① b) $\int\left(12x^2 - \sqrt{x^3} + \sin 5x + \frac{2}{x}\right)dx$

$= \int\left(12x^2 - x^{3/2} + \sin 5x + \frac{2}{x}\right)dx$

$= \frac{12x^3}{3} - \frac{x^{5/2}}{5/2} - \frac{1}{5}\cos 5x + 2\ln x + C$

$= 4x^3 - \frac{2}{5}x^{5/2} - \frac{1}{5}\cos 5x + 2\ln x + C$

① d) $\int\left(\frac{x^6 - x^4 + 3x^{1/2}}{x^{5/2}}\right)dx = \int\left(x^{7/2} - x^{3/2} + 3x^{-2}\right)dx$

$6 - \frac{5}{2}$	$4 - \frac{5}{2}$	$\frac{1}{2} - \frac{5}{2}$
$\frac{12}{2} - \frac{5}{2}$	$\frac{8}{2} - \frac{5}{2}$	$\frac{-4}{2}$
$\frac{7}{2}$	$\frac{3}{2}$	-2

$= \frac{2x^{9/2}}{9} - \frac{2x^{5/2}}{5} - 3x^{-1} + C$

① f) $\int\left(6\sec(2x)\tan(2x) - \frac{5}{x^3} + 4^{2x} - 1\right)dx$

$= \int\left(6\sec(2x)\tan(2x) - 5x^{-3} + 4^{2x} - 1\right)dx$

$= \frac{6}{2}\sec(2x) \; \frac{-5x^{-2}}{-2} + \frac{4^{2x}}{2\ln 4} - 1x + C$

$= 3\sec(2x) + \frac{5}{2x} + \frac{4^{2x}}{2\ln 4} - x + C$

② g) $\int \frac{3x^2-2}{x^3-2x+1}\,dx = \int \frac{1}{\underbrace{x^3-2x+1}}\underbrace{(3x^2-2)dx}$

$u = x^3-2x+1$

$du = (3x^2-2)dx$

$= \int \frac{1}{u}\,du$

$= \ln u + C$

$= \ln(x^3-2x+1) + C$

h) $\int (e^{\cos x} \sin x)\,dx = \int e^{u}(-1)\,du$

$u = \cos x$

$du = -\sin x\,dx$

$-1\,du = \sin x\,dx$

$= -(e^{u}) + C$

$= -e^{\cos x} + C$

i) $\int x\sqrt{4-x^2}\,dx = \int x(4-x^2)^{1/2}\,dx$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= \int u^{1/2}\left(-\frac{1}{2}\right)du$

$= -\frac{1}{2}\int u^{1/2}\,du$

$= -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$= -\frac{1}{3}u^{3/2} + C$

$= -\frac{1}{3}(4-x^2)^{3/2} + C$

④ a) $\int_1^2 (8x^2-4x+5)\,dx = \frac{8x^3}{3} - \frac{4x^2}{2} + 5x \Big|_1^2$

$= \frac{8(2)^3}{3} - 2(2)^2 + 5(2) - \left[\frac{8(1)^3}{3} - 2(1)^2 + 5(1)\right]$

$= \frac{64}{3} - 8 + 10 - \frac{8}{3} + 2 - 5$

$= \frac{56}{3} - 1$

$= \frac{56}{3} - \frac{3}{3} = \frac{53}{3}$

④ b) $\int_0^2 x^2\sqrt{x^3+1}\,dx = \int_0^2 x^2(x^3+1)^{1/2}\,dx$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

x	u
2	$(2)^3+1=9$
0	$(0)^3+1=1$

$= \int_1^9 u^{1/2} \cdot \frac{1}{3}\,du$

$= \frac{1}{3}\int_1^9 u^{1/2}\,du$

$= \frac{1}{3}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^9$

$= \frac{2}{9}u^{3/2}\Big|_1^9$

$= \frac{2}{9}(9)^{3/2} - \frac{2}{9}(1)^{3/2}$

$= \frac{2}{9}(27) - \frac{2}{9}(1)$

$= \frac{54}{9} - \frac{2}{9} = \frac{52}{9}$

$a=0$ $b=2$

⑥ a) $f(x) = x^2 + 1$ between $x=0$ + $x=2$

① $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

② $x_i^* = 0 + \frac{2i}{n} = \frac{2i}{n}$

③ $f\left(\frac{2i}{n}\right) = \left(\frac{2i}{n}\right)^2 + 1$

$= \frac{4i^2}{n^2} + 1$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^2}{n^2} + 1\right)\left(\frac{2}{n}\right)$$

quadratic

constant

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} + \frac{2}{n}$$

$$A = \lim_{n\to\infty} \left[\frac{\cancel{8}^{\,4}}{n^3} \cdot \left(\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right) + \frac{2}{\cancel{n}} \cdot \cancel{n}\right]$$

