Chapter Review For Final Exam:

Ch.1 → (Inverse Functions)

Ch.2 → (Radical Functions) $y = a\sqrt{b(x-h)} + k$

Ch.7 → (Exponential Functions) $y = 2^x$

Ch.8 → (Logarithmic Functions) $y = \log_2 x$

Ch.4 → (Trig & Unit Circle)

Ch.5 → (Trig Functions)

Ch.6 → (Trig Identities)

Ch.2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

$\{x|x \geq h, x \in R\}$ or $\{x|x \leq h, x \in R\}$

$\{y|y \geq k, y \in R\}$ or $\{y|y \leq k, y \in R\}$

Ch.7

$y = 3^x$

x	y
-2	1/9
-1	1/3
0	1
1	3
2	9

$\{x|x \in R\}$

$\{y|y > k, y \in R\}$ $\{y|y < k, y \in R\}$

Ch.8

$y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

$\{x|x > h, x \in R\}$ $\{x|x < h, x \in R\}$

$\{y|y \in R\}$

Ch.5

$y = \sin x$

x	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$y = \cos x$

x	y
0	1
π/2	0
π	-1
3π/2	0
2π	1

$\{x|x \in R\}$

$\{y|min \leq y \leq max, y \in R\}$

Chapter 3 Radical Functions

$$\sqrt{x+8} - 6 = x$$

Solve for x: $\left(\sqrt{x+8}\right)^2 = (x+6)^2$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2+12x+36$$

$$0 = x^2+11x+28$$

$7 \times 4 = 28$
$7 + 4 = 11$

$$0 = (x+7)(x+4)$$

$x+7=0$ | $x+4=0$

$x=-7$ | $x=-4$

Test $x=-4$ is a solution

$\sqrt{x+8} = x+6$

$\sqrt{-4+8}$ | $-4+6$

2 | 2 ✓

Test $x=-7$ is extraneous

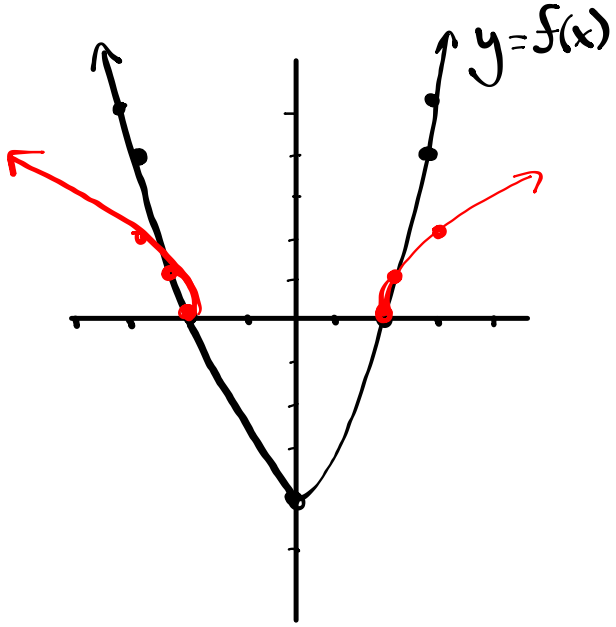
$\sqrt{x+8} = x+6$

$\sqrt{-7+8}$ | $-7+6$

1 | -1 X

Ch. 2

③ Using the graph of $y=f(x)$, sketch the graph of $y=\sqrt{f(x)}$. state the domain and range of each.



$y=f(x)$

D: $\{x|x\in\mathbb{R}\}$ or $(-\infty,\infty)$

R: $\{y|y\geq -4, y\in\mathbb{R}\}$ or $[-4,\infty)$

$y=\sqrt{f(x)}$

D: $\{x|x\leq -3, x\geq 3, x\in\mathbb{R}\}$

$(-\infty,-3]$ and $[3,\infty)$

R: $\{y|y\geq 0, y\in\mathbb{R}\}$ or $[0,\infty)$

Chapter 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

$\frac{\log 64}{\log 2} = 6$

$2^{2x+2} = 2^6$

$2x+2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

b) $9^{2x+1} = 81(27^x)$

$\frac{\log 9}{\log 3} = 2$ $\frac{\log 81}{\log 3} = 4$ $\frac{\log 27}{\log 3} = 3$

