Chapter Review For Final Exam:

Ch. 1 → (Inverse Functions)

Ch. 2 → (Radical Functions) $y = a\sqrt{b(x-h)} + k$

Ch. 7 → (Exponential Functions) $y = 2^x$

Ch. 8 → (Logarithmic Functions) $y = \log_2 x$

Ch. 4 → (Trig + Unit Circle)

Ch. 5 → (Trig Functions)

Ch. 6 → (Trig Identities)

Ch. 2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

Ch. 7

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

Ch. 8

$y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Ch. 5

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$y = \cos x$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

Chapter 2 Radical Functions

$$\sqrt{x+8}-6=x$$

Solve for x: $\left(\sqrt{x+8}\right)^2=(x+6)^2$

$$x+8=(x+6)(x+6)$$

$$x+8=x^2+12x+36$$

$$0=x^2+11x+28$$

$7 \times 4 = 28$
$7 + 4 = 11$

$$0=(x+7)(x+4)$$

$x+7=0$	$x+4=0$
$x=-7$	$x=-4$

Test $x=-4$ is a solution

$\sqrt{x+8} = x+6$

$\sqrt{-4+8}$	$-4+6$
2	2 ✓

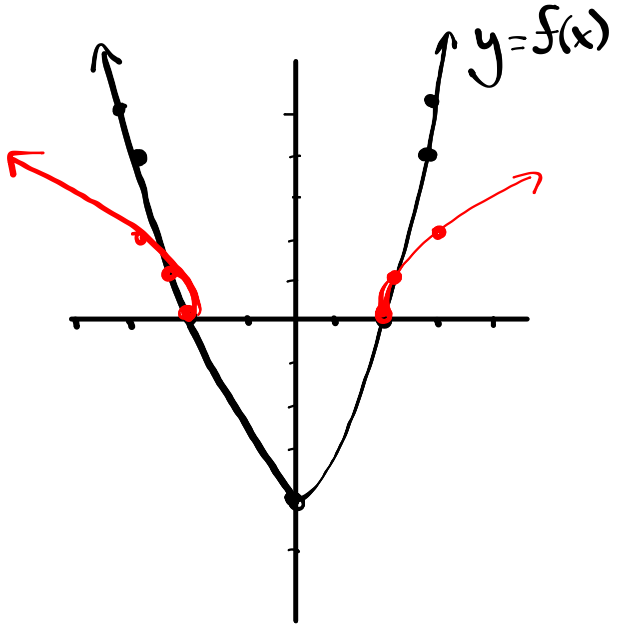
Test $x=-7$ is extraneous

$\sqrt{x+8} = x+6$

$\sqrt{-7+8}$	$-7+6$
1	-1 ✗

Ch. 2

③ Using the graph of $y=f(x)$, sketch the graph of $y=\sqrt{f(x)}$. State the domain and range of each.



$y=f(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty,\infty)$

R: $\{y \mid y \geq -4, y \in \mathbb{R}\}$ or $[4,\infty)$

$y=\sqrt{f(x)}$

D: $\{x \mid x \leq -2, x \geq 2, x \in \mathbb{R}\}$

$(-\infty,-2]$ and $[2,\infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0,\infty)$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

$\frac{\log 64}{\log 2} = 6$

$\cancel{2}^{2x+2} = \cancel{2}^6$

$2x + 2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

b) $9^{2x+1} = 81(27^x)$

$\frac{\log 9}{\log 3} = 2$ $\frac{\log 81}{\log 3} = 4$ $\frac{\log 27}{\log 3} = 3$

