

## Series &amp; Sequence

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\textcircled{1} \quad t_{12} = 15$$

$$t_{12} = a + (12-1)d$$

$$t_{12} = a + 11d$$

$$15 = a + 11d$$

$$a + 11d = 15$$

$$S_{15} = 105$$

$$S_{15} = \frac{15}{2}[2a + (15-1)d]$$

$$S_{15} = 7.5[2a + 14d]$$

$$S_{15} = 15a + 105d$$

$$105 = 15a + 105d$$

$$15a + 105d = 105$$

$$a + 7d = 7$$

## Elimination

$$a + 11d = 15$$

$$\Leftrightarrow a + 7d = 7$$

$$\frac{4d = 8}{4 \quad 4}$$

$$d = 2$$

↳

$$a + 7d = 7$$

$$a + 7(2) = 7$$

$$a + 14 = 7$$

$$a = -7$$

$$t_1 = -7$$

$$t_2 = -5$$

$$t_3 = -3$$

## Permutations + Combinations

- ① 3 digit # from 0, 1, 2, 8, and 9  
with no repetitions

$$\underline{4} \times \underline{4} \times \underline{3} = 48 \quad C$$

↑  
can't  
use 0  
as digit  
one

②  $\left(y - \frac{2}{y^2}\right)^5$

$${}^5C_0(y)^5\left(\frac{-2}{y^2}\right)^0 + {}^5C_1(y)^4\left(\frac{-2}{y^2}\right)^1 + {}^5C_2(y)^3\left(\frac{-2}{y^2}\right)^2 + {}^5C_3(y)^2\left(\frac{-2}{y^2}\right)^3 + {}^5C_4(y)^1\left(\frac{-2}{y^2}\right)^4 + {}^5C_5(y)^0\left(\frac{-2}{y^2}\right)^5$$

$$1(y^5)(1) + 5(y^4)\left(\frac{-2}{y^2}\right) + 10(y^3)\left(\frac{4}{y^4}\right) + 10(y^2)\left(\frac{-8}{y^6}\right) + 5(y)\left(\frac{16}{y^8}\right) + 1(1)\left(\frac{-32}{y^{10}}\right)$$

$$\boxed{y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}}$$

$$\textcircled{5} \quad (\underline{2x^2} + \underline{3y})^7 \rightarrow 3^{\text{rd}} \text{ term}$$

$${}_{7}C_2 (\underline{2x^2})^5 (\underline{3y})^2$$

$$21 (32x^{10})(9y^2)$$

$$\boxed{6048x^{10}y^2}$$

$$\textcircled{3} \quad y = \sqrt[7]{2x^2 + \sqrt{x^2 - 8x} \sqrt{3-x}} = \left[ 2x^2 + (x^2 - 8x)^{\frac{1}{2}} (3-x)^{\frac{1}{2}} \right]^{\frac{1}{7}}$$

$$y' = \frac{1}{7} \left[ 2x^2 + (x^2 - 8x)^{\frac{1}{2}} (3-x)^{\frac{1}{2}} \right]^{-\frac{6}{7}} \left[ 4x + \frac{1}{2} (x^2 - 8x)^{-\frac{1}{2}} (7x^6 - (8(3-x)^{\frac{1}{2}} + 8x \left(\frac{1}{2}\right) (3-x)^{-\frac{1}{2}}) \right)$$

$$\textcircled{2} \text{ Let } f(x) = \begin{cases} -x^2 + 2, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ -2x + 3, & \text{if } 1 < x \leq 3 \\ -3, & \text{if } x > 3 \end{cases}$$

