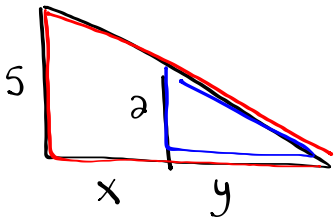
Questions From Homework

① 

$\frac{dx}{dt} = 1.5 m/s$

$\frac{x+y}{5} = \frac{y}{2}$

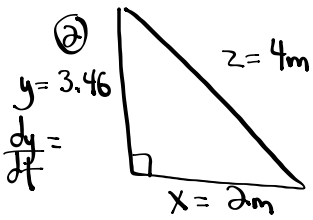
$2x+2y = 5y$

$2x = 3y$

$2\frac{dx}{dt} = 3\frac{dy}{dt}$

$2(1.5) = 3\frac{dy}{dt}$

$1 m/s = \frac{dy}{dt}$

② 

$\frac{dx}{dt} = 0.3 m/s$

(i) Find y

$x^2+y^2 = z^2$

$(2)^2+y^2 = (4)^2$

$y^2 = 12$

$y = 3.46 m$

(ii) Find $\frac{dy}{dt}$ (constant)

$x^2+y^2 = z^2$

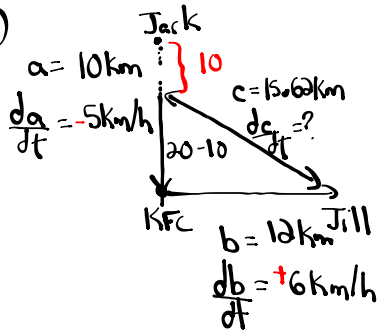
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(2)(0.3) + 2(3.46)\frac{dy}{dt} = 0$

$1.2 + 6.92\frac{dy}{dt} = 0$

$6.92\frac{dy}{dt} = -1.2$

$\frac{dy}{dt} = -0.1734 m/s$

⑤ 

(i) Find c

$a^2+b^2 = c^2$

$10^2+12^2 = c^2$

$100+144 = c^2$

$244 = c^2$

$15.62 = c$

(ii) Find $\frac{dc}{dt}$

$a^2+b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(10)(-5) + 2(12)(6) = 2(15.6)\frac{dc}{dt}$

$-100+144 = 31.24\frac{dc}{dt}$

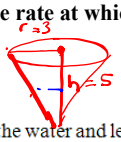
$44 = 31.24\frac{dc}{dt}$

$1.41 km/h = \frac{dc}{dt}$

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)

A water tank is built in the shape of a circular cone with height 5 m and diameter 6 m at the top. Water is being pumped into the tank at a rate of 1.6 m^3/min. Find the rate at which the water level is rising when the water is 2 m deep?



Let V be the volume of the water and let r and h be the radius of the surface and the height at time t, where t is measured in minutes. We are given the rate of increase of V, that is:

$\frac{dV}{dt} = 1.6m^3/\text{min}$

We are asked to find $\frac{dh}{dt}$ when $h = 2m$.

Given:
$\frac{dV}{dt} = 1.6 m^3/min \quad h = 2m$
$\frac{dh}{dt} = ?$

The quantities V and h are related by the equation:

$V = \frac{1}{3}\pi r^2 h$

But we have to express V as a function of h alone. To eliminate r we look for a relationship between r and h. We use similar triangles in the figure to write.

$\frac{r}{h} = \frac{3}{5}$ Thus $r = \frac{3}{5}h$ and we have:

$5r = 3h$

$r = \frac{3h}{5}$

$V = \frac{1}{3}\pi r^2 h$ (express V in terms of h only)

$V = \frac{1}{3}\pi\left(\frac{3h}{5}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{9h^2}{25}\right)h$

$V = \frac{9\pi h^3}{75}$

$V = \frac{3}{25}\pi h^3$

$\frac{dV}{dt} = \frac{9}{25}\pi h^2 \frac{dh}{dt}$

$1.6 = \frac{9}{25}\pi(2)^2 \frac{dh}{dt}$

$1.6 = \frac{36\pi}{25}\frac{dh}{dt}$

$1.6 \div \frac{36\pi}{25} = \frac{dh}{dt}$

$1.6 \times \frac{25}{36\pi} = \frac{dh}{dt}$

$0.3536 m/min = \frac{dh}{dt}$

Homework