$(3^2)^{2x+1} = 3^4(3^3)^x$

$3^{4x+2} = 3^4 \cdot 3^{3x}$

$\cancel{3}^{4x+2} = \cancel{3}^{3x+4}$

$4x + 2 = 3x + 4$

$x = 2$

Ex: $y = 3(2)^{2x+8} - 1$

$y = 3(2)^{2(x+4)} - 1$

a = 3 → vertical stretch by a factor of 3
no vertical reflection

b = 2 → horizontal compression by a factor of ½
no horizontal reflection

h = -4 → translate left 4 units

k = -1 → " down 1 unit

$(x, y) \rightarrow (\frac{1}{2}x - 4, 3y - 1)$

$y = 2^x$

x	y
-2	¼
-1	½
0	1
1	2
2	4

$y = 3(2)^{2(x+4)} - 1$

	x	y
	-5	-¼
(-4.5)	-9/2	½
	-4	2
(-3.5)	-7/2	5
	-3	11

$3(\frac{1}{4}) - 1$

$\frac{3}{4} - \frac{4}{4} = \frac{-1}{4}$

$3(\frac{1}{2}) - 1$

$\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

Ch. 8 → logarithms

4. Rewrite each expression as a single logarithm.

$$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$$

$$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5(x^2+1)$$

$$\log_5\left(\frac{x^3(x-1)^{1/2}}{x^2+1}\right)$$

$$\boxed{\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}}$$

$$\log_2\left(\frac{x^2}{y^3\sqrt[4]{z}}\right)$$

$$\log_2 x^2 - \log_2 y^3 - \log_2 z^{1/4}$$

$$\boxed{2\log_2 x - 3\log_2 y - \frac{1}{4}\log_2 z}$$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2)+\log_{10}(x-1)=1$$

$$\log_{10}(x+2)(x-1) = 1$$

$$\log_{10}(x^2-x+2x-2) = 1$$

$$\log_{10}(x^2+x-2) = 1$$ (log)

$$10^1 = x^2+x-2$$ (exp)

$$10 = x^2+x-2$$

$$0 = x^2+x-12$$

$-3 \times 4 = -12$

$-3 + 4 = 1$

12
1x12
2x6
3x4

$$0 = (x-3)(x+4)$$

$x-3=0$ | $x+4=0$
$x=3$ | $x=-4$ is extraneous

test $x=3$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$ ✓

$\log_{10}5+\log_{10}2$

$\log_{10}10$

1 ✓

$\therefore$

test $x=-4$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$

$\log_{10}(-2) + \log_{10}(-5)$

not possible

Ch.7 or Ch.8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

Base = $\frac{1}{2}$ = 0.5 exp = $\frac{t}{5.3}$ Initial Amount = 60

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$M(t) = 60(0.5)^{t/5.3}$ | $y = 60(0.5)^{t/5.3}$

b) What amount will be present in 10.6 years? [2] $t = 10.6$

$M(t) = 60(0.5)^{10.6/5.3}$

$M(t) = 60(0.5)^2$

$M(t) = 60(0.25) = 15$ mg

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

① 12.5% of 60mg

0.125 × 60

7.5 mg

$M(t) = 60(0.5)^{t/5.3}$

$\frac{7.5}{60} = \frac{\cancel{60}(0.5)^{t/5.3}}{\cancel{60}}$

$0.125 = (0.5)^{t/5.3}$

$(\cancel{0.5})^3 = (\cancel{0.5})^{t/5.3}$

$\frac{\log 0.125}{\log 0.5} = 3$

$5.3 \cdot 3 = \frac{t}{5.3} \cdot 5.3$

$\boxed{15.9 \text{ years} = t}$

③ b) $(32)^{-x+1} = \sqrt{256}\left(\frac{1}{8}\right)^{2x}$

$(2^5)^{-x+1} = 16\,(2^{-3})^{2x}$

$2^{-5x+5} = 2^4(2^{-6x})$

$\cancel{2}^{-5x+5} = \cancel{2}^{4-6x}$

$-5x+5 = 4-6x$

$\boxed{x=-1}$.

