Warm Up

Evaluate the following limits if they exist.

$$\lim_{x\to 0} \frac{2x^3}{\sin^3 3x}$$

$$\lim_{x\to 0} \frac{x^3}{\sin^3 3x} \cdot 2$$

$$\lim_{x\to 0} \left(\frac{3x}{\sin 3x}\right)^3 \cdot \frac{2}{27}$$

$$\frac{2}{27} \boxed{\lim_{x\to 0} \left(\frac{3x}{\sin 3x}\right)^3}$$

$$= \frac{2}{27}(1)^3 = \frac{2}{27}$$

$$\lim_{x\to -5^-} \frac{|x+5|}{4x+20}$$

$$\lim_{x\to -5^-} \frac{|x+5|}{4(x+5)}$$

$$\lim_{x\to -5^-} \frac{|-5.1+5|}{4(-5.1+5)}$$

$$\lim_{x\to -5^-} \frac{\cancel{0.1}}{4(-\cancel{0.1})} = -\frac{1}{4}$$

Questions from Homework

Identity

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{x}{\sin x}=1$$

⑨ $$\lim_{x\to 0}\frac{\sin^3 2x}{\sin^3 3x}$$

$$=\lim_{x\to 0}\frac{(\sin 2x)^3}{(\sin 3x)^3}$$

$$=\lim_{x\to 0}\frac{(\sin 2x)^3}{1}\cdot\frac{1}{(\sin 3x)^3}$$

$$=\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^3\left(\frac{3x}{\sin 3x}\right)^3\cdot\frac{8\cancel{x^3}}{27\cancel{x^3}}$$

$$=\frac{8}{27}\boxed{\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)}^3\cdot\boxed{\lim_{x\to 0}\left(\frac{3x}{\sin 3x}\right)}^3$$

$$=\frac{8}{27}(1)^3(1)^3=\frac{8}{27}$$

㉖ $$\lim_{x\to 0}\frac{2\boxed{\tan x}}{x\boxed{\sec x}}$$

$$=\lim_{x\to 0}\frac{2\,\frac{\sin x}{\cos x}}{x\,\frac{1}{\cos x}}$$

$$=\lim_{x\to 0}\frac{2\sin x}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{x}$$

$$=\lim_{x\to 0}\frac{2\sin x}{x}$$

$$=2\boxed{\lim_{x\to 0}\frac{\sin x}{x}}$$

$$=2(1)$$

$$=2$$

Questions from Homework

31 $\lim_{x\to 0} \frac{\sin^2 x \cos x}{(1-\cos x)} \frac{(1+\cos x)}{(1+\cos x)}$

$\lim_{x\to 0} \frac{\sin^2 x \cos x (1+\cos x)}{\boxed{(1-\cos^2 x)}} \rightarrow$ Pyth. $= \sin^2 x$

$\lim_{x\to 0} \frac{\cancel{\sin^2} x \cos x (1+\cos x)}{\cancel{\sin^2} x} = (1)(2) = \boxed{2}$

37 $\lim_{x\to 0} \frac{\sin 2x}{2x^2+x}$

$\lim_{x\to 0} \frac{\sin 2x}{x(2x+1)}$

$\lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{2x+1}\right)(2)$

$\lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right) \cdot \lim_{x\to 0} \left(\frac{1}{2x+1}\right) \cdot \lim_{x\to 0} 2$

$= (1)(1)(2)$

$= 2$

L'Hopital's Rule:

L'Hôpital's Rule: (if the limit is indeterminant) $\frac{0}{0}, \frac{\infty}{\infty}$

$$\lim_{x\to n}\frac{f(x)}{g(x)}=\lim_{x\to n}\frac{f'(x)}{g'(x)}$$

← derivatives (not the same as the quotient)

$$\lim_{x\to 3}\frac{2x^2-x-15}{x^2-9}=\frac{0}{0}$$

$$\lim_{x\to 3}\frac{4x-1}{2x}=\frac{11}{6}$$

$$\lim_{x\to \pi/2}\frac{1-\sin x}{\cos x}=0$$

$$\lim_{x\to \pi/2}\frac{-\cos x}{-\sin x}$$

$$\lim_{x\to \pi/2}\frac{\cos x}{\sin x}=\frac{0}{1}=0$$

$$\lim_{x\to 0}\frac{e^x-1}{x^3}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{e^x\cdot 1-0}{3x^2}$$

$$\lim_{x\to 0}\frac{e^x}{3x^2}=\frac{1}{0}\quad \text{DNE}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{0-(-\sin x\cdot 1)}{2x}$$

$$\lim_{x\to 0}\frac{\sin x}{2x}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\cos x}{2}=\frac{1}{2}$$

$$\lim_{x\to\infty}\frac{e^x}{x^2}=\frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{e^x}{2x}$$

$$\lim_{x\to\infty}\frac{e^x}{2}=\frac{\infty}{2}=\text{DNE}$$

$$\lim_{x \to 3} \frac{(x-\underset{2}{\cancel{6}})(x+\underset{2}{\cancel{5}})}{(x-3)(x+3)}$$

$$\lim_{x \to 3} \frac{\cancel{(x-3)}(2x+5)}{\cancel{(x-3)}(x+3)} = \frac{11}{6}$$

Homework

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"Limits of Trig Functions"