

Exponent Laws

$$2^{\frac{1}{2}} = \frac{5}{2}$$

$$\textcircled{1} X^a \cdot X^3 = X^5$$

$$2^{18} \cdot 2^{-3} = 2^{15}$$

$$X^2 \cdot X^{\frac{1}{2}} = X^{\frac{5}{2}}$$

$$\textcircled{2} \frac{X^5}{X^3} = X^2$$

$$\frac{b^8}{b^{-2}} = b^{10}$$

$$\textcircled{3} (X^5)^2 = X^{10}$$

$$(y^8)^{\frac{1}{2}} = y^4$$

$$\textcircled{4} \sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[4]{x} = x^{\frac{1}{4}}$$

$$\sqrt[5]{x^2} = x^{\frac{2}{5}}$$

$$\sqrt[7]{x^6} = x^{\frac{6}{7}}$$

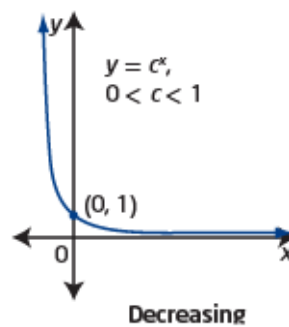
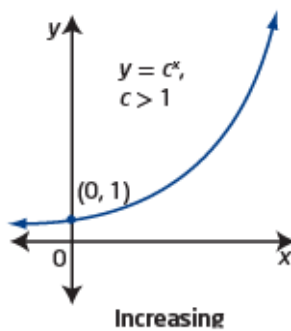
$$\textcircled{5} 3^2 = 9$$

$$\textcircled{6} 3^{-2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$$

Exponential Functions

The graph of an **exponential function**, such as $y = c^x$, is increasing for $c > 1$, decreasing for $0 < c < 1$, and neither increasing nor decreasing for $c = 1$. From the graph, you can determine characteristics such as domain and range, any intercepts, and any asymptotes.



exponential function

- a function of the form $y = c^x$, where c is a constant ($c > 0$) and x is a variable

Why is the definition of an exponential function restricted to positive values of c ?

Did You Know?

Any letter can be used to represent the base in an exponential function. Some other common forms are $y = a^x$ and $y = b^x$. In this chapter, you will use the letter c . This is to avoid any confusion with the transformation parameters, a , b , h , and k , that you will apply in Section 7.2.

Key Ideas

- An exponential function of the form $y = c^x, c > 0$,
 - is increasing for $c > 1$ (Base is greater 1)
 - is decreasing for $0 < c < 1$ (Between 0 + 1)
 - is neither increasing nor decreasing for $c = 1$ (Base equals 1)
 - has a domain of $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$
 - has a range of $\{y \mid y > 0, y \in \mathbb{R}\}$ or $(0, \infty)$
 - has a y-intercept of 1 (0, 1)
 - has no x-intercept
 - has a horizontal asymptote at $y = 0$

$\hookrightarrow y = k$

Example 1

Analyse the Graph of an Exponential Function

Graph each exponential function. Then identify the following:

- the domain and range
- the x -intercept and y -intercept, if they exist
- whether the graph represents an increasing or a decreasing function
- the equation of the horizontal asymptote

a) $y = 4^x$ $c = 4$ (Increasing)

b) $f(x) = \left(\frac{1}{2}\right)^x$ $c = \frac{1}{2}$ (Decreasing)

Solution

a) Method 1: Use Paper and Pencil

Use a table of values to graph the function.

Select integral values of x that make it easy to calculate the corresponding values of y for $y = 4^x$. $c=4$ (Increasing)

$$y = 4^x$$

x	y
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

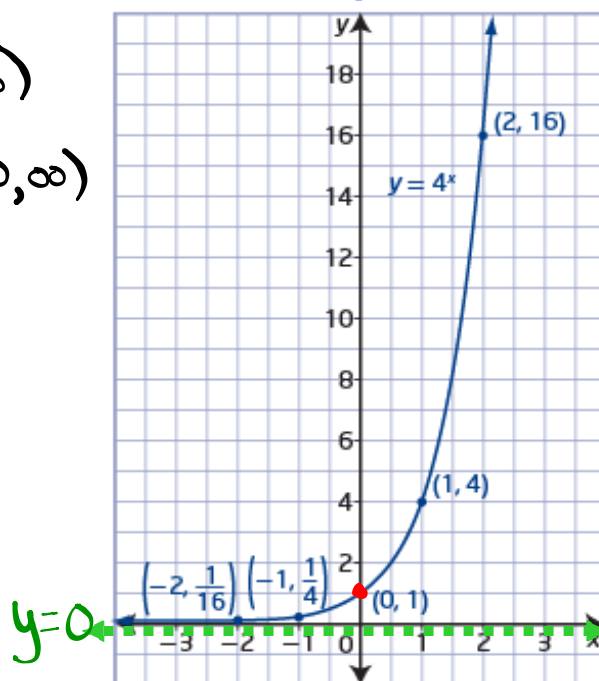
D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y > 0, y \in \mathbb{R}\}$ or $(0, \infty)$

x int: none

y int: $(0, 1)$

HA: $y = 0$



b) Method 1: Use Paper and Pencil

Use a table of values to graph the function.

