

Questions from Homework

4. Given $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x+4}$, and $h(x) = 4x - 2$, determine each combined function and state its domain.

- a) $y = (f + g)(x)$ b) $y = (h - g)(x)$
 c) $y = (g - h)(x)$ d) $y = (f + h)(x)$

$$\begin{array}{l|l|l}
 f(x) = \underline{3x^2 + 2} \quad (\checkmark \checkmark) & g(x) = \underline{\sqrt{x+4}} \quad (\checkmark \checkmark) & h(x) = 4x - 2 \quad (\checkmark \checkmark) \\
 \text{D: } \{x | x \in \mathbb{R}\} \text{ or } (-\infty, \infty) & \begin{array}{l} x+4 \geq 0 \\ x \geq -4 \\ \text{D: } \{x | x \geq -4, x \in \mathbb{R}\} \\ \text{or } [-4, \infty) \end{array} & \text{D: } \{x | x \in \mathbb{R}\} \text{ or } (-\infty, \infty)
 \end{array}$$

a) $y = f(x) + g(x)$
 $y = 3x^2 + 2 + \sqrt{x+4}$
 $y = 3x^2 + 2 + \sqrt{x+4}$
 D: $\{x | x \geq -4, x \in \mathbb{R}\}$
 $[-4, \infty)$

d) $y = f(x) + h(x)$
 $y = 3x^2 + 2 + (4x - 2)$
 $y = 3x^2 + 2 + 4x - 2$
 $y = 3x^2 + 4x$
 D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$

11. If $h(x) = (f - g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$.

- a) $h(x) = -x^2 + 5x + 3$
 b) $h(x) = \sqrt{x-4} + 5x + 2$
 c) $h(x) = -3x + 11$
 d) $h(x) = -2x^2 + 16x + 8$

Function Operations

To combine two functions, $f(x)$ and $g(x)$, multiply or divide as follows:

Product of Functions

$$h(x) = f(x)g(x)$$

$$h(x) = (f \cdot g)(x)$$

Quotient of Functions

$$h(x) = \frac{f(x)}{g(x)}$$

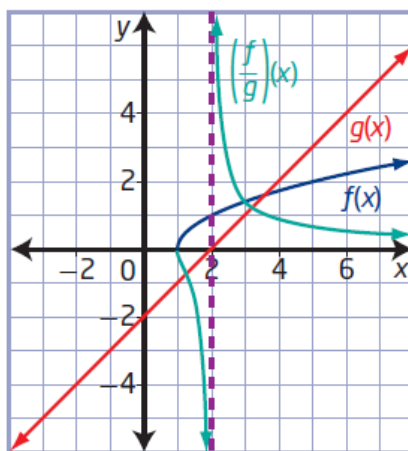
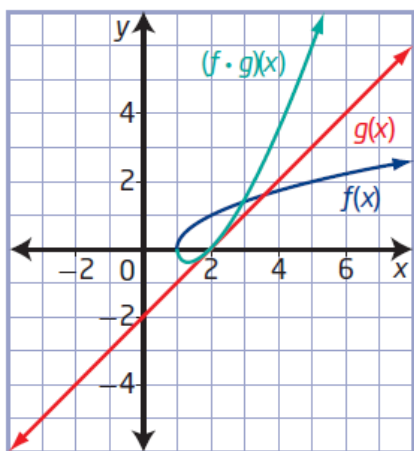
$$h(x) = \left(\frac{f}{g}\right)(x)$$

The domain of a product of functions is the domain common to the original functions. However, the domain of a quotient of functions must take into consideration that division by zero is undefined. The domain of a quotient, $h(x) = \frac{f(x)}{g(x)}$, is further restricted for values of x where $g(x) = 0$.

Example

Consider $f(x) = \sqrt{x-1}$ and $g(x) = x-2$.

