Chapter Review For Final Exam:

Ch. 1 → (Inverse Functions)

Ch. 2 → (Radical Functions) $y = a\sqrt{b(x-h)} + k$

Ch. 7 → (Exponential Functions) $y = 2^x$

Ch. 8 → (Logarithmic Functions) $y = \log_2 x$

Ch. 4 → (Trig + Unit Circle)

Ch. 5 → (Trig Functions)

Ch. 6 → (Trig Identities)

Ch. 2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

Ch. 7

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

Ch. 8

$y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Ch. 5

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

Chapter 2 Radical Functions

$$\sqrt{x+8} - 6 = x$$

Solve for x:

$$\left(\sqrt{x+8}\right)^2 = (x+6)^2$$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2+12x+36$$

$$0 = x^2+11x+28$$

$\underline{7} \times \underline{4} = 28$

$\underline{7} + \underline{4} = 11$

$$0 = (x+7)(x+4)$$

$x+7=0 \quad | \quad x+4=0$

$x=-7 \quad | \quad x=-4$

Test $x=-4$ is a solution

$\sqrt{x+8} = x+6$

$\sqrt{-4+8}$	$-4+6$
2	2 ✓

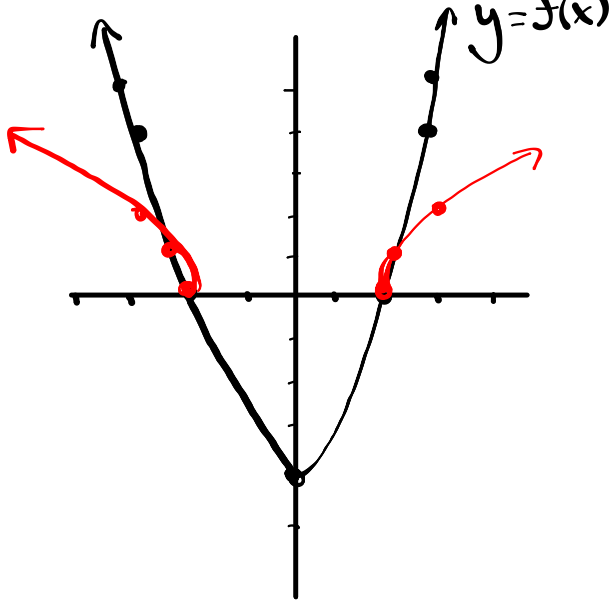
Test $x=-7$ is extraneous

$\sqrt{x+8} = x+6$

$\sqrt{-7+8}$	$-7+6$
1	-1 ✗

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. State the domain and range of each.



$y = f(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -4, y \in \mathbb{R}\}$ or $[4, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in \mathbb{R}\}$

$(-\infty, -2]$ and $[2, \infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

$\frac{\log 64}{\log 2} = 6$

$\cancel{2}^{2x+2} = \cancel{2}^6$

$2x + 2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

b) $9^{2x+1} = 81(27^x)$

$\frac{\log 9}{\log 3} = 2$ $\frac{\log 81}{\log 3} = 4$ $\frac{\log 27}{\log 3} = 3$

