Sketch the following function $f(x)=\frac{(x+2)^2}{(x^2+4)}$ if

$f'(x)=\frac{16-4x^2}{(x^2+4)^2}$ and $f''(x)=\frac{8x(x^2-12)}{(x^2+4)^3}$

- ✓ Intercepts
- ✓ Asymptotes
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values
- Intervals of Concavity $f''(x)$
- Points of Inflection

(i) y-int $(x=0)$

$f(0)=\frac{(0+2)^2}{(0^2+4)}=\frac{4}{4}=1$

$(0,1)$

x-int: $(y=0)$

$(x^2+4)\quad 0=\frac{(x+2)^2}{\cancel{(x^2+4)}}\quad \cancel{(x^2+4)}$

$0=(x+2)^2$

$0=(x+2)(x+2)$

$x+2=0$

$x=-2$

$(-2,0)$

(ii) VA (set denom = 0)

$x^2+4=0$

$x^2=-4$

Not possible

No VA

(iii) HA (compare degrees)

$\lim_{x\to\infty}\frac{x^2+4x+4}{x^2+4}=\frac{1}{1}=1$

HA: $\boxed{y=1}$

(iv) Intervals of Inc/Dec:

$f'(x)=\frac{16-4x^2}{(x^2+4)^2}$

CV: $16-4x^2=0$

$(4-2x)(4+2x)=0$

$4-2x=0$, $4=2x$, $2=x$ | $4+2x=0$, $2x=-4$, $x=-2$

$(x^2+4)^2=0$

$x^2+4=0$

$x^2=-4$

Not possible

min max

$-\quad+\quad-$

$(-3)\quad -2\quad (0)\quad 2\quad (3)$

Decreasing on $(-\infty,-2)+(2,\infty)$

Increasing on $(-2,2)$

(v) local min $(x=-2)$

$f(-2)=\frac{(-2+2)^2}{((-2)^2+4)}=\frac{0}{8}=0$

$(-2,0)$

Same as x-int

(vi) local max $(x=2)$

$f(2)=\frac{(2+2)^2}{((2)^2+4)}=\frac{16}{8}=2$

$(2,2)$

(vii) Concavity

$f''(x)=\frac{8x(x^2-12)}{(x^2+4)^3}$

IP IP IP

$-\quad+\quad-\quad+$

$(-4)\quad -\sqrt{12}\quad (-1)\quad 0\quad (1)\quad \sqrt{12}\quad (4)$

CV: $8x(x^2-12)=0$

$8x(x-\sqrt{12})(x+\sqrt{12})=0$

$8x=0$, $x=0$ | $x-\sqrt{12}=0$, $x=\sqrt{12}$, $x\doteq 3.46$ | $x+\sqrt{12}=0$, $x=-\sqrt{12}$, $x\doteq-3.46$

