

③ Given

$$\frac{dv}{dt} = 2 \text{ m}^3/\text{min}$$

$$\underline{r = 2h}$$

$$\frac{dh}{dt} = ?$$

$$h = 4 \text{ m}$$



$$V = \frac{1}{3} \pi r^2 h$$

Express in terms  
of  $h$  only

$$V = \frac{1}{3} \pi (2h)^2 h$$

$$V = \frac{1}{3} \pi 4h^2 \cdot h$$

$$V = \frac{4\pi h^3}{3}$$

$$\frac{dv}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$2 = 4\pi (4)^2 \frac{dh}{dt}$$

$$2 = 64\pi \frac{dh}{dt}$$

$$\frac{2}{64\pi} \text{ m/min} = \frac{dh}{dt}$$

$$0.009947 \text{ m/min} = \frac{dh}{dt}$$

⑤ Given:

$$r = \frac{3}{6} h$$

$$r = \frac{3}{6} h = \frac{1}{2} h$$

$$\frac{dV}{dt} = \pi \text{ m}^3/\text{min}$$

$$h = 3$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2} h\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^2}{4} \cdot h$$

$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\pi = \frac{1}{4} \pi (3)^2 \frac{dh}{dt}$$

$$\pi = \frac{9\pi}{4} \frac{dh}{dt}$$

$$\frac{4}{9} \pi \text{ m}^3/\text{min} = \frac{dh}{dt}$$

$$\boxed{0.4 \pi \text{ m}^3/\text{min} = \frac{dh}{dt}}$$

$$\textcircled{8} \quad x = y^3 + y \quad \text{show that } y'' = \frac{-6y}{(3y^2+1)^3}$$

$$1 = 3y^2 \frac{dy}{dx} + \frac{dy}{dx}$$

$$1 = \frac{dy}{dx} [3y^2 + 1]$$

$$\frac{1}{3y^2+1} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = y' = \frac{1}{3y^2+1} = (3y^2+1)^{-1}$$

$$y'' = -1(3y^2+1)^{-2} \left(6y \frac{dy}{dx}\right)$$

$$y'' = \frac{-6y \frac{dy}{dx}}{(3y^2+1)^2} = \frac{-6y \left(\frac{1}{3y^2+1}\right)}{(3y^2+1)^2} = \frac{-6y}{3y^2+1} \times \frac{1}{(3y^2+1)^2}$$

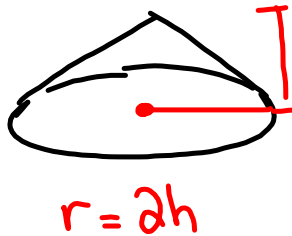
$$= \frac{-6y}{(3y^2+1)^3}$$

③ Given:

$$\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$$

$$h = 4 \text{ m}$$

$$\frac{dh}{dt} = ?$$



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (2h)^2 h$$

$$V = \frac{4}{3} \pi h^3$$

$$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$2 = 4\pi (4)^2 \frac{dh}{dt}$$

$$2 = 64\pi \frac{dh}{dt}$$

$$\frac{2}{64\pi} = \frac{dh}{dt}$$

$$\frac{1}{32\pi} \text{ m/min} = \frac{dh}{dt}$$

$$A = l^2 \text{ (Square)}$$

$$V = l^3 \text{ (Cube)}$$

$$A = \pi r^2 \text{ (Circle)}$$

$$A = 4\pi r^2 \text{ (Sphere)}$$

$$V = \frac{4}{3}\pi r^3 \text{ (Sphere)}$$

$$A = \frac{1}{2}bh \text{ (Triangle)}$$

$$V = \frac{1}{3}\pi r^2 h \text{ (Cone)}$$

$$x^2 + y^2 = z^2 \leftarrow z \text{ is constant (Ladder)}$$

$$x^2 + y^2 = z^2 \text{ (Intersection)}$$

$$\frac{h^{\text{of post}}}{x+y} = \frac{h^{\text{of man}}}{y} \text{ (Lamppost)}$$

$$x = y^3 + y$$

$$1 = 3y^2 \frac{dy}{dx} + \frac{dy}{dx}$$

$$1 = \frac{dy}{dx} (3y^2 + 1)$$

$$\frac{dy}{dx} = \boxed{\frac{1}{3y^2 + 1}}$$

$$\rightarrow y' = \frac{1}{3y^2 + 1} = (3y^2 + 1)^{-1}$$

$$y'' = -(3y^2 + 1)^{-2} (6y \frac{dy}{dx})$$

$$y'' = \frac{-6y \boxed{\frac{dy}{dx}}}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y \left[ \frac{1}{3y^2 + 1} \right]}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y}{3y^2 + 1} \cdot \frac{1}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y}{(3y^2 + 1)^3}$$

$$\textcircled{6} \quad \frac{dV}{dt} = 6 \text{ m}^3/\text{min}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dr}{dt} = ?$$