$(3^2)^{2x+1} = 3^4(3^3)^x$

$3^{4x+2} = 3^4 \cdot 3^{3x}$

$\cancel{3}^{4x+2} = \cancel{3}^{3x+4}$

$4x + 2 = 3x + 4$

$x = 2$

Ex: $y = 3(2)^{2x+8} - 1$

$y = 3(2)^{2(x+4)} - 1$

a = 3 → vertical stretch by a factor of 3
no vertical reflection

b = 2 → horizontal compression by a factor of ½
no horizontal reflection

h = -4 → translate left 4 units

k = -1 → " down 1 unit

$(x, y) \rightarrow (\frac{1}{2}x - 4, 3y - 1)$

$y = 2^x$

x	y
-2	¼
-1	½
0	1
1	2
2	4

$y = 3(2)^{2(x+4)} - 1$

	x	y
	-5	-¼
(-4.5)	-9/2	½
	-4	2
(-3.5)	-7/2	5
	-3	11

$3(\frac{1}{4}) - 1$

$\frac{3}{4} - \frac{4}{4} = \frac{-1}{4}$

$3(\frac{1}{2}) - 1$

$\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

Ch. 8 → logarithms

4. Rewrite each expression as a single logarithm.

$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$

$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5 (x^2+1)$

$\log_5\left(\frac{x^3(x-1)^{1/2}}{x^2+1}\right)$

$\boxed{\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}}$

$\log_2\left(\frac{x^2}{y^3\sqrt[4]{z}}\right)$

$\log_2 x^2 - \log_2 y^3 - \log_2 z^{1/4}$

$\boxed{2\log_2 x - 3\log_2 y - \frac{1}{4}\log_2 z}$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}(x^2-x+2x-2) = 1$$

$$\log_{10}(x^2+x-2) = 1 \quad \text{(log)}$$

$$10^1 = x^2+x-2 \quad \text{(exp)}$$

$$10 = x^2+x-2$$

$$0 = x^2+x-12 \qquad \underline{-3} \times \underline{4} = -12 \qquad \underline{-3} + \underline{4} = 1$$

$$0=(x-3)(x+4)$$

12
1x12
2x6
3x4

$x-3=0$ | $x+4=0$

$x=3$ | $x=-4$ is extraneous

test $x=3$

$$\log_{10}(x+2)+\log_{10}(x-1)=1 \checkmark$$

$$\log_{10}5+\log_{10}2$$

$$\log_{10}10$$

$$1 \checkmark$$

$\therefore$

test $x=-4$

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(-2)+\log_{10}(-5)$$

not possible

Ch.7 or Ch.8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

Base = ½ = 0.5   exp = $t/5.3$ Initial Amount = 60

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$$M(t) = 60(0.5)^{t/5.3} \quad \Big| \quad y = 60(0.5)^{t/5.3}$$

b) What amount will be present in 10.6 years? [2] $t = 10.6$

$$M(t) = 60(0.5)^{10.6/5.3}$$

$$M(t) = 60(0.5)^2$$

$$M(t) = 60(0.25) = 15\text{ mg}$$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

① 12.5% of 60mg

0.125 × 60

7.5 mg

$$M(t) = 60(0.5)^{t/5.3}$$

$$\frac{7.5}{60} = \frac{\cancel{60}(0.5)^{t/5.3}}{\cancel{60}}$$

$$0.125 = (0.5)^{t/5.3}$$

$$(\cancel{0.5})^3 = (\cancel{0.5})^{t/5.3}$$

$$5.3 \cdot 3 = \frac{t}{5.3} \cdot 5.3$$

$$\boxed{15.9 \text{ years} = t}$$

$$\frac{\log 0.125}{\log 0.5} = 3$$

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$

e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$

$$\frac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$$

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$0 \le \theta \le 2\pi$

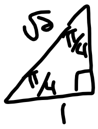
12 a) $\sin\theta = \sin\theta\tan\theta$

Common Factor

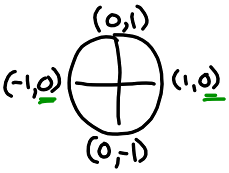
$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$	$\tan\theta - 1 = 0$
$\theta = 0, \pi, 2\pi$	$\tan\theta = 1$
	$\theta_R = \frac{\pi}{4}$



S | A
T | C



Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

13 b) $3\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$

$\frac{-3}{} \times \frac{1}{} = -3$

$\frac{-3}{} + \frac{1}{} = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

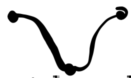
$3\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{3}$	$\sin\theta = 1$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$	$\theta = 90°$
$\theta_R = 19$	

Unit Circle

Where is sine negative: S | A / T | C

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 ~~cm~~ above the floor, occurs at 1.8 sec.