$(x^2+4)^3=0$

$x^2+4=0$

$x^2=-4$

Not Possible

CU on $(-\sqrt{12},0)$ and $(\sqrt{12},\infty)$

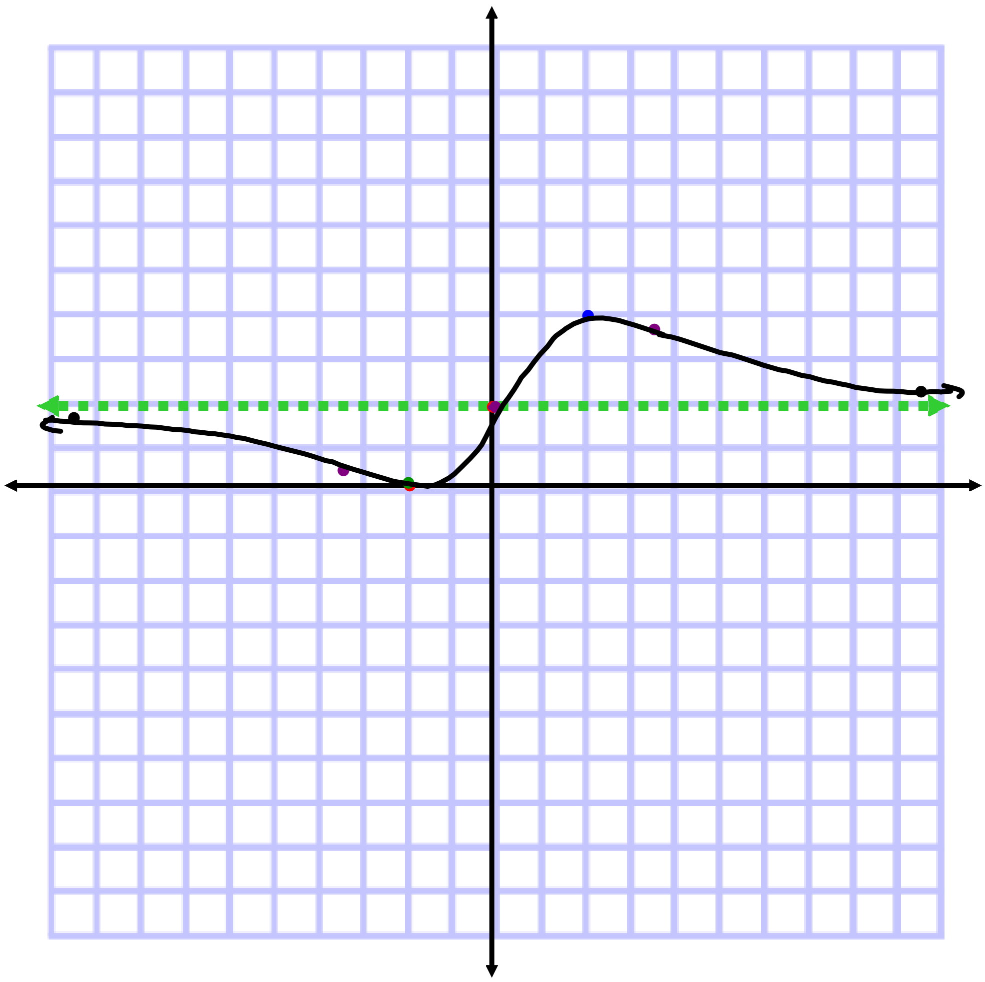
CD on $(-\infty,-\sqrt{12})$ and $(0,\sqrt{12})$

(viii) IP.

$f(-3.46)=\frac{(-3.46+2)^2}{(-3.46)^2+4}=\frac{2.1316}{15.9716}\doteq 0.13$ $(-3.46,\ 0.13)$

$f(0)=\frac{(0+2)^2}{(0)^2+4}=\frac{4}{4}=1$ $(0,1)$ Same as y-int

$f(3.46)=\frac{(3.46+2)^2}{(3.46)^2+4}=\frac{29.8116}{15.9716}\doteq 1.87$ $(3.46,\ 1.87)$



$f(x) = 3x^5 - 5x^3$

$f(x) = x^3(3x^2 - 5)$

$x^3 = 0$	$3x^2 - 5 = 0$
$x = 0$	$3x^2 = 5$
	$x^2 = 5/3$
$(0,0)$	$x = \pm\sqrt{5/3}$

$(\sqrt{5/3}, 0)$ + $(-\sqrt{5/3}, 0)$

$f(x) = x^3 - 9x^2 + 24x - 10$

① y-intercept $(x=0)$

$f(0) = (0)^3 - 9(0)^2 + 24(0) - 10$
$= -10$
$(0,-10)$

② Asymptotes: None
(Polynomial Function)

③ $f(x) = x^3 - 9x^2 + 24x - 10$

$f'(x) = 3x^2 - 18x + 24$

$f'(x) = 3(x^2 - 6x + 8)$

$f'(x) = 3(x-4)(x-2)$

CV: $x = 2, 4$

max min
$+ \quad - \quad +$
(1) 2 (3) 4 (5)

Increasing on $(-\infty, 2) + (4, \infty)$
Decreasing on $(2, 4)$

Max/min

$f(x) = x^3 - 9x^2 + 24x - 10$

$f(2) = (2)^3 - 9(2)^2 + 24(2) - 10 = 10$ $(2,10)$ max

$f(4) = (4)^3 - 9(4)^2 + 24(4) - 10 = 6$ $(4,6)$ min

Concavity:

$f(x) = x^3 - 9x^2 + 24x - 10$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18$

$f''(x) = 6(x-3)$

CV: $x = 3$

IP
$- \quad +$
(0) 3 (4)

CD on $(-\infty, 3)$
CU on $(3, \infty)$

IP:

$f(x) = x^3 - 9x^2 + 24x - 10$

$f(3) = (3)^3 - 9(3)^2 + 24(3) - 10 = 8$ $(3,8)$ IP