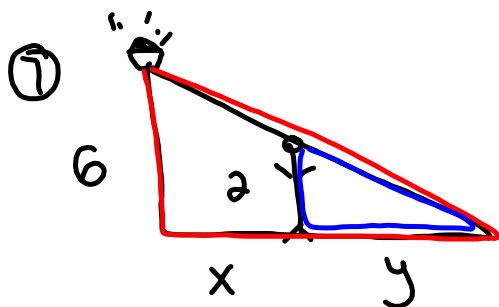
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$r = 2 \text{ m}$$

$$6 = 4\pi (2)^2 \frac{dr}{dt}$$

$$6 = 16\pi \frac{dr}{dt}$$

$$\frac{3}{8\pi} \text{ m/min} = \frac{dr}{dt}$$



Let  $x$  = distance from post

Let  $y$  = shadow length

Given:

$$\frac{dy}{dt} = ?$$

$$\frac{dx}{dt} = \underline{-1.5 \text{ m/s}}$$

Similar  $\triangle$ 's

$$\frac{6}{x+y} = \frac{2}{y}$$

$$2x + 2y = 6y$$

$$2x = 4y$$

$$2 \frac{dx}{dt} = 4 \frac{dy}{dt}$$

$$2(-1.5) = 4 \frac{dy}{dt}$$

$$-3 = 4 \frac{dy}{dt}$$

$$\boxed{-0.75 \text{ m/s} = \frac{dy}{dt}}$$



⑧  $x^2 y + y^2 = 22$  find  $\frac{dx}{dt}$  when  $y = 2$  and  $\frac{dy}{dt} = 4$

$$x^2 y' + 2x x' y + 2y y' = 0$$

$$(3)^2(4) + 2(2)x'(3) + 2(2)(4) = 0$$

$$36 + 12x' + 16 = 0$$

$$12x' = -52$$

$$x' = \pm 4.3$$

To find x:

$$x^2 y + y^2 = 22$$

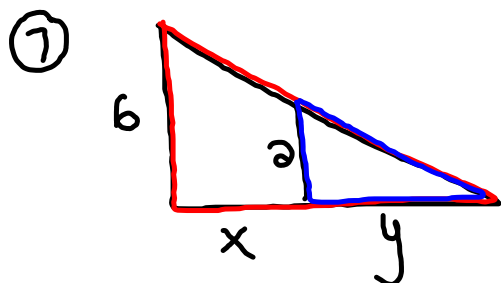
$$x^2(2) + (2)^2 = 22$$

$$2x^2 + 4 = 22$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$



Given:

$$\frac{dx}{dt} = -1.5 \text{ m/s}$$

$$x = 5 \text{ m}$$

$$\frac{6}{x+y} = \frac{2}{y}$$

$$2x + 2y = 6y$$

$$2x = 4y$$

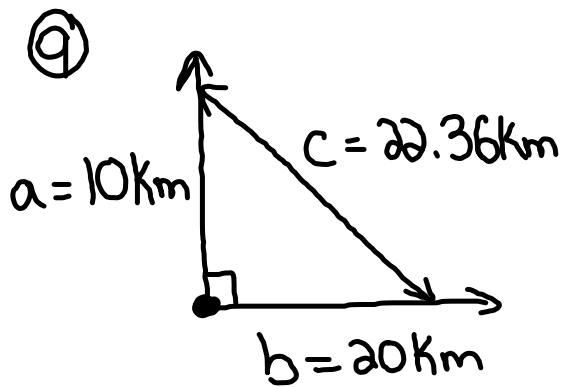
$$2 \frac{dx}{dt} = 4 \frac{dy}{dt}$$

$$2(-1.5) = 4 \frac{dy}{dt}$$

$$-3 = 4 \frac{dy}{dt}$$

$$-0.75 \text{ m/s} = \frac{dy}{dt}$$

$\therefore$  His shadow is decreasing at a rate of 0.75 m/s



Given:

$$\frac{da}{dt} = 5 \text{ km/h} \quad a = 10 \text{ km}$$

$$\frac{db}{dt} = 10 \text{ km/h} \quad b = 20 \text{ km}$$

$$\frac{dc}{dt} = ?$$

$$c = \sqrt{a^2 + b^2}$$

$$c = \sqrt{100 + 400}$$

$$c = 22.36$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(10)(5) + 2(20)(10) = 2(22.36) \frac{dc}{dt}$$

$$100 + 400 = 44.72 \frac{dc}{dt}$$

$$11.18 \text{ km/h} = \frac{dc}{dt}$$

$\therefore$  The distance between the two is increasing at a rate of 11.18 km/h.