Select integral values of x that make it easy to calculate the

corresponding values of y for $f(x) = \left(\frac{1}{2}\right)^x$. $c = \frac{1}{2}$ (Decreasing)

$$y = \left(\frac{1}{2}\right)^x$$

x	$f(x)$
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

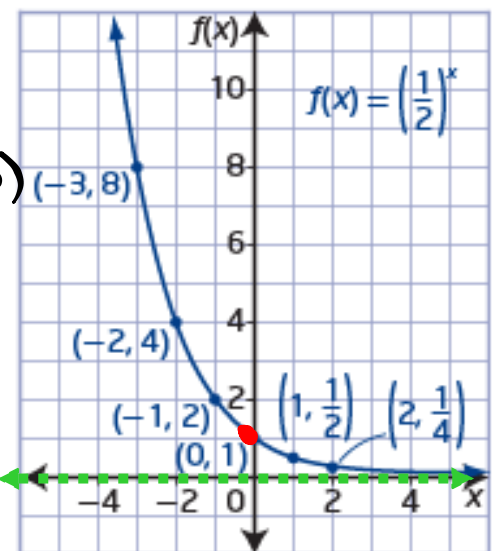
D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y > 0, y \in \mathbb{R}\}$ or $(0, \infty)$

x int: none

y int: $(0, 1)$

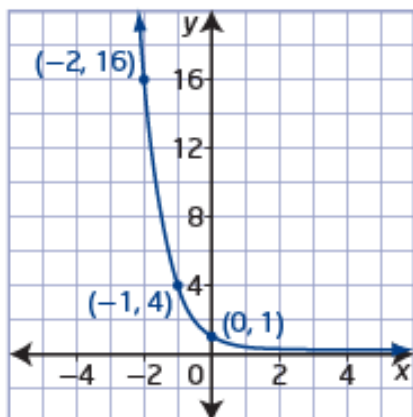
HA: $y = 0$



Example 2

Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?



Decreasing $0 < c < 1$
 • Base is between 0 + 1

Solution

Look for a pattern in the ordered pairs from the graph.

x	y
-2	16
-1	4
0	1

As x increases by 1
 the y values decrease by a factor
 of 4 $c = \frac{1}{4}$

Choose a point other than $(0, 1)$ to substitute into the function $y = \left(\frac{1}{4}\right)^x$ to verify that the function is correct. Try the point $(-2, 16)$. $x = -2$ $y = 16$

$(0, 1)$ is on all exponential functions of the form $y = c^x$

Why should you not use the point $(0, 1)$ to verify that the function is correct?

Check:

Left Side

y
 16 ✓

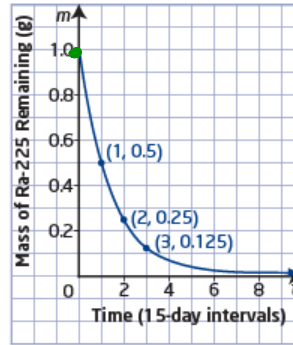
Right Side

$\left(\frac{1}{4}\right)^x$
 $\left(\frac{1}{4}\right)^{-2}$
 $(4)^2$
 16 ✓

Example 3

Application of an Exponential Function

A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass, m , in grams, of Ra-225 remaining over time, t , in 15-day intervals, can be modelled using the exponential graph shown.



- What is the initial mass of Ra-225 in the sample? What value does the mass of Ra-225 remaining approach as time passes?
- What are the domain and range of this function?
- Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals.
- Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.

a) Initial Amount = 1g
As time passes the radium approaches a mass of 0g.

b) D: $\{x | x \geq 0, x \in \mathbb{R}\}$ or $[0, \infty)$
R: $\{y | 0 < y \leq 1, y \in \mathbb{R}\}$ or $(0, 1]$

c) $m = (\text{Initial Amount})(\text{Base})^{\frac{t}{\text{time it takes to } \dots = 15}}$
 $m = (1)\left(\frac{1}{2}\right)^{\frac{t}{15}}$
Base = $\frac{1}{2}$ (Half life)

d) $\frac{1}{30} = \cancel{(1)} \left(\frac{1}{2}\right)^{\frac{t}{15}}$ (Divide both sides by Initial Amount)