$$-x^2 + 2$$

x	y
1	1
0	2
-1	1
-2	-2

$$1$$

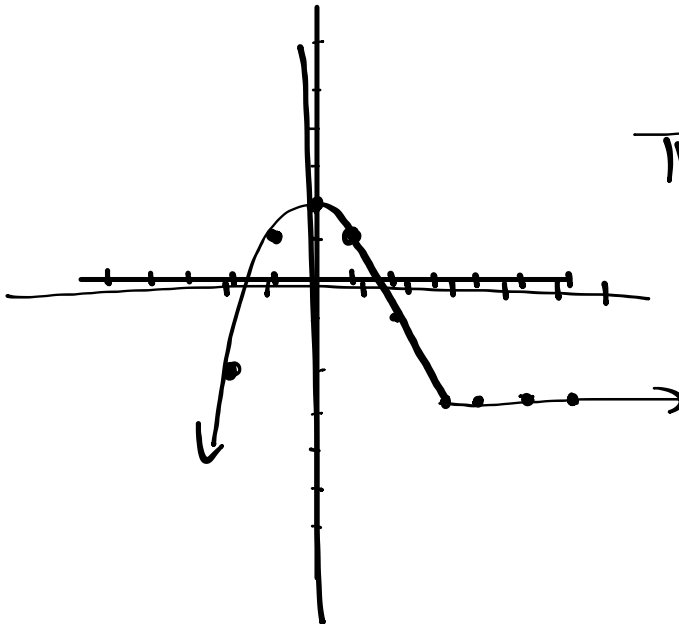
x	y
1	1

$$-2x + 3$$

x	y
1	1
2	-1
3	-3

$$-3$$

x	y
3	-3
4	-3
5	-3
6	-3



The function is continuous

$$\textcircled{a} \left(x^2 - \frac{x}{2}\right)^5$$

$${}_5C_0(x^2)^5\left(\frac{-x}{2}\right)^0 + {}_5C_1(x^2)^4\left(\frac{-x}{2}\right)^1 + {}_5C_2(x^2)^3\left(\frac{-x}{2}\right)^2 + {}_5C_3(x^2)^2\left(\frac{-x}{2}\right)^3 + {}_5C_4(x^2)^1\left(\frac{-x}{2}\right)^4 + {}_5C_5(x^2)^0\left(\frac{-x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(\frac{-x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(\frac{-x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(\frac{-x^5}{32}\right)$$

$$x^{10} - \frac{5x^9}{2} + \frac{5x^8}{2} - \frac{5x^7}{4} + \frac{5x^6}{16} - \frac{x^5}{32}$$

④ c) Case 1 (All Black)

$${}_{26}C_5 = \boxed{65\,780}$$

Case 2 (4 Black + 1 Red)

$${}_{26}C_4 \times {}_{26}C_1 = 14\,950 \times 26 = \boxed{388\,700}$$

Case 3 (3 black + 2 red)

$${}_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845\,000}$$

$$\text{Total} = 845\,000 + 388\,700 + 65\,780 = \boxed{1\,299\,480}$$

⑤ 14 Letters

- 5 are burnt
- 9 are good

3 good + 2 bad

$${}^9C_3 \times {}^5C_2$$

$$84 \times 10$$

$$840$$



## Derivatives Exam Review:

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{\frac{3}{2}x^{-1/2} + \frac{1}{2}x^{3/2} - 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3+x^2-4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

$$\textcircled{5} \text{ c) } \underline{1} + \underline{5} + \underline{9} + \dots + \underline{77}$$

$\begin{array}{cc} \vee & \vee \\ 4 & 4 \end{array}$

$$a = 1$$

$$t_n = 77$$

$$d = 4$$

① Find  $n$ :

$$t_n = a + (n-1)d$$

$$77 = 1 + (n-1)4$$

$$76 = 4(n-1)$$

$$19 = n-1$$

$$\boxed{20 = n}$$

② Find  $S_{20}$

$$S_{20} = \frac{20}{2} (1 + 77)$$

$$= 10(78)$$

$$\boxed{= 780}$$

$$\textcircled{5} \text{ b) } \sum_{n=1}^{\infty} \underline{3} \left( \frac{1}{2} \right)^{n-1} = \underline{3} + \frac{3}{2} + \frac{3}{4} + \dots$$