Ch. 4 → Trig Equation

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0$, $0 \le \theta \le 2\pi$ (Radians)

$\tan\theta(\tan\theta + 1) = 0$

S | A
T | C

$\tan\theta = 0$
(Unit Circle)
$\boxed{\theta = 0, \pi, 2\pi}$

$\tan\theta + 1 = 0$
$\tan\theta = -1$
(Special Triangle)
$\bar{\theta} = \tan^{-1}(1)$
$\bar{\theta} = \frac{\pi}{4}$

where is $\tan\theta < 0$

Q2	Q4
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4}$
$\boxed{\theta = \frac{3\pi}{4}}$	$\boxed{\theta = \frac{7\pi}{4}}$

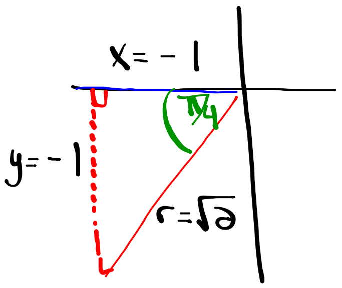
e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta < 360°$

Ch. 4 → Trig Expression

$$\frac{5\tan^2 \frac{5\pi}{4}}{6\sin \frac{5\pi}{6} + 4\sin \frac{4\pi}{3}}$$

$\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$

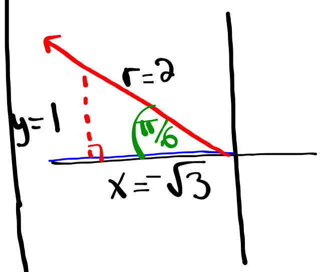
1π



$\tan \frac{5\pi}{4} = \frac{-1}{-1} = 1$

$\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

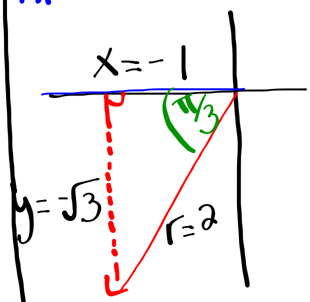
1π



$\sin \frac{5\pi}{6} = \frac{1}{2}$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

1π



$\sin \frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$

$$\frac{5\tan^2 \frac{5\pi}{4}}{6\sin \frac{5\pi}{6} + 4\sin \frac{4\pi}{3}}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right) + 4\left(\frac{-\sqrt{3}}{2}\right)} \rightarrow -\frac{4\sqrt{3}}{2} = -2\sqrt{3}$$