$\frac{1}{30} = \left(\frac{1}{2}\right)^{\frac{t}{15}}$ (Get a common base)
 $\hookrightarrow \frac{\log(\frac{1}{30})}{\log(\frac{1}{2})} = 4.91$

$15 \cdot 4.91 = \frac{t}{15} \cdot 15$ (Multiply both sides by 15)

$73.6 = t$

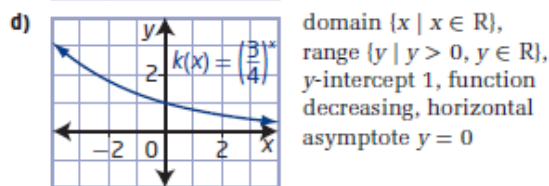
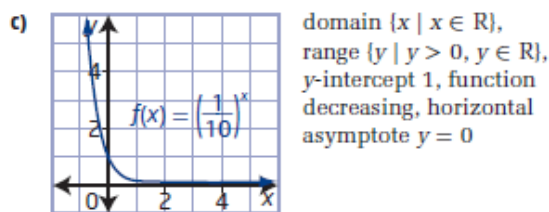
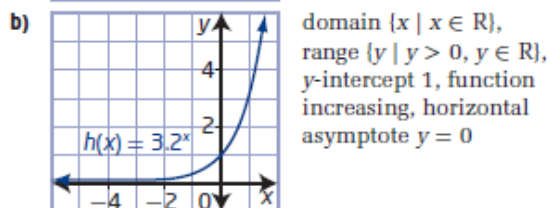
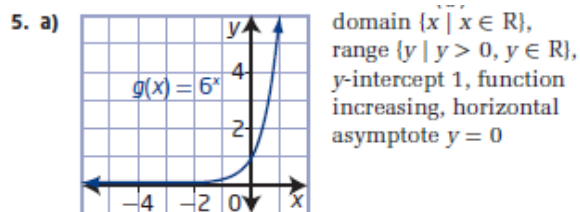
73.6 days to reach $\frac{1}{30}$ of its initial amount

Homework

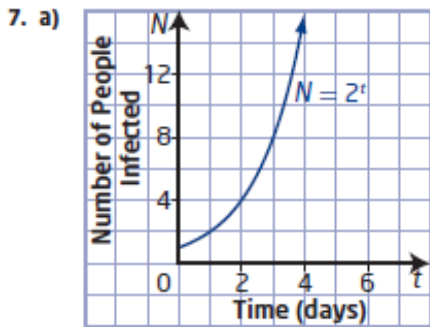
#1-8 on page 343

7.1 Characteristics of Exponential Functions, pages 342 to 345

1. a) No, the variable is not the exponent.
 b) Yes, the base is greater than 0 and the variable is the exponent.
 c) No, the variable is not the exponent.
 d) Yes, the base is greater than 0 and the variable is the exponent.
2. a) $f(x) = 4^x$ b) $g(x) = \left(\frac{1}{4}\right)^x$
 c) $x = 0$, which is the y-intercept
3. a) B b) C c) A
4. a) $f(x) = 3^x$ b) $f(x) = \left(\frac{1}{5}\right)^x$



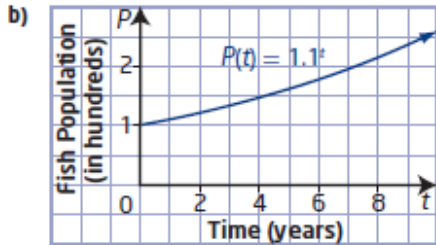
- 6. a) $c > 1$; number of bacteria increases over time
- b) $0 < c < 1$; amount of actinium-225 decreases over time
- c) $0 < c < 1$; amount of light decreases with depth
- d) $c > 1$; number of insects increases over time



The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.

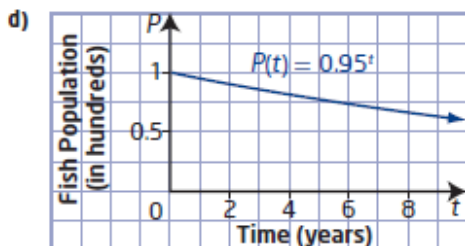
- b) i) 1 person ii) 2 people
- iii) 16 people iv) 1024 people

- 8. a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.



domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid P \geq 100, P \in \mathbb{R}\}$

- c) The base of the exponent would become $100\% - 5\%$ or 95%, written as 0.95 in decimal form.



domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid 0 < P \leq 100, P \in \mathbb{R}\}$