

Function Operations

To combine two functions, $f(x)$ and $g(x)$, add or subtract as follows:

Sum of Functions

$$h(x) = f(x) + g(x)$$

$$h(x) = (f + g)(x)$$

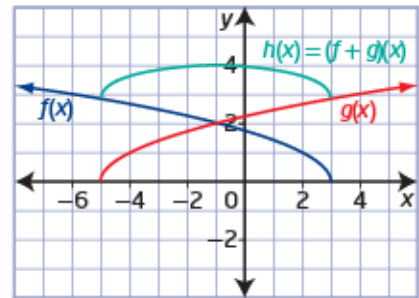
Difference of Functions

$$h(x) = f(x) - g(x)$$

$$h(x) = (f - g)(x)$$

Key Ideas

- You can add two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f + g)(x)$.
- You can subtract two functions, $f(x)$ and $g(x)$, to form the combined function $h(x) = (f - g)(x)$. $h(x) = f(x) - g(x)$
- The domain of the combined function formed by the sum or difference of two functions is the domain common to the individual functions. For example,
 - Domain of $f(x)$: $\{x \mid x \leq 3, x \in \mathbb{R}\}$
 - Domain of $g(x)$: $\{x \mid x \geq -5, x \in \mathbb{R}\}$
 - Domain of $h(x)$: $\{x \mid -5 \leq x \leq 3, x \in \mathbb{R}\}$
- The range of a combined function can be determined using its graph.
- To sketch the graph of a sum or difference of two functions given their graphs, add or subtract the y-coordinates at each point.



Example 1

Determine the Sum of Two Functions

Consider the functions $f(x) = 2x + 1$ and $g(x) = x^2$.

- a) Determine the equation of the function $h(x) = (f + g)(x)$.
- b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.
- c) State the domain and range of $h(x)$.
- d) Determine the values of $f(x)$, $g(x)$, and $h(x)$ when $x = 4$.

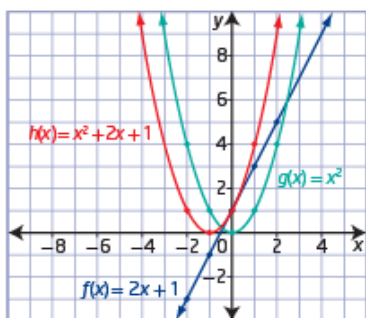
a) $h(x) = (f + g)(x)$
 $h(x) = \underline{f(x)} + \underline{g(x)}$ Domain: $f(x) = 2x + 1$ | Domain: $g(x) = x^2$
 $h(x) = 2x + 1 + x^2$ $\{x | x \in \mathbb{R}\}$ | $\{x | x \in \mathbb{R}\}$
 $h(x) = x^2 + 2x + 1$

b)

$f(x) = 2x + 1$	
x	y
-2	-3
-1	-1
0	1
1	3
2	5

$g(x) = x^2$	
x	y
-2	4
-1	1
0	0
1	1
2	4

$h(x) = x^2 + 2x + 1$	
x	y
-2	1
-1	0
0	1
1	4
2	9



How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

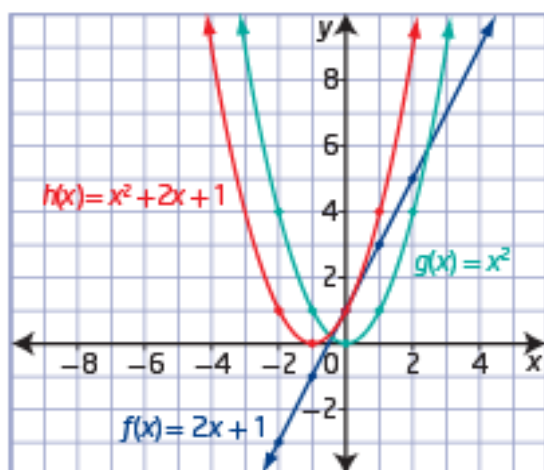
c) $h(x) = x^2 + 2x + 1$

D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

d) when $x = 4$

$f(x) = 2x + 1$	$g(x) = x^2$	$h(x) = x^2 + 2x + 1$
$f(4) = 2(4) + 1$	$g(4) = (4)^2$	$h(4) = (4)^2 + 2(4) + 1$
$f(4) = 9$	$g(4) = 16$	$h(4) = 16 + 8 + 1$
		$h(4) = 25$



How are the y -coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

- c) The function $f(x) = 2x + 1$ has domain $\{x \mid x \in \mathbb{R}\}$.
 The function $g(x) = x^2$ has domain $\{x \mid x \in \mathbb{R}\}$.
 The function $h(x) = (f + g)(x)$ has domain $\{x \mid x \in \mathbb{R}\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.
 The range of $h(x)$ is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

- d) Substitute $x = 4$ into $f(x)$, $g(x)$, and $h(x)$.

$$\begin{array}{lll}
 f(x) = 2x + 1 & g(x) = x^2 & h(x) = x^2 + 2x + 1 \\
 f(4) = 2(4) + 1 & g(4) = 4^2 & h(4) = 4^2 + 2(4) + 1 \\
 f(4) = 8 + 1 & g(4) = 16 & h(4) = 16 + 8 + 1 \\
 f(4) = 9 & & h(4) = 25
 \end{array}$$

Example 2

Determine the Difference of Two Functions

Consider the functions $f(x) = \sqrt{x-1}$ and $g(x) = x-2$.