The domain of $f(x)$ is $\{x \mid x \geq 1, x \in \mathbb{R}\}$, and the domain of $g(x)$ is $\{x \mid x \in \mathbb{R}\}$. So, the domain of $(f \cdot g)(x)$ is $\{x \mid x \geq 1, x \in \mathbb{R}\}$, while the domain of $\left(\frac{f}{g}\right)(x)$ is $\{x \mid x \geq 1, x \neq 2, x \in \mathbb{R}\}$



$$f(x) = \sqrt{x-1} \quad (\curvearrowright)$$

$$x-1 \geq 0$$

$$x \geq 1$$

$$D: \{x \mid x \geq 1, x \in \mathbb{R}\}$$

$$\text{or } [1, \infty)$$

$$g(x) = x-2 \quad (\curvearrowright)$$

$$D: \{x \mid x \in \mathbb{R}\}$$

$$\text{or } (-\infty, \infty)$$

Key Ideas

- The combined function $h(x) = (f \cdot g)(x)$ represents the product of two functions, $f(x)$ and $g(x)$.
- The combined function $h(x) = \left(\frac{f}{g}\right)(x)$ represents the quotient of two functions, $f(x)$ and $g(x)$, where $g(x) \neq 0$.
- The domain of a product or quotient of functions is the domain common to both $f(x)$ and $g(x)$. The domain of the quotient $\left(\frac{f}{g}\right)(x)$ is further restricted by excluding values where $g(x) = 0$.
- The range of a combined function can be determined using its graph.

Example 1

Determine the Product of Functions

Given $f(x) = (x + 2)^2 - 5$ and $g(x) = 3x - 4$, determine $h(x) = (f \cdot g)(x)$. State the domain and range of $h(x)$.

Solution $f(x) = (x+2)(x+2) - 5$
 $f(x) = x^2 + 2x + 2x + 4 - 5 = x^2 + 4x - 1$

To determine $h(x) = (f \cdot g)(x)$, multiply the two functions.

$$h(x) = (f \cdot g)(x)$$

$$h(x) = f(x)g(x)$$

$$h(x) = ((x + 2)^2 - 5)(3x - 4)$$

$$h(x) = (x^2 + 4x - 1)(3x - 4)$$

$$h(x) = 3x^3 - 4x^2 + 12x^2 - 16x - 3x + 4$$

$$h(x) = 3x^3 + 8x^2 - 19x + 4$$

How can you tell from the original functions that the product is a cubic function?

$$f(x) = x^2 + 4x - 1$$

$$D: \{x | x \in \mathbb{R}\}$$

$$\text{or } (-\infty, \infty)$$

$$g(x) = 3x - 4$$

$$D: \{x | x \in \mathbb{R}\}$$

$$\text{or } (-\infty, \infty)$$

$$h(x) = 3x^3 + 8x^2 - 19x + 4$$

$$D: \{x | x \in \mathbb{R}\}$$

$$\text{or } (-\infty, \infty)$$

$$R: \{y | y \in \mathbb{R}\}$$

$$\text{or } (-\infty, \infty)$$

Example 2

Determine the Quotient of Functions

Consider the functions $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

- Determine the equation of the function $h(x) = \left(\frac{g}{f}\right)(x)$.
- Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.
- State the domain and range of $h(x)$.

Solution

$$f(x) = x^2 + x - 6 \quad \left| \quad g(x) = 2x + 6 \quad \left| \quad h(x) = \frac{2}{x-2}\right.\right.$$

$$D: \{x \mid x \in \mathbb{R}\} \quad \left| \quad D: \{x \mid x \in \mathbb{R}\} \quad \left| \quad D: \{x \mid x \neq -3, 2, x \in \mathbb{R}\}\right.\right.$$

- To determine $h(x) = \left(\frac{g}{f}\right)(x)$, divide the two functions.

$$h(x) = \left(\frac{g}{f}\right)(x)$$

$$h(x) = \frac{g(x)}{f(x)}$$

$$h(x) = \frac{2x + 6}{x^2 + x - 6}$$

Common
Simple Trinomial
 $\frac{-2 \times 3}{-2 + 3} = -6$
 $\frac{-2 \times 3}{-2 + 3} = 1$

$$h(x) = \frac{2(x + 3)}{(x + 3)(x - 2)}$$

Factor.