$$\frac{5}{(3 - 2\sqrt{3})} \cdot \frac{(3+2\sqrt{3})}{(3+2\sqrt{3})}$$

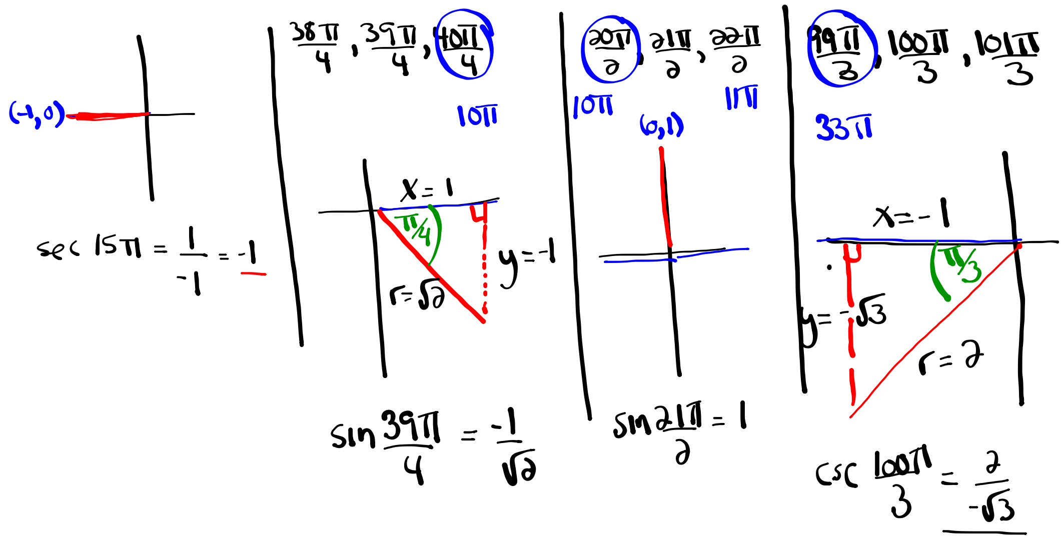
$(-2\sqrt{3})(2\sqrt{3}) = -4\sqrt{9}$

$$\frac{15 + 10\sqrt{3}}{9 + \cancel{6\sqrt{3}} - \cancel{6\sqrt{3}} - 4(3)}$$

$$\frac{15 + 10\sqrt{3}}{-3} \quad \text{or} \quad \frac{-15 - 10\sqrt{3}}{3}$$

Ch. 4

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$



$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \cancel{\sqrt{2}}\left(-\frac{1}{\cancel{\sqrt{2}}}\right)(1) - \left(-\frac{2}{\sqrt{3}}\right)^2$$

$$-1 - 1 - \frac{4}{3}$$

$$-2 - \frac{4}{3}$$

$$-\frac{6}{3} - \frac{4}{3}$$

$$-\frac{10}{3}$$

$0 \le \theta \le 2\pi$

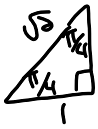
12 a) $\sin\theta = \sin\theta\tan\theta$

Common Factor

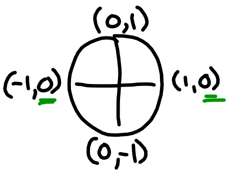
$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$	$\tan\theta - 1 = 0$
$\theta = 0, \pi, 2\pi$	$\tan\theta = 1$
	$\theta_R = \frac{\pi}{4}$



$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$



Where is $\tan\theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

13 b) $3\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta)(+\sin\theta - 1) = 0$

$-3 \times 1 = -3$

$-3 + 1 = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$	$\sin\theta - 1 = 0$
$\sin\theta = -\frac{1}{3}$	$\sin\theta = 1$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$	$\theta = 90°$
$\theta_R = 19$	

Unit Circle

Where is sine negative: $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

(b) How high was the weight above the floor when the stopwatch was initially started?

Count by $\frac{P}{4}$ =

① Amp = 11 P = 16 min = -4

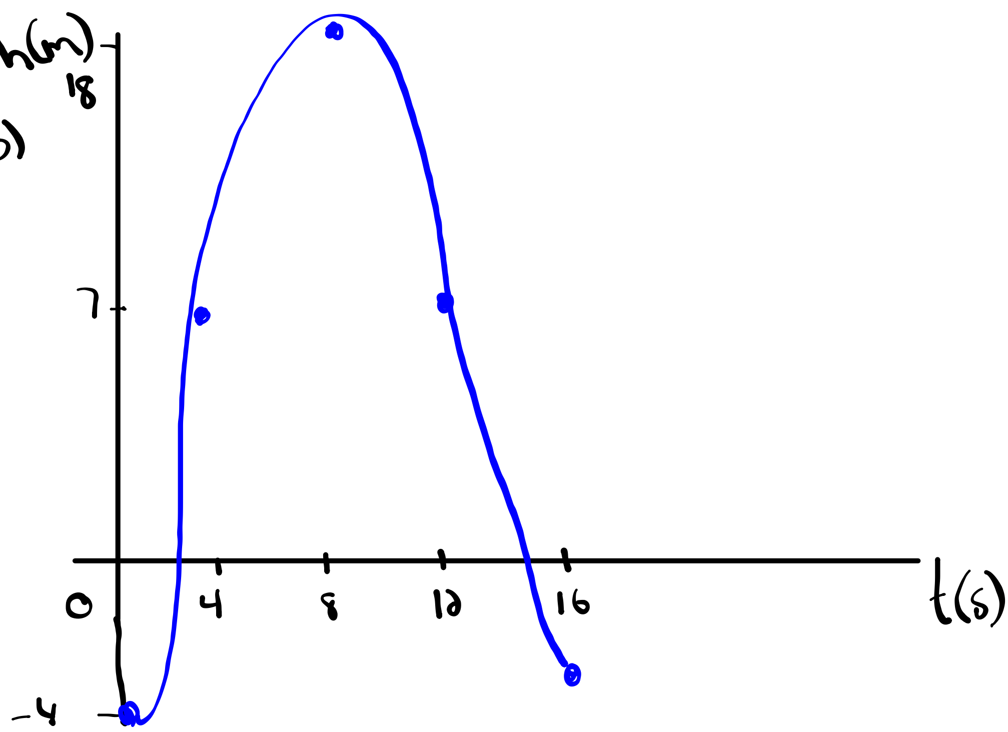
a = ±11 $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