$(3^2)^{2x+1} = 3^4(3^3)^x$

$3^{4x+2} = 3^4 \cdot 3^{3x}$

$3^{4x+2} = 3^{3x+4}$

$4x+2 = 3x+4$

$x = 2$

Ex: $y = 3(2)^{2x+8} - 1$

$y = 3(2)^{2(x+4)} - 1$

$a = 3$ → vertical stretch by a factor of 3
no vertical reflection

$b = 2$ → horizontal compression by a factor of $\frac{1}{2}$
no horizontal reflection

$h = -4$ → translate left 4 units

$k = -1$ → " down 1 unit

$(x, y) \rightarrow \left(\frac{1}{2}x - 4, 3y - 1\right)$

$y = 2^x$

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

$y = 3(2)^{2(x+4)} - 1$

x	y
-5	$-\frac{1}{4}$
(-4.5) $-\frac{9}{2}$	$\frac{1}{2}$
-4	2
(-3.5) $-\frac{7}{2}$	5
-3	11

$3\left(\frac{1}{4}\right) - 1$

$\frac{3}{4} - \frac{4}{4} = \frac{-1}{4}$

$3\left(\frac{1}{2}\right) - 1$

$\frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

Ch. 8 → logarithms

4. Rewrite each expression as a single logarithm.

$3\log_5 x + \frac{1}{2}\log_5(x-1) - \log_5(x^2+1)$

$\log_5 x^3 + \log_5 (x-1)^{1/2} - \log_5 (x^2+1)$

$\log_5 \left(\frac{x^3 (x-1)^{1/2}}{x^2+1} \right)$

$\log_5 \frac{x^3\sqrt{x-1}}{x^2+1}$

$\log_2 \left(\frac{x^2}{y^3 \sqrt[4]{z}} \right)$

$\log_2 x^2 - \log_2 y^3 - \log_2 z^{1/4}$

$2\log_2 x - 3\log_2 y - \frac{1}{4}\log_2 z$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$\log_{10}(x+2)+\log_{10}(x-1)=1$

$\log_{10}(x+2)(x-1) = 1$

$\log_{10}(x^2-x+2x-2) = 1$

$\log_{10}(x^2+x-2) = 1$ (log)

$10^1 = x^2+x-2$ (exp)

$10 = x^2+x-2$

$0 = x^2+x-12$ $\quad -3 \times 4 = -12$ $\quad -3+4=1$

b
1x12
2x6
3x4

$0 = (x-3)(x+4)$

$x-3=0$ | $x+4=0$

$x=3$ | $x=-4$ is extraneous

test $x=3$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$ ✓

$\log_{10}5+\log_{10}2$

$\log_{10}10$

1 ✓

test $x=-4$

$\log_{10}(x+2)+\log_{10}(x-1) = 1$

$\log_{10}(-2)+\log_{10}(-5)$

not possible

$\therefore$

Ch.7 or Ch.8

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

Base = ½ = 0.5 exp = $t/5.3$ Initial Amount = 60

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$M(t) = 60(0.5)^{t/5.3}$ | $y = 60(0.5)^{x/5.3}$

b) What amount will be present in 10.6 years? $t = 10.6$ [2]

$M(t) = 60(0.5)^{10.6/5.3}$

$M(t) = 60(0.5)^{2}$

$M(t) = 60(0.25) = 15$ mg

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

- 12.5% of 60mg

0.125×60

7.5 mg

$M(t) = 60(0.5)^{t/5.3}$

$\frac{7.5}{60} = \frac{60(0.5)^{t/5.3}}{60}$

$0.125 = (0.5)^{t/5.3}$

$\frac{\log 0.125}{\log 0.5} = 3$

$(0.5)^3 = (0.5)^{t/5.3}$

$5.3 \cdot 3 = \frac{t}{5.3} \cdot 5.3$

15.9 years = t

③ b) $(32)^{-x+1} = \sqrt{256}\left(\frac{1}{8}\right)^{2x}$

$(2^5)^{-x+1} = 16\,(2^{-3})^{2x}$

$2^{-5x+5} = 2^4(2^{-6x})$

$2^{-5x+5} = 2^{4-6x}$

$-5x+5 = 4-6x$

$\boxed{x = -1}$.