$$h(x) = \frac{2\cancel{(x + 3)}}{\underset{1}{\cancel{(x + 3)}}(x - 2)}$$

$$h(x) = \frac{2}{x - 2}, x \neq -3, 2 \quad \text{Identify any non-permissible values.}$$

hole @ $x = -3$

$$h(x) = \frac{2}{x-2}$$

$$h(-3) = \frac{2}{-3-2} = \frac{2}{-5} = -\frac{2}{5}$$

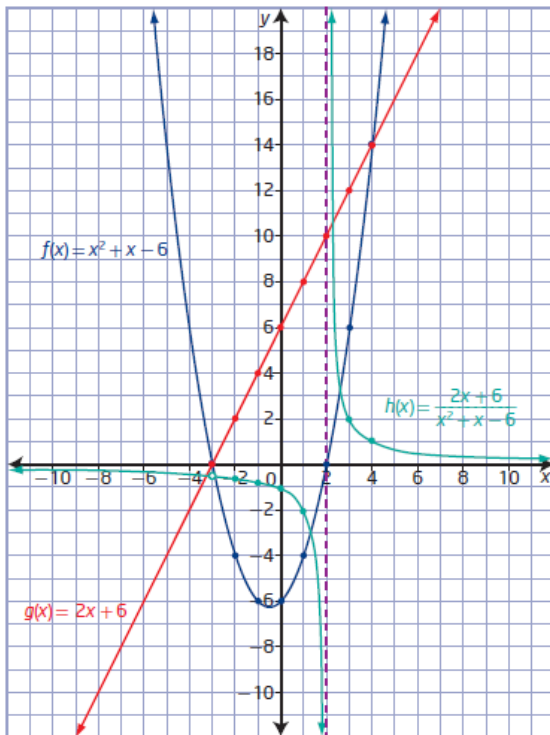
$$\boxed{(-3, -\frac{2}{5})}$$

b) Method 1: Use Paper and Pencil

x	$f(x) = x^2 + x - 6$	$g(x) = 2x + 6$	$h(x) = \frac{2}{x-2}, x \neq -3, 2$
-3	0	0	does not exist
-2	-4	2	$-\frac{1}{2}$
-1	-6	4	$-\frac{2}{3}$
0	-6	6	-1
1	-4	8	-2
2	0	10	undefined
3	6	12	2
4	14	14	1

hole @ (-3, -2/3)

VA @ x=2

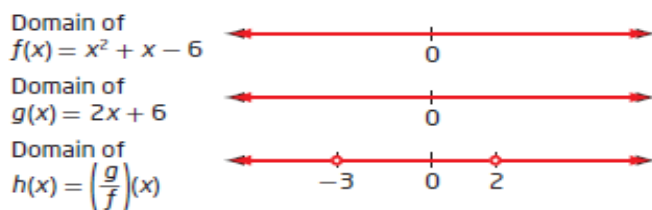


How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

- c) The function $f(x) = x^2 + x - 6$ is quadratic with domain $\{x \mid x \in \mathbb{R}\}$.
 The function $g(x) = 2x + 6$ is linear with domain $\{x \mid x \in \mathbb{R}\}$.
 The domain of $h(x) = \left(\frac{g}{f}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $f(x) = 0$.

Since the function $h(x)$ does not exist at $\left(-3, -\frac{2}{5}\right)$ and is undefined at $x = 2$, the domain is $\{x \mid x \neq -3, x \neq 2, x \in \mathbb{R}\}$. This is shown in the graph by the point of discontinuity at $\left(-3, -\frac{2}{5}\right)$ and the vertical asymptote that appears at $x = 2$.

How do you know there is a point of discontinuity and an asymptote?