$y = \cos x$

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

max = 50cm

min = 30cm

$K = \frac{50+30}{2} = 40$

Amp = 10 $\quad P = 2(1.8-0.4) = 2.8 \quad h = 0.4$

$a = \pm 10 \quad b = \frac{360}{2.8} = 128.57$

$y = 10\cos[128.57(17.2-0.4)]+40$

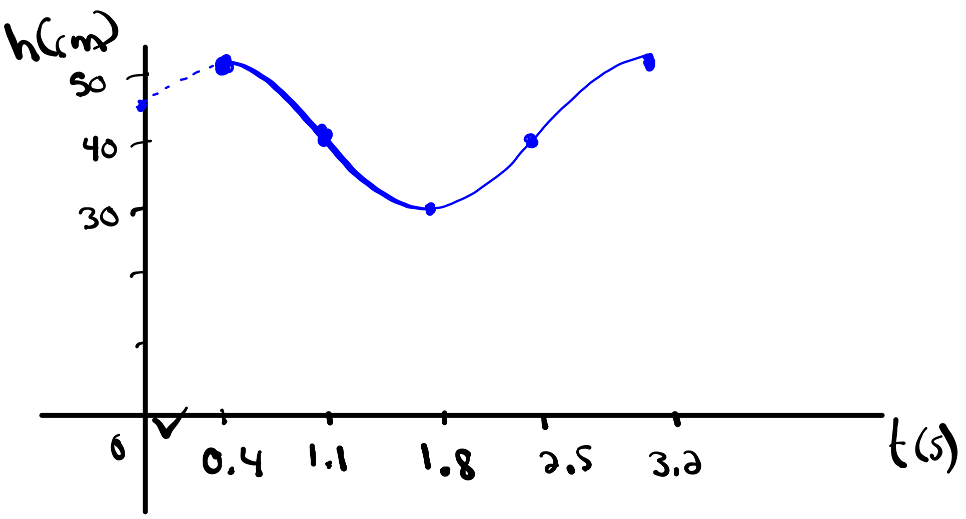
$y = 49.9$ cm

(b) How high was the weight above the floor when the stopwatch was initially started?

$t = 0$

$y = 10\cos[128.57(0-0.4)]+40$

$y = 46.23$cm



Count by $\frac{P}{4} = \frac{2.8}{4} = 0.7$

① Amp = 11 P = 16 min = -4

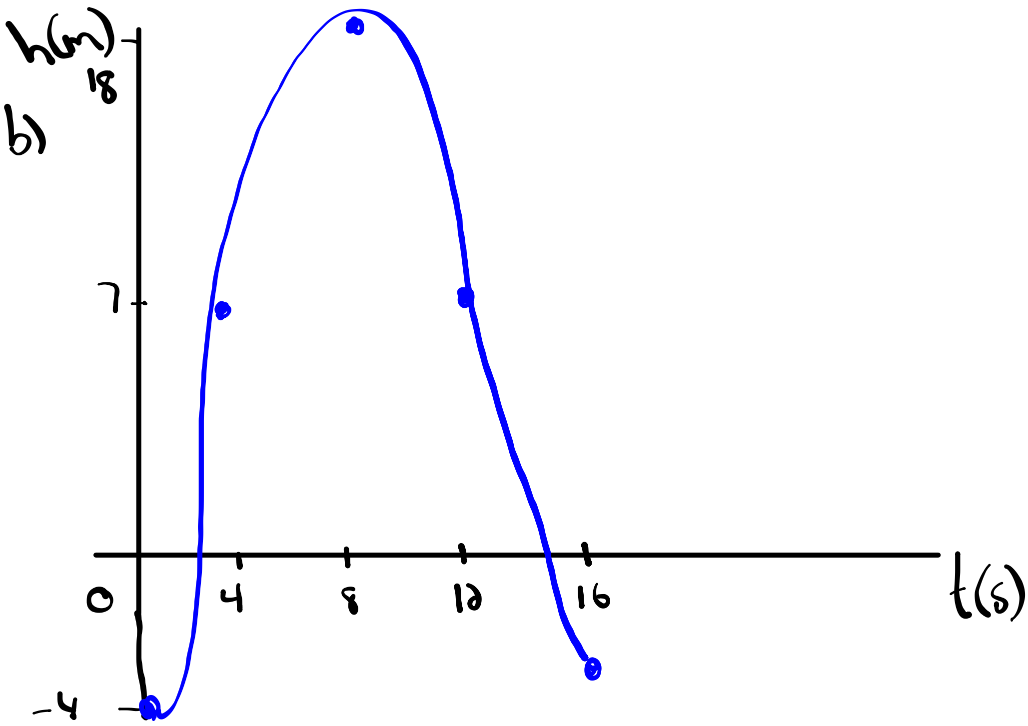
a = ± 11 $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