1

4

max = 68

min = 24

$k = \frac{68+24}{2} = 46$

Amp = 68 − 46 = 22

$a = \pm 22$

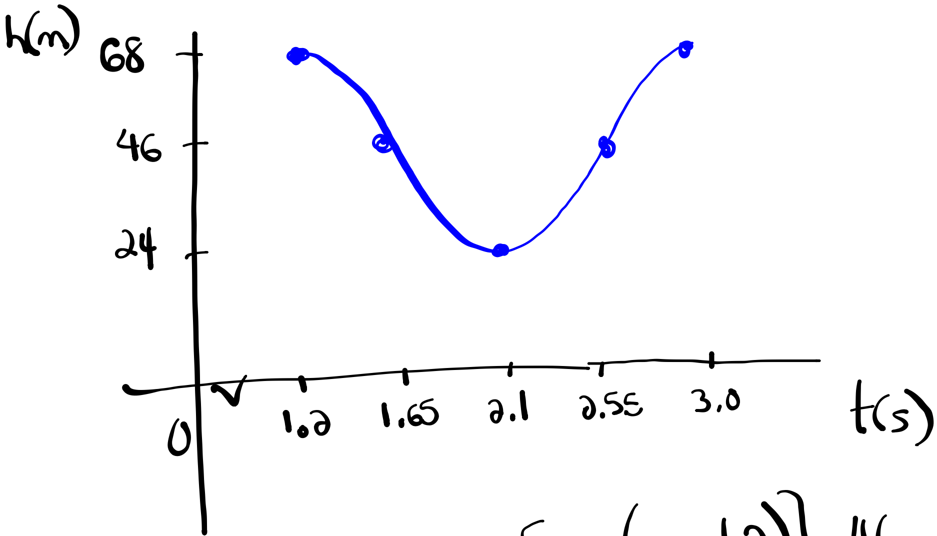
$P = 2(2.1 - 1.2)$

$P = 1.8$

$b = \frac{360}{1.8} = 200$

$h = 1.2$

$\frac{P}{4} = \frac{1.8}{4} = 0.45$



$y = 22\cos[200(x - 1.2)] + 46$

⑤ c) $y = \frac{1}{2}\cos(1\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

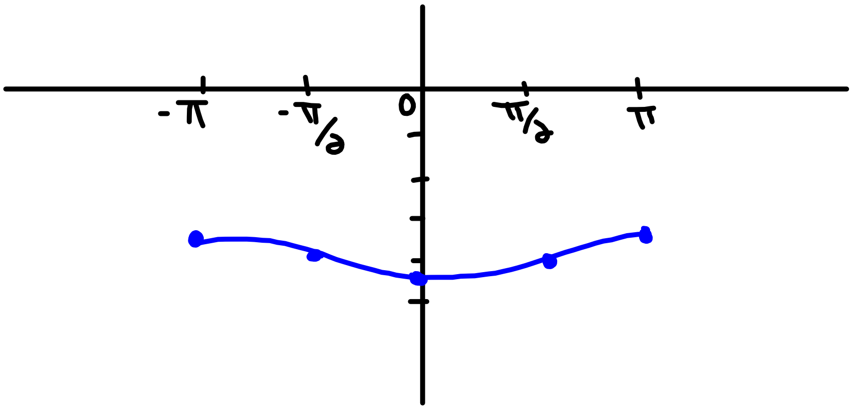
$(x, y) \rightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$\rightarrow$

x	y	
$-\pi$	$-7/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5



$$\frac{1}{\sec^{2}\theta\cot\theta}=\frac{\sin\theta-\sin^{3}\theta}{\cos\theta}$$