The range of $h(x)$ is $\{y \mid y \neq 0, -\frac{2}{5}, y \in \mathbb{R}\}$.

hole @ $x = -3$

$h(x) = \frac{a}{x-a}$

$h(-3) = \frac{a}{-3-a} = \frac{a}{-5} = -\frac{a}{5}$

$\left(-3, -\frac{2}{5}\right)$

↑ horizontal asymptote

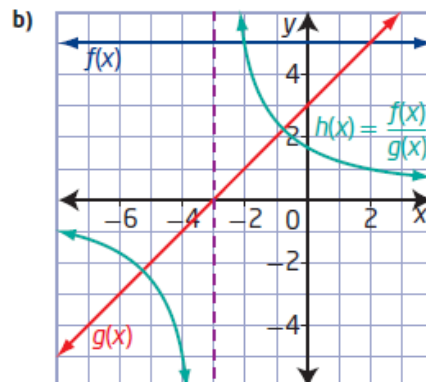
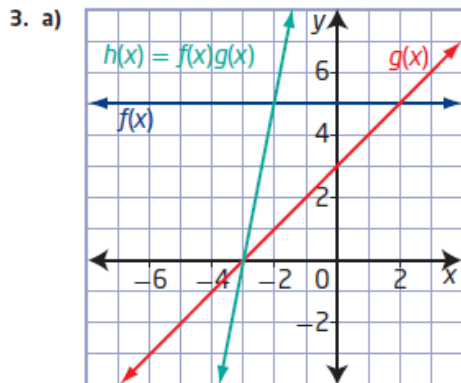
Homework

finish #1-9 on page 496-497

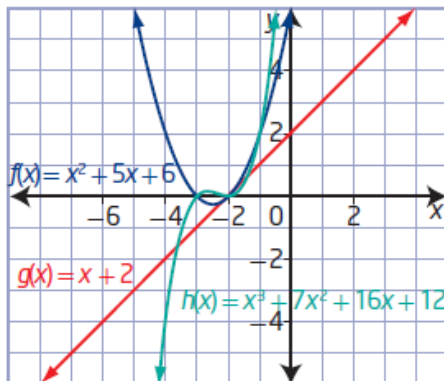
10.2 Products and Quotients of Functions, pages 496 to 498

1. a) $h(x) = x^2 - 49$, $k(x) = \frac{x+7}{x-7}$, $x \neq 7$
- b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \frac{2x-1}{3x+4}$, $x \neq -\frac{4}{3}$
- c) $h(x) = (x+2)\sqrt{x+5}$, $k(x) = \frac{\sqrt{x+5}}{x+2}$, $x \geq -5$, $x \neq -2$
- d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \frac{\sqrt{x-1}}{\sqrt{6-x}}$, $1 \leq x < 6$

2. a) -3 b) 0 c) -1 d) 0

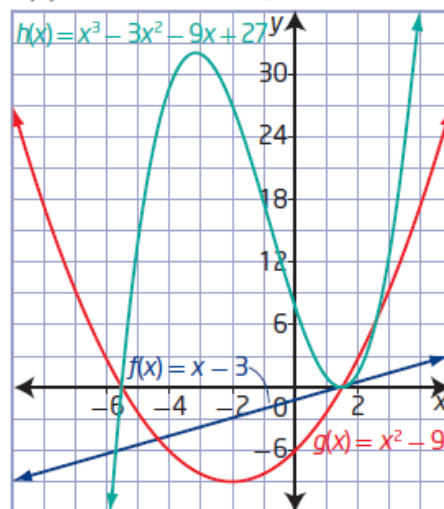


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



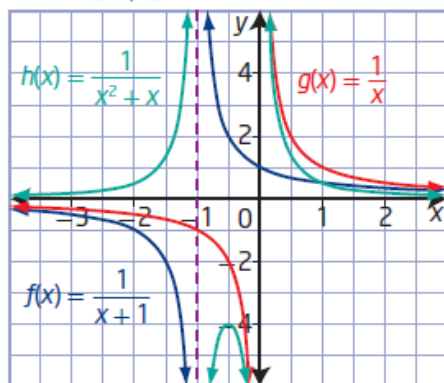
domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

- b) $h(x) = x^3 - 3x^2 - 9x + 27$



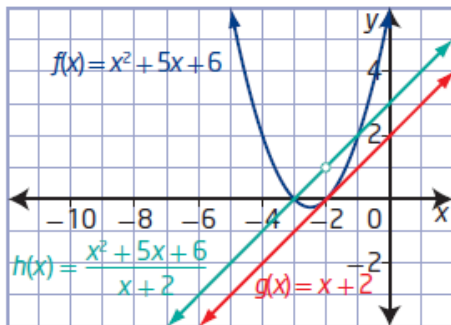
domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

