(x+y)} - \underline{\cos(x-y)} = -2\sin x \sin y$$

$$\cos x \cos y - \sin x \sin y - (\cos x \cos y + \sin x \sin y) = -2\sin x \sin y$$

$$\cancel{\cos x \cos y} - \sin x \sin y - \cancel{\cos x \cos y} - \sin x \sin y$$

$$\boxed{-2\sin x \sin y}$$

$$\textcircled{31} \quad \frac{1 + \boxed{\cos 2\theta}}{\boxed{\sin 2\theta}} = \boxed{\cot \theta}$$

$$\frac{1 + (\cos^2 \theta - \sin^2 \theta)}{2 \sin \theta \cos \theta} \quad \left| \quad \frac{\cos \theta}{\sin \theta}$$

$$\frac{1 + \cos^2 \theta - \sin^2 \theta}{2 \sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta + \cos^2 \theta}{2 \sin \theta \cos \theta}$$

$$\frac{\cancel{2} \cos^2 \theta}{\cancel{2} \sin \theta \cancel{\cos \theta}}$$

$$\boxed{\frac{\cos \theta}{\sin \theta}}$$

$$\textcircled{32} \quad \frac{\cos y}{1 + \sin y} + \frac{1 + \sin y}{\cos y} = 2 \boxed{\sec y}$$

FOIL

$$\frac{\cos^2 y + (1 + \sin y)(1 + \sin y)}{\cos y (1 + \sin y)}$$

$$\frac{\cos^2 y + 1 + 2\sin y + \sin^2 y}{\cos y (1 + \sin y)}$$

$$2 \left(\frac{1}{\cos y} \right)$$

$$\frac{2}{\cos y}$$

$$\frac{2 + 2\sin y}{\cos y (1 + \sin y)}$$

$$\frac{2 \cancel{(1 + \sin y)}}{\cos y \cancel{(1 + \sin y)}}$$

$$\textcircled{8} \quad \frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \underline{\underline{\cot^2 x}}$$

$$\frac{\cos x \tan^2 x}{\tan^2 x (\sec x - 1)} - \frac{\cos x (\sec x - 1)}{\tan^2 x (\sec x - 1)}$$

$$\frac{1}{\tan^2 x}$$

C.F. →

$$\frac{\cos x \tan^2 x - \cos x \sec x + \cos x}{\tan^2 x (\sec x - 1)}$$

Pyth. ↙

$$\frac{\cos x (\tan^2 x - \sec x + 1)}{\tan^2 x (\sec x - 1)}$$

← C.F.

$$\frac{\cos x (\sec^2 x - \sec x)}{\tan^2 x (\sec x - 1)}$$

$$\frac{\cos x \cancel{\sec x} (\cancel{\sec x - 1})}{\tan^2 x (\cancel{\sec x - 1})}$$

$$\frac{\cos x \left(\frac{1}{\cos x} \right)}{\tan^2 x}$$

$$\frac{1}{\tan^2 x}$$

$$\textcircled{5} \tan^4 \theta = \sec^4 \theta (1 - 2\cos^2 \theta + \cos^4 \theta)$$
$$\frac{\sin^4 \theta}{\cos^4 \theta} \quad \left| \quad \frac{1}{\cos^4 \theta} (1 - \cos^2 \theta)(1 - \cos^2 \theta) \right.$$
$$\left. \frac{1}{\cos^4 \theta} (\sin^2 \theta)(\sin^2 \theta) \right.$$
$$\frac{\sin^4 \theta}{\cos^4 \theta}$$