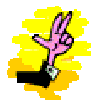

The Fundamental Counting Principle

The principle which states that all possible outcomes in a *sample space* can be found by multiplying the number of ways each event can occur.

Example 1: A deli has a lunch special which consists of a sandwich, soup, dessert and drink for $4.99.

They offer the following choices:

(4) **Sandwich**: chicken salad, ham, tuna, and roast beef
(3) **Soup**: tomato, chicken noodle, vegetable
(2) **Dessert**: cookie and pie
(5) **Drink**: tea, coffee, coke, diet coke and sprite

How many lunch specials are there?

$4 \times 3 \times 2 \times 5 = 120$

Let's use the basic counting principle

There are 4 stages or events: choosing a sandwich, choosing a soup, choosing a dessert and choosing a drink.

There are **4 choices for the sandwich, 3 choices for the soup, 2 choices for the dessert and 5 choices for the drink**

Putting that all together we get:

Sand.		Soup		Dessert		Drink		# of lunch specials
4	**x**	**3**	**x**	**2**	**x**	**5**	**=**	**120**

So there are 120 lunch specials possible

Example 2: A company places a 6-symbol code on each unit of product. The code consists of 4 digits, the first of which is the number 5, follov by 2 letters, the first of which is NOT a vowel. How many different codes are possible?

Let's use the basic counting principle

There are 6 stages or events: digit 1, digit 2, digit 3, digit 4, letter 1, and letter 2.

In general there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The first digit is limited being the number 5, so there is only one possibility for that one. There are no restriction on digits 2 - 4, so each one of those has 10 possibilities.
In general, there are 26 letters in the alphabet. The first letter, cannot be a vowel (a i, o, u), so that means there are 21 possible letters that could go there. The second letter has no restriction, so there are 26 possibilities for that one.

Putting that all together we get:

digit 1		digit 2		digit 3		digit 4		letter 1		letter 2		# of codes
1	x	10	x	10	x	10	x	21	x	26	=	546000

So there are 546000 different 6-symbol codes possible

Factorial Notation

(multiply) (in a row) (counting #'s)

The product of consecutive natural numbers, in decreasing order to the number 1, can be represented using ***factorial notation.***

(largest to smallest)

The symbol for factorial is the exclamation mark, !.

Ex: $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$3! = 3 \times 2 \times 1 = 6$

Example 1

Calculate the following:

a) 5! $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

b) 3! $= 3 \times 2 \times 1 = 6$

c) 10! $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$

Solution

a) 5! is read as "five factorial" b) 3! is read as "three factorial"

5! = 5 x 4 x 3 x 2 x 1
= 120

3! = 3 x 2 x 1
= 6

c) 10! is read as "ten factorial"

10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
= 3 628 800

Fortunately, all scientific calculators have a factoral key so we don't have to manually do the calculations.

Example 2

Simplify the following:

a) $\frac{14!}{8!} = \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times \cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{8} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$

NOTE: The factors 8, 7, 6, ... , 1 in the denominator of the fraction can cancel with the like factors in the numerator of the fraction. So all we are left with is:
$14 \times 13 \times 12 \times 11 \times 10 \times 9 = 2\ 162\ 160$

b) $\frac{9!}{4!} = 9 \times 8 \times 7 \times 6 \times 5$

$= 15\ 120$

ex: ① $8 \times 7 \times 6 = \frac{8!}{5!}$

② $10 \times 9 \times 8 \times 5 \times 4 \times 3 \times 2 \times 1 = \frac{10!}{7!} \times 5!$

$= \frac{10!\,5!}{7!}$

$16 \times 15 \times 14 \times 10 \times 9 \times 8 \times 3 \times 2 \times 1 = \frac{16!}{13!} \times \frac{10!}{7!} \times 3!$

$= \frac{16!\,10!\,3!}{13!\,7!}$

Example 3

Write 12 x 11 x 10 x 9 as a ratio/quotient of factorials.

Solution

The expression contains the first four terms of 12! We can write an expression that would divide the remaining terms.

$$\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= \frac{12!}{8!}$$

Example 4

Write 10 x 9 x 8 x 3 x 2 x 1 as a ratio of factorials.