1

4)

max = 68
min = 24

$k = \frac{68+24}{2} = 46$

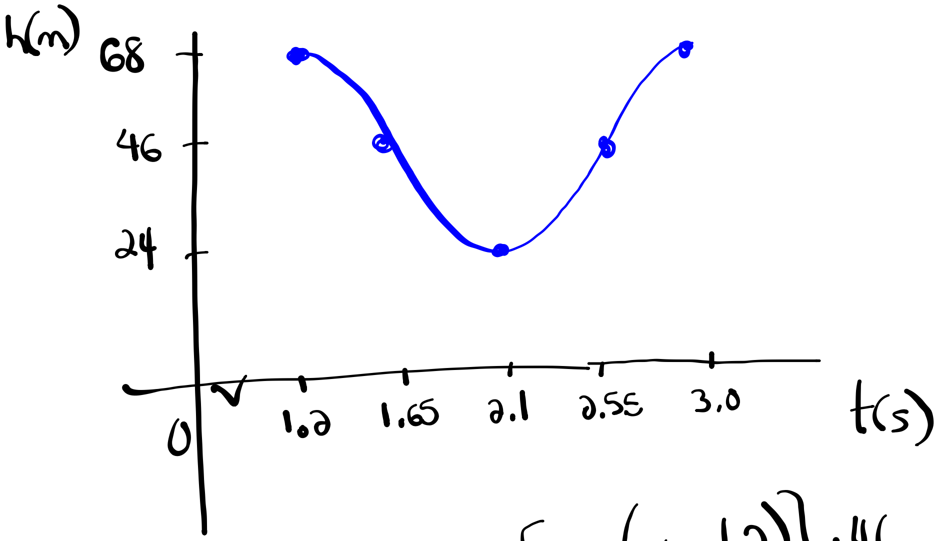
Amp = 68 - 46 = 22
a = ± 22

P = 2(2.1 - 1.2)
P = 1.8

$b = \frac{360}{1.8} = 200$

h = 1.2

$\frac{P}{4} = \frac{1.8}{4} = 0.45$



$y = 22\cos[200(x - 1.2)] + 46$

5 c) $y = \frac{1}{2}\cos(1\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

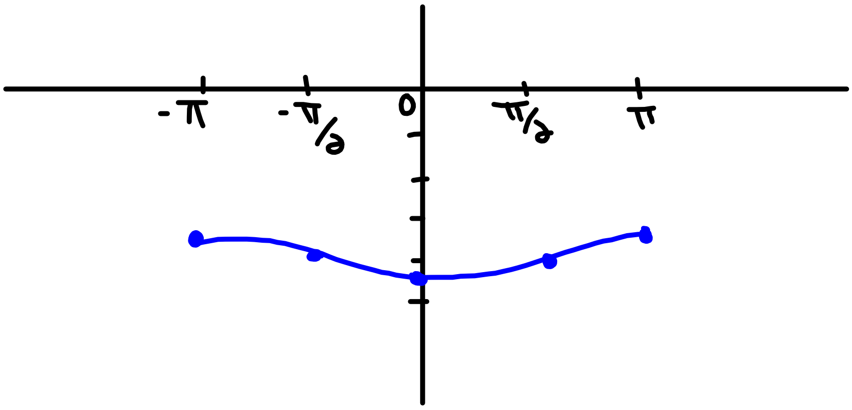
$(x, y) \rightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\longrightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5



$$\frac{1}{\sec^2\theta\cot\theta}=\frac{\sin\theta-\sin^3\theta}{\cos\theta}$$