Ch. 4 → Trig Equation

2. Solve for all values of θ in the specified domain.

$\tan^2\theta + \tan\theta = 0,\ 0 \le \theta \le 2\pi$ (Radians)

$\tan\theta(\tan\theta + 1) = 0$

S	A
T	C

$\tan\theta = 0$ (Unit Circle)

$\theta = 0, \pi, 2\pi$

$\tan\theta + 1 = 0$

$\tan\theta = -1$ (Special Triangle)

$\bar{\theta} = \tan^{-1}(1)$

$\bar{\theta} = \frac{\pi}{4}$

where is $\tan\theta < 0$

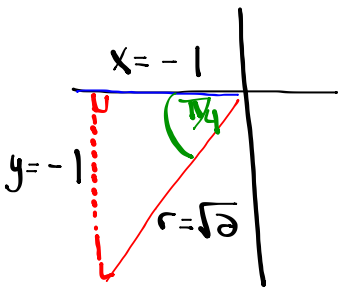
Q2	Q4
$\theta = \pi - \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4}$
$\theta = \frac{3\pi}{4}$	$\theta = \frac{7\pi}{4}$

e. $\cos^2\theta + \frac{1}{2}\cos\theta = 0,\ 0° \le \theta < 360°$

Ch. 4 → Trig Expression

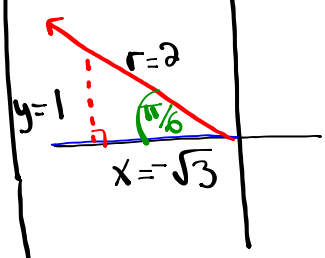
* $\dfrac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$

$\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$ → 1π



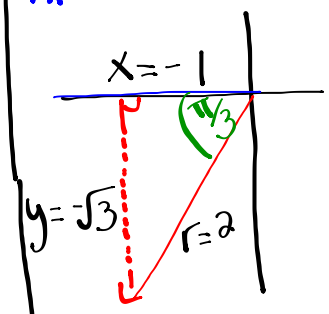
$\tan\frac{5\pi}{4} = \frac{-1}{-1} = 1$

$\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$ → 1π



$\sin\frac{5\pi}{6} = \frac{1}{2}$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ → 1π



$\sin\frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$

$$\frac{5\tan^2 5\pi/4}{6\sin 5\pi/6 + 4\sin 4\pi/3}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right) + 4\left(\frac{-\sqrt{3}}{2}\right)} \rightarrow -\frac{4\sqrt{3}}{2} = -2\sqrt{3}$$