- a) Determine the equation of the function $h(x) = (f-g)(x)$.
- b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.
- c) State the domain of $h(x)$.
- d) Use the graph to approximate the range of $h(x)$.

a) $h(x) = (f-g)(x)$
 $h(x) = \underline{f(x)} - \underline{g(x)}$
 $h(x) = \sqrt{x-1} - (x-2)$
 $h(x) = \sqrt{x-1} - x + 2$

Domain:
 $f(x) = \sqrt{x-1}$
 $x-1 \geq 0$
 $x \geq 1$
 $\{x | x \geq 1, x \in \mathbb{R}\}$

Domain:
 $g(x) = x-2$
 $\{x | x \in \mathbb{R}\}$

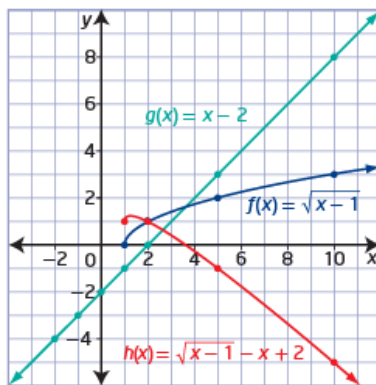
b) **Method 1: Use Paper and Pencil**

For the function $f(x) = \sqrt{x-1}$, the value of the radicand must be greater than or equal to zero: $x-1 \geq 0$ or $x \geq 1$.

x	$f(x) = \sqrt{x-1}$	$g(x) = x-2$	$h(x) = \sqrt{x-1} - x + 2$
-2	undefined	-4	undefined
-1	undefined	-3	undefined
0	undefined	-2	undefined
1	0	-1	1
2	1	0	1
5	2	3	-1
10	3	8	-5

Why is the function $h(x)$ undefined when $x < 1$?

How could you use the values in the columns for $f(x)$ and $g(x)$ to determine the values in the column for $h(x)$?



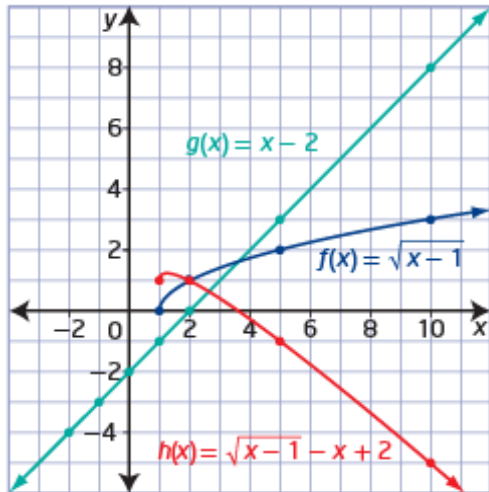
How could you use the y-coordinates of points on the graphs of $f(x)$ and $g(x)$ to create the graph of $h(x)$?

c) $h(x) = \sqrt{x-1} - x + 2$

D: $\{x | x \geq 1, x \in \mathbb{R}\}$ or $[1, \infty)$

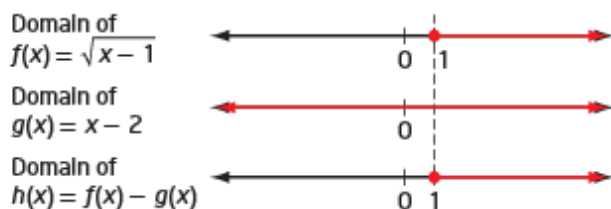
d) $h(x) = \sqrt{x-1} - x + 2$

R: $\{y | y \leq 1.2, y \in \mathbb{R}\}$ or $(-\infty, 1.2]$



How could you use the y-coordinates of points on the graphs of $f(x)$ and $g(x)$ to create the graph of $h(x)$?

- c) The function $f(x) = \sqrt{x - 1}$ has domain $\{x \mid x \geq 1, x \in \mathbb{R}\}$.
 The function $g(x) = x - 2$ has domain $\{x \mid x \in \mathbb{R}\}$.
 The function $h(x) = (f - g)(x)$ has domain $\{x \mid x \geq 1, x \in \mathbb{R}\}$, which consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.



What values of x belong to the domains of both $f(x)$ and $g(x)$?

- d) From the graph, the range of $h(x)$ appears to be approximately $\{y \mid y \leq 1.2, y \in \mathbb{R}\}$.