- c) $h(x) = \frac{1}{x^2 + x}$



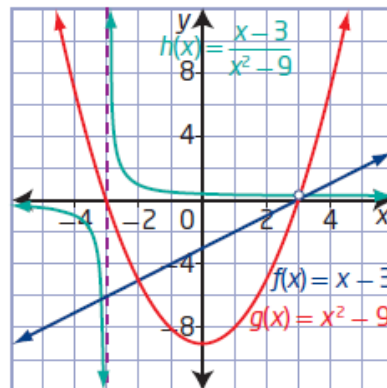
domain $\{x \mid x \neq 0, -1, x \in \mathbb{R}\}$,
range $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathbb{R}\}$

5. a) $h(x) = x + 3, x \neq -2$



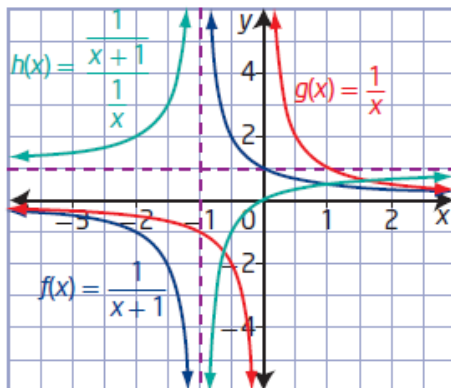
domain $\{x \mid x \neq -2, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 1, y \in \mathbb{R}\}$

b) $h(x) = \frac{1}{x + 3}, x \neq \pm 3$



domain $\{x \mid x \neq \pm 3, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 0, \frac{1}{6}, y \in \mathbb{R}\}$

c) $h(x) = \frac{x}{x + 1}, x \neq -1, 0$



domain $\{x \mid x \neq -1, 0, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 0, 1, y \in \mathbb{R}\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$

b) $y = \frac{x^2 - x - 6}{x + 4}, x \neq -4$ c) $y = \frac{2x - 1}{x + 4}, x \neq -4$

d) $y = \frac{x^2 - x - 6}{x^2 + 8x + 16}, x \neq -4$

7. a) $g(x) = 3$

b) $g(x) = -x$

c) $g(x) = \sqrt{x}$

d) $g(x) = 5x - 6$

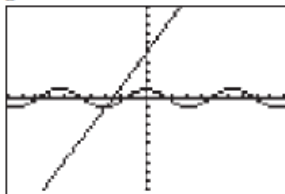
8. a) $g(x) = x + 7$

b) $g(x) = \sqrt{x + 6}$

c) $g(x) = 2$

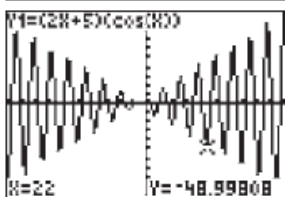
d) $g(x) = 3x^2 + 26x - 9$

9. a)

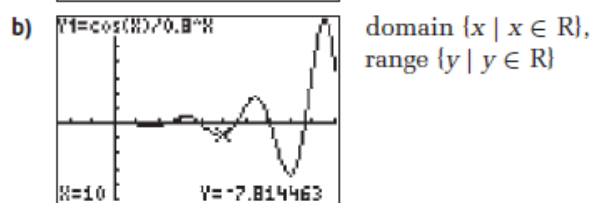


$f(x)$:
domain $\{x \mid x \in \mathbb{R}\}$,
range $\{y \mid y \in \mathbb{R}\}$
 $g(x)$: domain $\{x \mid x \in \mathbb{R}\}$,
range
 $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

b)



domain $\{x \mid x \in \mathbb{R}\}$,
range $\{y \mid y \in \mathbb{R}\}$



11. a) $y = \frac{f(x)}{g(x)}$ b) $y = f(x)f(x)$
 c) The graphs of $y = \frac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.