Solution

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} \times 3 \times 2 \times 1$$

$$= \frac{10!}{7!} \times 3! \text{ or } \frac{10!\,3!}{7!}$$

NOTE: By definition $0! = 1$

Fundamental Counting Principle

1. The Tiffany Restaurant offers the following menu:

Main Course	Dessert	Beverage
Chicken	Cheesecake	Coffee
Ham	Ice Cream	Tea
Steak	Apple Pie	Milk
Prime Rib		Lemonade
Roast Pork		
5	3	4

$5 \times 3 \times 4 = \underline{60}$

In how many ways can a customer order a meal consisting of one choice from each category?

2. How many license plates can be made using 3 letters followed by 3 digits if repetitions are allowed?
   $26 \times 26 \times 26 \times 10 \times 10 \times 10 = \underline{17\,576\,000}$
3. In how many ways can a 15-member club elect a president, a vice-president and a secretary?
   $15 \times 14 \times 13 = \underline{2730}$
4. A club consists of 10 grade 12 students and 8 grade 11 students. In how many ways can this club elect a president, a vice-president and a secretary if:

   a) the president must be in grade 12 and the other officers in grade 11?
   $10 \times 8 \times 7 = \underline{560}$

   b) the president and vice-president must be in grade 12 and the secretary must be in grade 11?
   $10 \times 9 \times 8 = \underline{720}$
5. In a combination lock there are 60 different positions. To open the lock you move to a certain number in the clockwise direction, then to a number in the counterclockwise direction and finally to a third number in the clockwise direction. How many different combinations are there?
   $60 \times 60 \times 60 = \underline{216\,000}$

Factorial Notation

6. Evaluate. Which express on produces the largest number? Smallest number?

   a) $6! = 720$
   b) $\frac{15!}{12!} = 2730$
   c) $\frac{9!}{2!} = 181\,440$
   d) $\frac{9!}{7!} = 72$
   e) $4! - 3! = 24 - 6 = 18$
   f) $11! = 39\,916\,800$
   g) $\frac{3!}{4} = \frac{6}{4} = \frac{3}{2}$
   h) $\frac{10!}{2!} = 1\,814\,400$
   i) $\frac{7!}{6!} = 7$

   $11!$ is the largest

   $\frac{3!}{4}$ is the smallest

$10 \times 8 \times 7 = \underline{560}$

b) the president and vice-president must be in grade 12 and the secretary must be in grade 11?

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11! is the largest

$\frac{3!}{4}$ is the smallest

7. Write each of the following as a ratio of factorials.

a) 7 x 6 x 5
b) 10 x 9 x 8 x 7 x 6
c) 10 x 9 x 8
d) 12 x 11 x 10 x 9
e) 30 x 29 x 28 x 3 x 2 x 1
f) 25 x 24 x 23 x 22 x 4 x 3 x 2 x 1
g) 40 x 39 x 38 x 9 x 8 x 7
h) 66 x 65 x 64 x 63 x 27 x 26
i) 101 x 100 x 99 x 60 x 59 x 58
j) 85 x 84 x 83 x 11 x 9 x 8 x 4 x 3 x 2 x 1
k) 50 x 49 x 48 x 16 x 15 x 14 x 2 x 1
l) 41 x 40 x 39 x 27 x 26 x 25 x 8 x 7 x 6

a) $\frac{7!}{4!}$ b) $\frac{10!}{5!}$ c) $\frac{10!}{7!}$ d) $\frac{12!}{8!}$

e) $\frac{30!\,3!}{27!}$ f) $\frac{25!\,4!}{21!}$ g) $\frac{40!\,9!}{37!\,6!}$

h) $\frac{66!\,27!}{62!\,25!}$ i) $\frac{101!\,60!}{98!\,57!}$ j) $\frac{85!\,11!\,9!\,4!}{82!\,10!\,7!}$

k) $\frac{50!\,16!\,2!}{47!\,13!}$ l) $\frac{41!\,27!\,8!}{38!\,24!\,5!}$