$\begin{array}{cc} \vee & \vee \\ 2 & 4 \\ \frac{1}{2} & \frac{1}{2} \end{array}$

$$a = 3$$

$$r = \frac{1}{2}$$

$$S_n = \frac{3}{1 - \frac{1}{2}} = \frac{3}{\frac{1}{2}} = 3 \cdot 2 = \textcircled{6}$$

$$\textcircled{4} \text{ c) } \lim_{n \rightarrow \infty} (-1)^{n+1} n^2$$

$$= 1, -4, 9, -16, 25, -36, 49, -64, 81, -100$$

= Diverging (has no limit)

$$- , \frac{1}{5} , - , - , \frac{25}{5} , - , -$$

$$t_2 = \frac{1}{5} \quad , \quad t_5 = 25$$

$$t_2 = ar^{2-1} \quad t_5 = ar^{5-1}$$

$$t_2 = ar^1 \quad t_5 = ar^4$$

$$\frac{1}{5} = ar \quad 25 = ar^4$$

$$ar = \frac{1}{5}$$

$$ar^4 = 25$$

$$\frac{ar^4 = 25}{ar = \frac{1}{5}}$$

$$r^3 = 125$$

$$r = 5$$

$$ar = \frac{1}{5}$$

$$a(5) = \frac{1}{5}$$

$$5a = \frac{1}{5}$$

$$a = \frac{1}{25}$$

Expand:  $(\underline{x} - \underline{\frac{1}{2}})^{\textcircled{4}}$

$${}^4C_0(\cancel{x})^4\left(\frac{1}{2}\right)^0 + {}^4C_1(\cancel{x})^3\left(\frac{1}{2}\right)^1 + {}^4C_2(\cancel{x})^2\left(\frac{1}{2}\right)^2 + {}^4C_3(\cancel{x})^1\left(\frac{1}{2}\right)^3 + {}^4C_4(\cancel{x})^0\left(\frac{1}{2}\right)^4$$

$$(1)(x^4)(1) + (4)(x^3)\left(-\frac{1}{2}\right) + (6)(x^2)\left(\frac{1}{4}\right) + (4)(x)\left(-\frac{1}{8}\right) + (1)(1)\left(\frac{1}{16}\right)$$

$$x^4 - \frac{4x^3}{2} + \frac{6x^2}{4} - \frac{4x}{8} + \frac{1}{16}$$

$$x^4 - 2x^3 + \frac{3x^2}{2} - \frac{1}{2}x + \frac{1}{16}$$

$$f(x) = 3x^2 + 2x - 7$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 7$$

$$= 3(x^2 + 2xh + h^2) + 2x + 2h - 7$$

$$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 7$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 7 - (3x^2 + 2x - 7)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(6x + 3h + 2)}{h} = 6x + 2$$

$$f(x) = \frac{(3x^2+5)^3}{\sqrt{2x-7}}$$

$$f'(x) = \frac{\overbrace{3(3x^2+5)^2}^{f'} \cdot \overbrace{(6x)(2x-7)^{1/2}}^g} - \overbrace{(3x^2+5)^3}^f \cdot \overbrace{(\frac{1}{2})(2x-7)^{-1/2}}^{g'}}{[\sqrt{2x-7}]^2}$$

$$f'(x) = \frac{18x(3x^2+5)^2(2x-7)^{1/2} - (3x^2+5)^3(2x-7)^{-1/2}}{2x-7}$$

← Factor

$$f'(x) = \frac{(3x^2+5)^2(2x-7)^{-1/2} \left[ 18x(2x-7) - (3x^2+5) \right]}{(2x-7)}$$

$$f'(x) = \frac{(3x^2+5)^2(33x^2-126x-5)}{(2x-7)^{3/2}}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{3(x+3)} \frac{1}{\cancel{x+3}} - \frac{1}{\cancel{3}} \cancel{3(x+3)}}{x \cdot 3(x+3)} \quad \text{CD: } 3(x+3)$$

$$\lim_{x \rightarrow 0} \frac{3 - (x+3)}{3x(x+3)}$$

$$\lim_{x \rightarrow 0} \frac{3 - x - 3}{3x(x+3)}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{-x}}{\underline{3x}(\underline{x+3})} = \frac{-1}{3(3)} = \frac{-1}{9}$$