Questions from Homework

Velocity

④ $h = 24.5t - 4.9t^2$

$v = 24.5 - 9.8t$

b) Let $v = 0$

$0 = 24.5 - 9.8t$

$9.8t = 24.5$

$t = 2.5s$

c) Find $h(2.5)$

$h = 24.5(2.5) - 4.9(2.5)^2$

$h = 30.6m$

d) Let $h = 0$

$0 = 24.5t - 4.9t^2$

$0 = t(24.5 - 4.9t)$

$t = 0$ | $24.5 - 4.9t = 0$

$24.5 = 4.9t$

$\boxed{5 = t}$

④ d) $s = \frac{5t}{1+t}$

$$v = \frac{(1+t)(5) - 5t(1)}{(1+t)^2}$$

$$v = \frac{5+5t-5t}{(1+t)^2} = \frac{5}{(1+t)^2}$$

$$a = \frac{(1+t)^2(0) - 5(2)(1+t)(1)}{[(1+t)^2]^2}$$

$$a = \frac{-10(1+t)}{(1+t)^4} = \frac{-10}{(1+t)^3}$$

$$a(4) = \frac{-10}{(1+4)^3} = \frac{-10}{125} = \boxed{\frac{-2}{25}\ m/s^2}$$

③ d) $s = \sqrt{t^2+t} = (t^2+t)^{1/2}$

$$v = \frac{1}{2}(t^2+t)^{-1/2}(2t+1)$$

$$v = \frac{2t+1}{2(t^2+t)^{1/2}}$$

$$a = \frac{2(t^2+t)^{1/2}(2) - (2t+1)(1)(t^2+t)^{-1/2}(2t+1)}{\left[2(t^2+t)^{1/2}\right]^2}$$

$$a = \frac{4(t^2+t)^{1/2} - (2t+1)^2(t^2+t)^{-1/2}}{4(t^2+t)}$$

FOIL

$$a = \frac{(t^2+t)^{-1/2}\left[4(t^2+t) - (2t+1)^2\right]}{4(t^2+t)}$$

$$a = \frac{4t^2+4t-4t^2-4t-1}{4(t^2+t)^{3/2}}$$

$$a = \frac{-1}{4(t^2+t)^{3/2}}$$

⑧ $s = t^3 - 15t^2 + 63t$

$v = 3t^2 - 30t + 63$

a) Let $v = 0$

$0 = 3t^2 - 30t + 63$

$0 = 3(t^2 - 10t + 21)$

$0 = 3(t-7)(t-3)$

$t-7=0$ | $t-3=0$

$t = 7s$ | $t = 3s$

b) $v > 0$

$3t^2 - 30t + 63 > 0$

$3(t-7)(t-3) > 0$

① $t-7>0$ | $t-3>0$

$t>7$ | $t>3$

② $t-7<0$ | $t-3<0$

$t<7$ | $t<3$

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Differentiate with respect to time

If $xy^2 = 12$ and $\frac{dy}{dt} = 6$, Find $\frac{dx}{dt}$ when $y = 2$

$xy^2 = 12$

$x(2)^2 = 12$

$4x = 12$

$x = 3$

$xy^2 = 12$

$x\,2y\frac{dy}{dt} + \frac{dx}{dt}y^2 = 0$

$(3)2(2)(6) + \left(\frac{dx}{dt}\right)(2)^2 = 0$

$72 + 4\frac{dx}{dt} = 0$

$4\frac{dx}{dt} = -72$

$\frac{dx}{dt} = -18$

If $x^3 + y^3 = 9$ and $\frac{dx}{dt} = 4$, Find $\frac{dy}{dt}$ when $x = 2$

$x^3+y^3=9$

$(2)^3+y^3=9$

$8+y^3=9$

$y^3=1$

$\boxed{y=1}$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$

$3(2)^2(4) + 3(1)^2\frac{dy}{dt} = 0$

$48 + 3\frac{dy}{dt} = 0$

$3\frac{dy}{dt} = -48$

$\boxed{\frac{dy}{dt} = -16}$