$$\frac{5}{(3-2\sqrt{3})} \cdot \frac{(3+2\sqrt{3})}{(3+2\sqrt{3})}$$

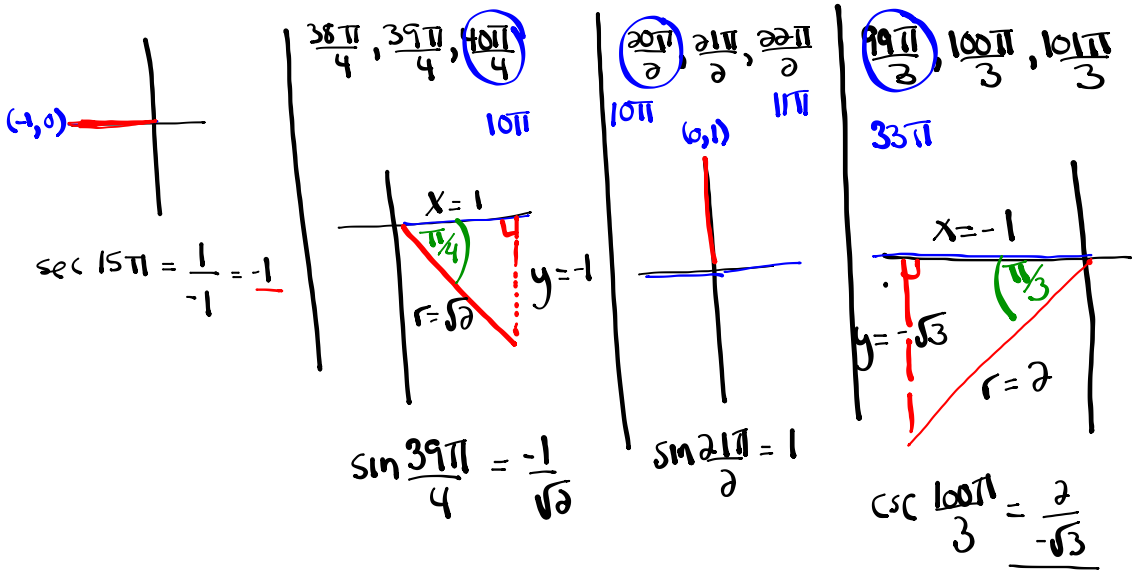
$(-2\sqrt{3})(2\sqrt{3}) = -4\sqrt{9}$

$$\frac{15+10\sqrt{3}}{9+6\sqrt{3}-6\sqrt{3}-4(3)}$$

$$\frac{15+10\sqrt{3}}{-3} \quad \text{or} \quad \frac{-15-10\sqrt{3}}{3}$$

Ch. 4

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$



$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)(1) - \left(-\frac{2}{\sqrt{3}}\right)^2$$

$$-1 - 1 - \frac{4}{3}$$

$$-2 - \frac{4}{3}$$

$$-\frac{6}{3} - \frac{4}{3}$$

$$\boxed{-\frac{10}{3}}$$

$0 \le \theta \le 2\pi$

⑫ a) $\sin\theta = \sin\theta\tan\theta$ ← Common Factor

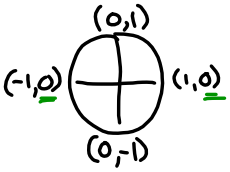
$0 = \sin\theta\tan\theta - \sin\theta$

$0 = (\sin\theta)(\tan\theta - 1)$

$\sin\theta = 0$ | $\tan\theta - 1 = 0$

$\theta = 0, \pi, 2\pi$ | $\tan\theta = 1$

$\theta_R = \frac{\pi}{4}$



Where is tanθ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$

Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

⑬ b) $3\sin^2\theta - 2\sin\theta - 1 = 0$, $0 \le \theta \le 360°$

$(3\sin^2\theta - 3\sin\theta) + (+\sin\theta - 1) = 0$

$-3 \times 1 = -3$

$-3 + 1 = -2$

$3\sin\theta(\sin\theta - 1) + 1(\sin\theta - 1) = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$3\sin\theta + 1 = 0$ | $\sin\theta - 1 = 0$

$\sin\theta = -\frac{1}{3}$ | $\sin\theta = 1$ → Unit Circle

$\theta = 90°$

$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$

$\theta_R = 19$

Where is sine negative:

Q3	Q4
$\theta = 180° + \theta_R$	$\theta = 360° - \theta_R$
$\theta = 180° + 19$	$\theta = 360° - 19$
$\theta = 199$	$\theta = 340$

