① b) $\int\left(12x^2 - \sqrt{x^3} + \sin 5x + \frac{2}{x}\right)dx$

$= \int\left(12x^2 - x^{3/2} + \sin 5x + \frac{2}{x}\right)dx$

$= \frac{12x^3}{3} - \frac{x^{5/2}}{5/2} - \frac{1}{5}\cos 5x + 2\ln x + C$

$= 4x^3 - \frac{2}{5}x^{5/2} - \frac{1}{5}\cos 5x + 2\ln x + C$

① f) $\int\left(6\sec(2x)\tan(2x) - \frac{5}{x^3} + 4^{2x} - 1\right)dx$

$= \int\left(6\sec(2x)\tan(2x) - 5x^{-3} + 4^{2x} - 1\right)dx$

$= \frac{6}{2}\sec(2x) - \frac{5x^{-2}}{-2} + \frac{4^{2x}}{2\ln 4} - 1x + C$

$= 3\sec(2x) + \frac{5}{2x} + \frac{4^{2x}}{2\ln 4} - x + C$

2) g) $\int \frac{3x^2-2}{x^3-2x+1}\,dx = \int \frac{1}{x^3-2x+1}(3x^2-2)\,dx$

$u = x^3-2x+1$

$du = (3x^2-2)\,dx$

$= \int \frac{1}{u}\,du$

$= \ln u + C$

$= \ln(x^3-2x+1) + C$

h) $\int (e^{\cos x}\sin x)\,dx = \int e^{u}(-1)\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-1\,du = \sin x\,dx$

$= -(e^{u}) + C$

$= -e^{\cos x} + C$

i) $\int x\sqrt{4-x^2}\,dx = \int x(4-x^2)^{1/2}\,dx$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \int u^{1/2}\left(-\frac{1}{2}\right)du$

$= -\frac{1}{2}\int u^{1/2}\,du$

$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$= -\frac{1}{3}u^{3/2} + C$

$= -\frac{1}{3}(4-x^2)^{3/2} + C$

$a=0$ $b=2$

⑥ a) $f(x) = x^2 + 1$ between $x=0$ + $x=2$

① $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

③ $f\left(\frac{2i}{n}\right) = \left(\frac{2i}{n}\right)^2 + 1$

$= \frac{4i^2}{n^2} + 1$

② $x_i^* = 0 + \frac{2i}{n} = \frac{2i}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + 1\right)\left(\frac{2}{n}\right)$$

quadratic

constant

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} + \frac{2}{n}$$

$$A = \lim_{n\to\infty} \left[\frac{\cancel{8}^{4}}{n^{\cancel{3}2}} \cdot \left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}\right) + \frac{2}{\cancel{n}} \cdot \cancel{n}\right]$$

$2n^2 + 3n + 1$

$$A = \lim_{n\to\infty} \frac{8n^2 + 12n + 4}{3n^2} + 2$$

$$A = \frac{8}{3} + 2$$

$$A = \frac{8}{3} + \frac{6}{3} = \frac{14}{3}$$

Integration Exam Review

① a) $f(x) = x^3 - 3x^2 + 5$

$$F(x) = \frac{x^4}{4} - \frac{3x^3}{3} + \frac{5x}{1} + C$$

$$\boxed{F(x) = \frac{1}{4}x^4 - x^3 + 5x + C}$$

c) $\int (x^{5/6} - 3x^{9/2} + x^{-6} - 3x^{-1/2})dx$

$$= \frac{x^{11/6}}{\frac{11}{6}} - \frac{3x^{11/2}}{\frac{11}{2}} + \frac{x^{-5}}{-5} - \frac{3x^{1/2}}{\frac{1}{2}} + C$$

$$\boxed{= \frac{6}{11}x^{11/6} - \frac{6}{11}x^{11/2} - \frac{1}{5x^5} - 6\sqrt{x} + C}$$

② b) $\int_{-1}^{2} (x^3 - 2x^2 + 6) = \frac{x^4}{4} - \frac{2x^3}{3} + 6x \Big]_{-1}^{2}$

* $A = F(b) - F(a)$

$$= \frac{(2)^4}{4} - \frac{2(2)^3}{3} + 6(2) - \left[\frac{(-1)^4}{4} - \frac{2(-1)^3}{3} + 6(-1)\right]$$

$$= 4 - \frac{16}{3} + 12 - \left[\frac{1}{4} + \frac{2}{3} - 6\right]$$

$$= \frac{32}{3} - \left[-\frac{61}{12}\right]$$

$$= \frac{32}{3} + \frac{61}{12}$$

$$= \frac{128 + 61}{12}$$

$$= \frac{189}{12}$$

$$\boxed{= \frac{63}{4}}$$

② a) $f(x) = x^3 - 2x^2 + 6$ from $x=-1$ to $x=2$

$a=-1$ $b=2$

① $\Delta x = \frac{b-a}{n}$

$= \frac{3}{n}$

② $f(x_i^*) = f(a + i\Delta x)$

$= f\left(-1 + \frac{3i}{n}\right)$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1+\frac{3i}{n}\right)\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1+\frac{3i}{n}\right)^3 - 2\left(-1+\frac{3i}{n}\right)^2 + 6\right]\frac{3}{n}$$

$\downarrow$

Summation:

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{n^2(n+1)^2}{4}\right]$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$