$2n^2+3n+1$

$$A = \lim_{n\to\infty} \frac{8n^2+12n+4}{3n^2} + 2$$

$$A = \frac{8}{3} + 2$$

$$A = \frac{8}{3} + \frac{6}{3} = \frac{14}{3}$$

⑧ Area between $y = 6 - x^2$ and $y = 3 - 2x$

① Intersection

$$6 - x^2 = 3 - 2x$$
$$0 = x^2 - 2x - 3$$
$$0 = (x-3)(x+1)$$
$$x - 3 = 0 \quad \Big| \quad x + 1 = 0$$
$$x = 3 \quad \Big| \quad x = -1$$

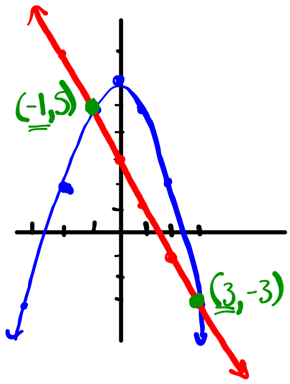
② Quick Sketch

• $y = 6 - x^2$

x	y
-2	2
-1	5
0	6
1	5
2	2
3	-3

• $y = 3 - 2x$

x	y
-2	7
-1	5
0	3
1	1
2	-1
3	-3



③ $A = \int_{-1}^{3} (6 - x^2) - (3 - 2x)\,dx$

$$A = \int_{-1}^{3} (6 - x^2 - 3 + 2x)\,dx$$

$$A = \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$$

$$A = -\frac{x^3}{3} + x^2 + 3x \Big|_{-1}^{3}$$

$$A = -\frac{(3)^3}{3} + (3)^2 + 3(3) - \left[\frac{(-1)^3}{3} + (-1)^2 + 3(-1)\right]$$

$$A = -\cancel{9} + \cancel{9} + 9 - \frac{1}{3} - 1 + 3$$

$$A = 11 - \frac{1}{3}$$

$$A = \frac{33}{3} - \frac{1}{3} = \frac{32}{3}$$

Integration Exam Review

① a) $f(x) = x^3 - 3x^2 + 5$

$$F(x) = \frac{x^4}{4} - \frac{3x^3}{3} + \frac{5x}{1} + C$$

$$\boxed{F(x) = \frac{1}{4}x^4 - x^3 + 5x + C}$$

c) $\int (x^{5/6} - 3x^{9/2} + x^{-6} - 3x^{-1/2})\,dx$

$$= \frac{x^{11/6}}{\frac{11}{6}} - \frac{3x^{11/2}}{\frac{11}{2}} + \frac{x^{-5}}{-5} - \frac{3x^{1/2}}{\frac{1}{2}} + C$$

$$\boxed{= \frac{6}{11}x^{11/6} - \frac{6}{11}x^{11/2} - \frac{1}{5x^5} - 6\sqrt{x} + C}$$

② b) $\int_{-1}^{2} (x^3 - 2x^2 + 6) = \frac{x^4}{4} - \frac{2x^3}{3} + 6x \Big]_{-1}^{2}$

* $A = F(b) - F(a)$

$$= \frac{(2)^4}{4} - \frac{2(2)^3}{3} + 6(2) - \left[\frac{(-1)^4}{4} - \frac{2(-1)^3}{3} + 6(-1)\right]$$

$$= 4 - \frac{16}{3} + 12 - \left[\frac{1}{4} + \frac{2}{3} - 6\right]$$

$$= \frac{32}{3} - \left[-\frac{61}{12}\right]$$

$$= \frac{32}{3} + \frac{61}{12}$$

$$= \frac{128 + 61}{12}$$

$$= \frac{189}{12}$$

$$\boxed{= \frac{63}{4}}$$

② a) $f(x) = x^3 - 2x^2 + 6$ from $x = -1$ to $x = 2$

$a = -1$ $b = 2$

① $\Delta x = \frac{b-a}{n}$

$= \frac{3}{n}$

② $f(x_i^*) = f(a + i\Delta x)$

$= f\left(-1 + \frac{3i}{n}\right)$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1 + \frac{3i}{n}\right)\frac{3}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(-1 + \frac{3i}{n}\right)^3 - 2\left(-1 + \frac{3i}{n}\right)^2 + 6\right]\frac{3}{n}$$

$\downarrow$

Summation:

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{n^2(n+1)^2}{4}\right]$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$