Ch. 5 → Trig Functions

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

max to min = half the P

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

max = 50
min = 30

$k = \frac{50+30}{2} = 40$

Amp = max − k
Amp = 50 − 40
Amp = 10
$a = \pm 10$

$P = 2(1.8 - 0.4)$
$P = 2(1.4)$
$P = 2.8$

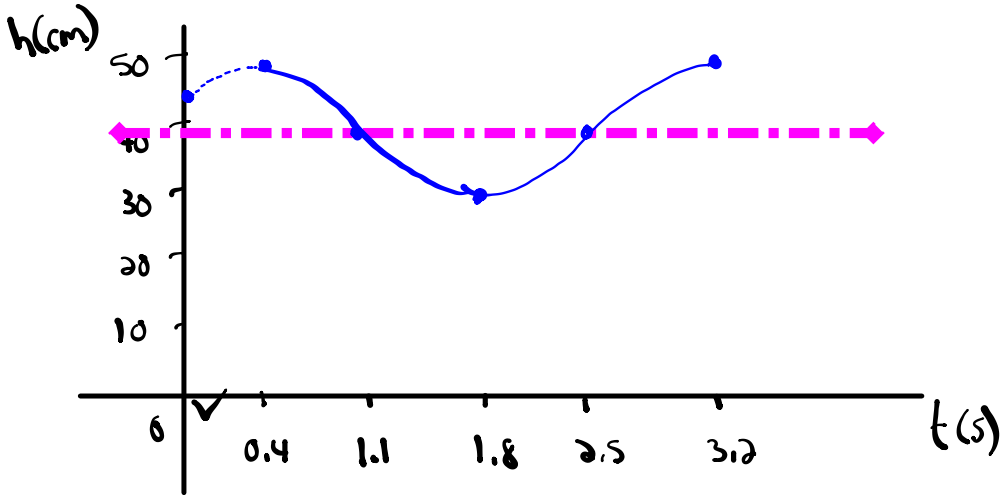
$b = \frac{360}{2.8} = 128.57$

$h = 0.4$

$y = 10\cos[128.57(17.2 - 0.4)] + 40 = 50 \text{ cm}$

(b) How high was the weight above the floor when the stopwatch was initially started? (t = 0)

$y = 10\cos[128.57(0 - 0.4)] + 40 = 46.2 \text{ cm}$



Count by $\frac{P}{4} = \frac{2.8}{4} = 0.7$

① Amp = 11 P = 16 min = -4

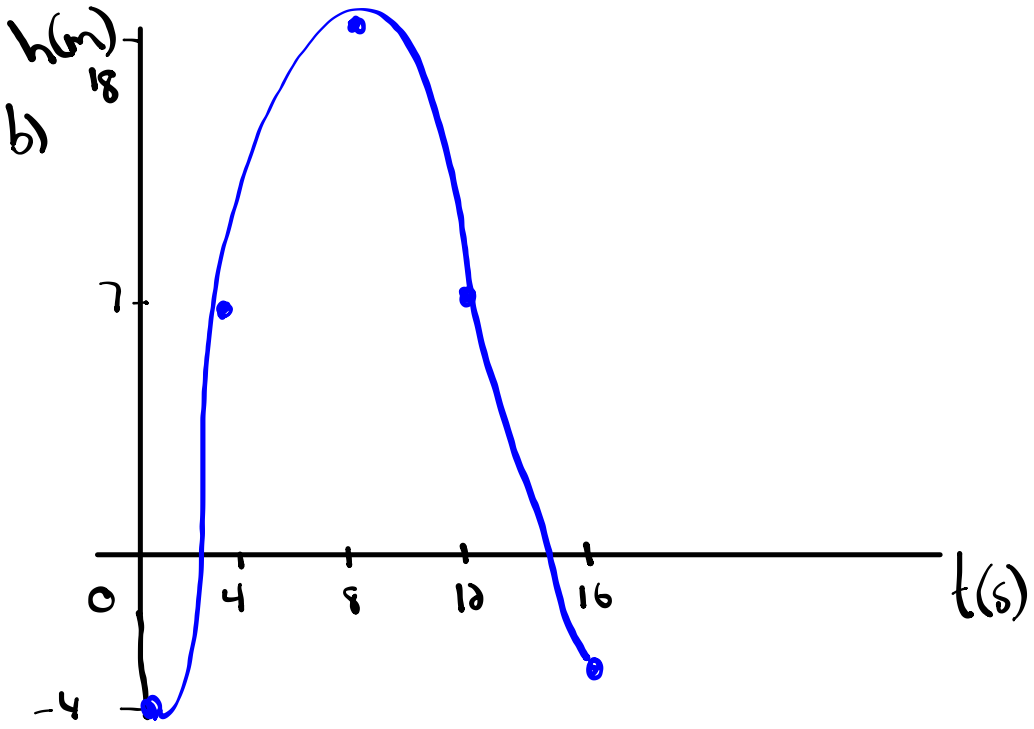
$a = \pm 11$ $b = \frac{360}{16} = 22.5$ max = -4 + 22 = 18

K = -4 + 11 = 7

h = 0

a) equation: $y = -11\cos[22.5(x)] + 7$

b)