⑥ Given:

$$\frac{dV}{dt} = 6 \text{ m}^3/\text{min}$$

$$\frac{dr}{dt} = ?$$

$$r = 2 \text{ m}$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$6 = 4\pi (2)^2 \frac{dr}{dt}$$

$$6 = 16\pi \frac{dr}{dt}$$

$$0.119 \text{ m/min} = \frac{dr}{dt}$$

$$\text{Square: } A = l^2 \quad \rightarrow \quad \frac{dA}{dt} = 2l \frac{dl}{dt}$$

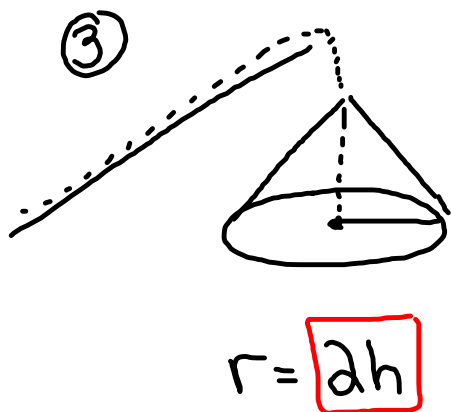
$$\text{Cube: } V = l^3 \quad \rightarrow \quad \frac{dV}{dt} = 3l^2 \frac{dl}{dt}$$

$$\text{Circle: } A = \pi r^2 \quad \rightarrow \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\text{Cone: } V = \frac{1}{3} \pi r^2 h$$

$$\text{Sphere: } V = \frac{4}{3} \pi r^3 \quad \rightarrow \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$A = 4\pi r^2 \quad \rightarrow \quad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$



Given:

$$\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$$

$$\frac{dh}{dt} = ?$$

$$h = 4 \text{ m}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (2h)^2 h$$

$$V = \frac{4\pi}{3} h^3$$

$$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$2 = 4\pi (4)^2 \frac{dh}{dt}$$

$$2 = 64\pi \frac{dh}{dt}$$

$$\frac{1}{32\pi} \text{ m/min} = \frac{dh}{dt}$$

## Related Rates Review #2

$$\textcircled{8} \quad x = y^3 + y \quad * \text{ Use Implicit Diff.}$$

$$1 = 3y^2 y' + y'$$

$$1 = y'(3y^2 + 1)$$

$$\frac{1}{3y^2 + 1} = y'$$

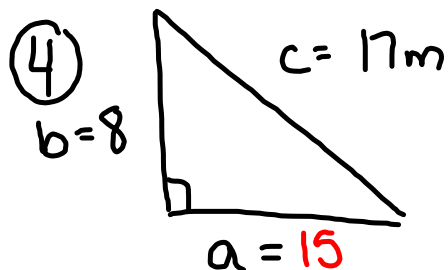
$$y' = \frac{1}{3y^2 + 1}$$

$$y'' = \frac{(\cancel{3y^2 + 1})(0) - 1(6yy')}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6yy'}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y \left( \frac{1}{3y^2 + 1} \right)}{(3y^2 + 1)^2}$$

$$y'' = \frac{-6y}{3y^2 + 1} \times \frac{1}{(3y^2 + 1)^2} = \frac{-6y}{(3y^2 + 1)^3}$$



Given

$$\frac{db}{dt} = -4\text{m/s} \quad \frac{da}{dt} = ?$$

$$b=8$$

$$a=\underline{15}$$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = 17^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 0$$

$$2(15) \frac{da}{dt} + 2(8)(-4) = 0$$

$$30 \frac{da}{dt} = 64$$

$$\frac{da}{dt} = 2.13\text{m/s}$$



$$\textcircled{5} \quad \frac{dV}{dt} = \pi \text{ m}^3/\text{min}$$

$$\frac{dh}{dt} = ?$$

$$h = 3$$

$$* \quad \frac{r}{h} = \frac{3}{6}$$

$$6r = 3h$$

$$r = \frac{3}{6}h$$

$$r = \frac{1}{2}h$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{4}h^2\right) h$$

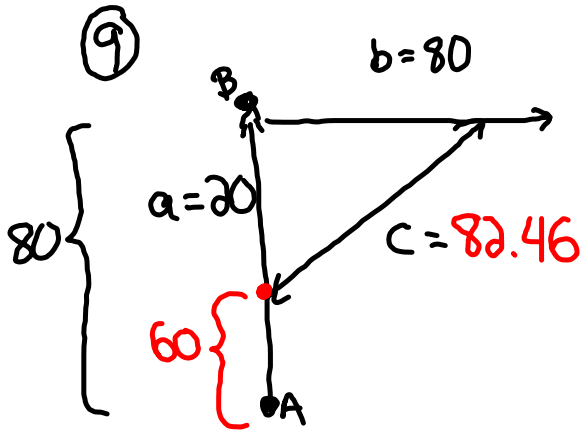
$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\pi = \frac{1}{4} \pi (3)^2 \frac{dh}{dt}$$

$$\pi = \frac{9\pi}{4} \frac{dh}{dt}$$

$$0.4 \text{ m/min} = \frac{dh}{dt}$$



Given:

$$\frac{da}{dt} = -30 \text{ km/h}$$

$$a = 20$$

$$\frac{db}{dt} = 40 \text{ km/h}$$

$$b = 80 \text{ km}$$

$$\frac{dc}{dt} = ?$$

$$c = \underline{\underline{82.46 \text{ km}}}$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(20)(-30) + 2(80)(40) = 2(82.46) \frac{dc}{dt}$$

$$-1200 + 6400 = 164.92 \frac{dc}{dt}$$

$$5200 = 164.92 \frac{dc}{dt}$$

$$31.5 \text{ km/h} = \frac{dc}{dt}$$