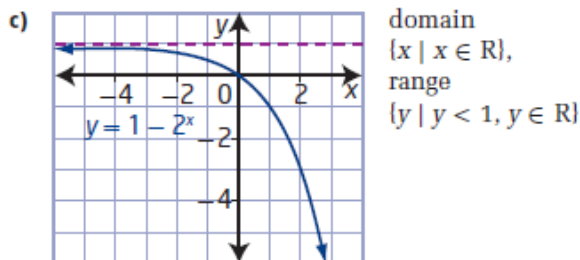
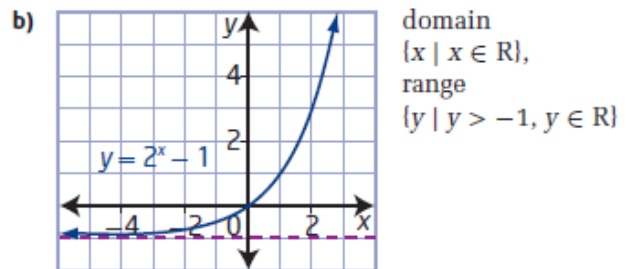
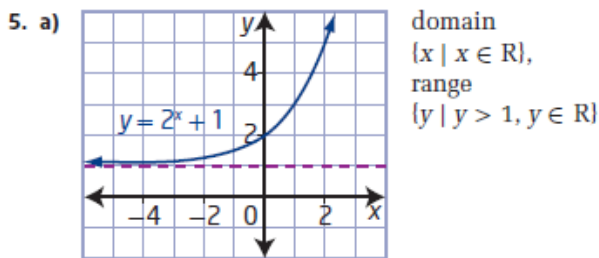
How can you use a graphing calculator to verify the range?

Homework

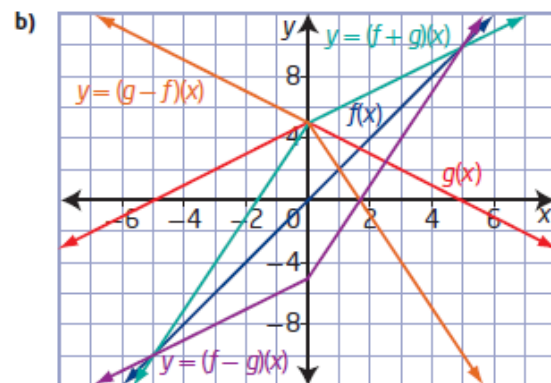
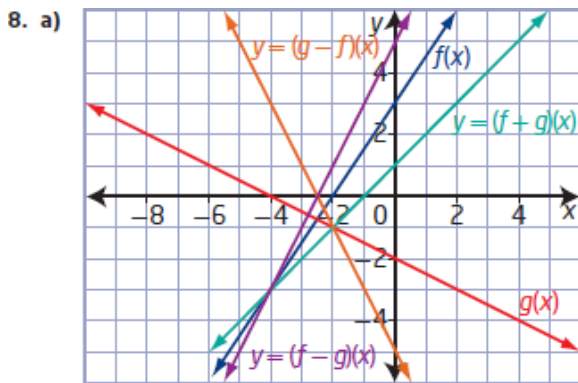
finish #1-11 on page 483-484

10.1 Sums and Differences of Functions, pages 483 to 487

1. a) $h(x) = |x - 3| + 4$ b) $h(x) = 2x - 3$
 c) $h(x) = 2x^2 + 3x + 2$ d) $h(x) = x^2 + 5x + 4$
2. a) $h(x) = 5x + 2$ b) $h(x) = -3x^2 - 4x + 9$
 c) $h(x) = -x^2 - 3x + 12$ d) $h(x) = \cos x - 4$
3. a) $h(x) = x^2 - 6x + 1; h(2) = -7$
 b) $m(x) = -x^2 - 6x + 1; m(1) = -6$
 c) $p(x) = x^2 + 6x - 1; p(1) = 6$
4. a) $y = 3x^2 + 2 + \sqrt{x+4};$ domain $\{x \mid x \geq -4, x \in \mathbb{R}\}$
 b) $y = 4x - 2 - \sqrt{x+4};$ domain $\{x \mid x \geq -4, x \in \mathbb{R}\}$
 c) $y = \sqrt{x+4} - 4x + 2;$ domain $\{x \mid x \geq -4, x \in \mathbb{R}\}$
 d) $y = 3x^2 + 4x;$ domain $\{x \mid x \in \mathbb{R}\}$



6. a) 8 b) 6 c) 7
 d) not in the domain
7. a) B b) C c) A



- | | |
|----------------------------|----------------------------|
| 9. a) $y = 3x^2 + 11x + 1$ | b) $y = 3x^2 - 3x + 3$ |
| c) $y = 3x^2 + 3x + 1$ | d) $y = 3x^2 - 11x + 3$ |
| 10. a) $g(x) = x^2$ | b) $g(x) = \sqrt{x+7}$ |
| c) $g(x) = -3x + 1$ | d) $g(x) = 3x^2 - x - 4$ |
| 11. a) $g(x) = x^2 - 1$ | b) $g(x) = -\sqrt{x-4}$ |
| c) $g(x) = 8x - 9$ | d) $g(x) = 2x^2 - 11x - 6$ |