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④ max = 68

min = 24

$k = \frac{68+24}{2} = 46$

Amp = 68 − 46 = 22

$a = \pm 22$

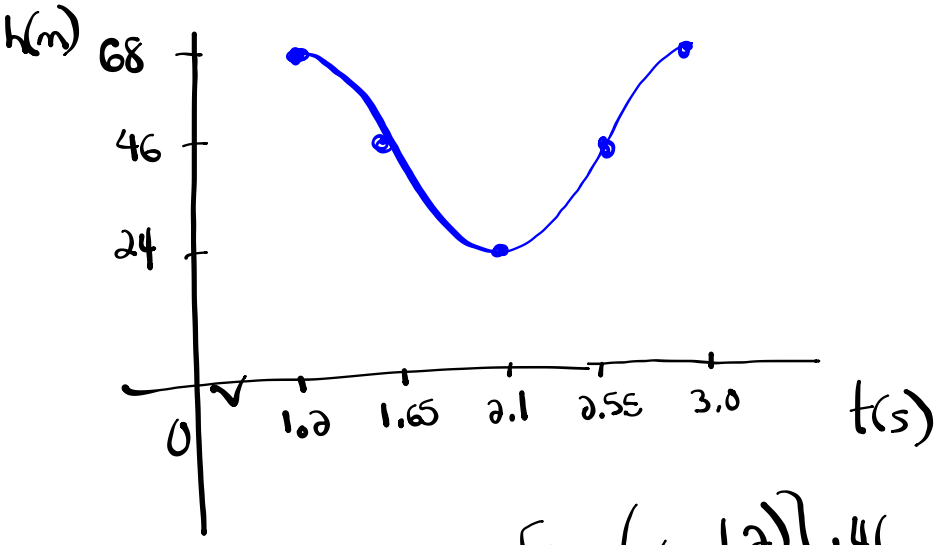
$P = 2(2.1 - 1.2)$

$P = 1.8$

$b = \frac{360}{1.8} = 200$

$h = 1.2$

$\frac{P}{4} = \frac{1.8}{4} = 0.45$



$y = 22\cos[200(x - 1.2)] + 46$

⑤ c) $y = \frac{1}{2}\cos(\theta + \pi) - 4$

$a = \frac{1}{2}$

$b = 1$

$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

$c = -\pi$

$d = -4$

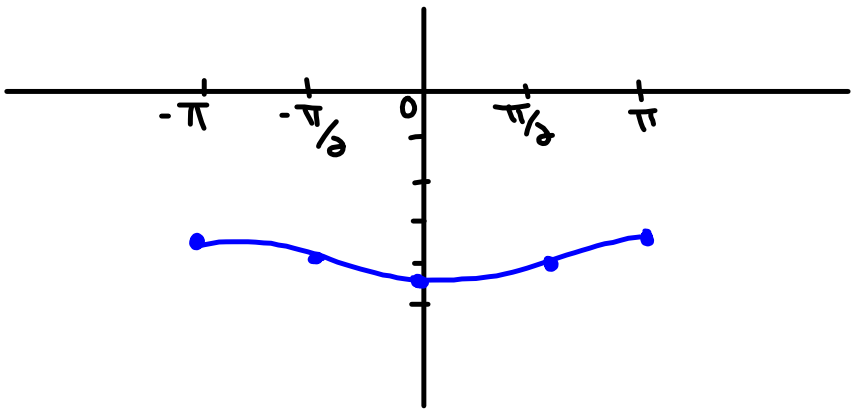
$(x, y) \rightarrow \left(\frac{1}{1}x - \pi, \frac{1}{2}y - 4\right)$

$y = \cos\theta$

x	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

→

x	y	
$-\pi$	$-1/2$	-3.5
$-\frac{\pi}{2}$	-4	-4
0	$-9/2$	-4.5
$\pi/2$	-4	-4
π	$-7/2$	-3.5



* $\dfrac{1}{\sec^2\theta\cot\theta} = \dfrac{\sin\theta - \sin^3\theta}{\cos\theta}$

$\dfrac{1}{\sec^2\theta} \cdot \dfrac{1}{\cot\theta}$

$\cos^2\theta \quad \tan\theta$

$\cos^2\theta \quad \dfrac{\sin\theta}{\cos\theta}$

$\sin\theta\cos\theta$

$\dfrac{\sin\theta(1-\sin^2\theta)}{\cos\theta}$

$\dfrac{\sin\theta(\cos^2\theta)}{\cos\theta}$

$\sin\theta\cos\theta$