② b) $xy^2 = 45$ if $y = 3$ and $\frac{dy}{dt} = 6$ Find $\frac{dx}{dt}$

(i) $xy^2 = 45$

$x(3)^2 = 45$

$9x = 45$

$x = 5$

(ii) $(x)(y^2) = 45$

$\frac{dx}{dt} y^2 + x 2y \frac{dy}{dt} = 0$

(iii) $\frac{dx}{dt}(3)^2 + 2(5)(3)(6) = 0$

$9\frac{dx}{dt} + 180 = 0$

$9\frac{dx}{dt} = -180$

$\frac{dx}{dt} = -20$

Review Sheet:

① Find $\frac{dy}{dx}$ for $x^2+2xy+2y^2=4$

$$2x+2y+2x\frac{dy}{dx}+4y\frac{dy}{dx}=0$$

$$2x\frac{dy}{dx}+4y\frac{dy}{dx}=-2x-2y$$

$$\frac{dy}{dx}(2x+4y)=-2x-2y$$

$$\frac{dy}{dx}=\frac{-2x-2y}{2x+4y}=\frac{-2(x+y)}{2(x+2y)}=\boxed{-\frac{x+y}{x+2y}}$$

Review Sheet

② $y = \sqrt{x^2+2x} = (x^2+2x)^{1/2}$

$$y' = \frac{1}{2}(x^2+2x)^{-1/2}(2x+2)$$

$$y' = \frac{x+1}{(x^2+2x)^{1/2}}$$

$$y'' = \frac{(x^2+2x)^{1/2}(1) - (x+1)\left(\frac{1}{2}\right)(x^2+2x)^{-1/2}(2x+2)}{\left[(x^2+2x)^{1/2}\right]^2}$$

$$y'' = \frac{(x^2+2x)^{1/2} - (x+1)^2(x^2+2x)^{-1/2}}{(x^2+2x)}$$

$$y'' = \frac{(x^2+2x)^{-1/2}\left[(x^2+2x)' - (x+1)^2\right]}{(x^2+2x)}$$

$$y'' = \frac{x^2+2x-(x^2+2x+1)}{(x^2+2x)^{3/2}}$$

$$y'' = \frac{-1}{(x^2+2x)^{3/2}}$$

② $y = \sqrt{x^2+2x} = (x^2+2x)^{1/2}$

$y' = \frac{1}{2}(x^2+2x)^{-1/2}(2x+2)$

$y' = (x+1)(x^2+2x)^{-1/2}$

$y'' = (x+1)\left(\frac{-1}{2}\right)(x^2+2x)^{-3/2}(2x+2) + 1(x^2+2x)^{-1/2}$

$y'' = -(x+1)^2(x^2+2x)^{-3/2} + (x^2+2x)^{-1/2}$

$y'' = (x^2+2x)^{-3/2}\left[-(x+1)^2 + (x^2+2x)\right]$

$y'' = \frac{-x^2-2x-1+x^2+2x}{(x^2+2x)^{3/2}}$

$y'' = \frac{-1}{(x^2+2x)^{3/2}}$

Review Sheet

③ $xy^2 = 12$, $\frac{dy}{dt} = 6$, $y = 2$, Find $\frac{dx}{dt}$

(i) $xy^2 = 12$

$x(2)^2 = 12$

$4x = 12$

$x = 3$

(ii) $xy^2 = 12$

$\frac{dx}{dt}y^2 + 2xy\frac{dy}{dt} = 0$

(iii) $\frac{dx}{dt}(2)^2 + 2(3)(2)(6) = 0$

$4\frac{dx}{dt} + 72 = 0$

$4\frac{dx}{dt} = -72$

$\frac{dx}{dt} = -18$

④ $-3xy^2 - x^3 = 3y$

$-3x2yy' - 3y^2 - 3x^2 = 3y'$

$-6xyy' - 3y' = 3x^2 + 3y^2$

$y'(-6xy - 3) = 3x^2 + 3y^2$

$$y' = \frac{3x^2 + 3y^2}{-6xy - 3}$$

$$y' = \frac{3(x^2 + y^2)}{-3(2xy + 1)}$$

$$\boxed{y' = -\frac{x^2 + y^2}{2xy + 1}}$$

Review Sheet

⑤ $s = -2t^3 + t^2 + 4t + 1$

$s' = -6t^2 + 2t + 4$ (velocity)

$s'' = -12t + 2$ (acceleration)

a) $s'(3) = -6(3)^2 + 2(3) + 4$

$= -54 + 6 + 4$

$= -44 \text{ m/s}$

$s''(3) = -12(3) + 2$

$= -36 + 2$

$= -34 \text{ m/s}^2$

b) $a = -10 \text{ m/s}^2$

$s'' = -12t + 2$

$-10 = -12t + 2$

$12t = 12$

$t = 1 \text{ sec}$

c) $s(1) = -2(1)^3 + (1)^2 + 4(1) + 1$

$= -2 + 1 + 4 + 1$

$= 4 \text{ m}$

$s'(1) = -6(1)^2 + 2(1) + 4$

$= -6 + 2 + 4$

$= 0 \text{ m/s}$

Review Sheet.

⑥ $h = 100 + 15t - 5t^2$

$h' = 15 - 10t$ (velocity)

$h'' = -10$ (acceleration)

a) max height ($h' = 0$)

$0 = 15 - 10t$

$10t = 15$

$t = 1.5 \text{ sec}$

b) hits ground ($h = 0$)

$0 = 100 + 15t - 5t^2$

$0 = 5(20 + 3t - t^2)$

$0 = 5(-t^2 + 3t + 20)$

$a = -1 \quad b = 3 \quad c = 20$

c) s' @ 6.2 sec

$s'(6.2) = 15 - 10(6.2)$

$s'(6.2) = 15 - 62 = -47 \text{ m/s}$

d) constant acceleration of -10 m/s^2

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-3 \pm \sqrt{(3)^2 - 4(-1)(20)}}{2(-1)}$$

$$t = \frac{-3 \pm \sqrt{9 + 80}}{-2}$$

$t = \frac{-3 + \sqrt{89}}{-2}$ $\quad t = -3.$ sec

$t = \frac{-3 - \sqrt{89}}{-2}$

$t = 6.2 \text{ sec}$

Review Sheet

7) $s = 2t^3 - 6t^2 + 9t$

$s' = 6t^2 - 12t + 9$

$s'' = 12t - 12$

a) $12t - 12 = 0$

$12t = 12$

$t = 1 \text{ sec}$

b) $12t - 12 > 0$

$12t > 12$

$t > 1$