Sketch the following function $f(x)=\frac{(x+2)^2}{(x^2+4)}$ if

$f'(x)=\frac{16-4x^2}{(x^2+4)^2}$ and $f''(x)=\frac{8x(x^2-12)}{(x^2+4)^3}$

- ✓ Intercepts
- ✓ Asymptotes
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values
- Intervals of Concavity $f''(x)$
- Points of Inflection

(i) y-int ($x=0$)

$f(0)=\frac{(0+2)^2}{(0^2+4)}=\frac{4}{4}=1$

$(0,1)$

x-int: ($y=0$)

$(x^2+4)\; 0=\frac{(x+2)^2}{\cancel{(x^2+4)}}\;\cancel{(x^2+4)}$

$0=(x+2)^2$

$0=(x+2)(x+2)$

$x+2=0$

$x=-2$

$(-2,0)$

(ii) VA (set denom = 0)

$x^2+4=0$

$x^2=-4$

Not possible

No VA

(iii) HA (compare degrees)

$\lim_{x\to\infty}\frac{x^2+4x+4}{x^2+4}=\frac{1}{1}=1$

HA: $\boxed{y=1}$

(iv) Intervals of Inc/Dec:

$f'(x)=\frac{16-4x^2}{(x^2+4)^2}$

CV: $16-4x^2=0$

$(4-2x)(4+2x)=0$

$4-2x=0$, $4=2x$, $2=x$ | $4+2x=0$, $2x=-4$, $x=-2$

$(x^2+4)^2=0$, $x^2+4=0$, $x^2=-4$, Not possible

min max

$-$ | $+$ | $-$

(-3) -2 (0) 2 (3)

Decreasing on $(-\infty,-2)+(2,\infty)$

Increasing on $(-2,2)$

(v) local min ($x=-2$)

$f(-2)=\frac{(-2+2)^2}{((-2)^2+4)}=\frac{0}{8}=0$

$(-2,0)$

Same as x-int

(vi) local max ($x=2$)

$f(2)=\frac{(2+2)^2}{((2)^2+4)}=\frac{16}{8}=2$

$(2,2)$

(vii) Concavity

$f''(x)=\frac{8x(x^2-12)}{(x^2+4)^3}$

IP IP IP

$-$ | $+$ | $-$ | $+$

(-4) $-\sqrt{12}$ (-1) 0 (1) $\sqrt{12}$ (4)

CV: $8x(x^2-12)=0$

$8x(x-\sqrt{12})(x+\sqrt{12})=0$

$8x=0$, $x=0$ | $x-\sqrt{12}=0$, $x=\sqrt{12}$, $x\doteq 3.46$ | $x+\sqrt{12}=0$, $x=-\sqrt{12}$, $x\doteq -3.46$

$(x^2+4)^3=0$, $x^2+4=0$, $x^2=-4$, Not Possible

CU on $(-\sqrt{12},0)$ and $(\sqrt{12},\infty)$

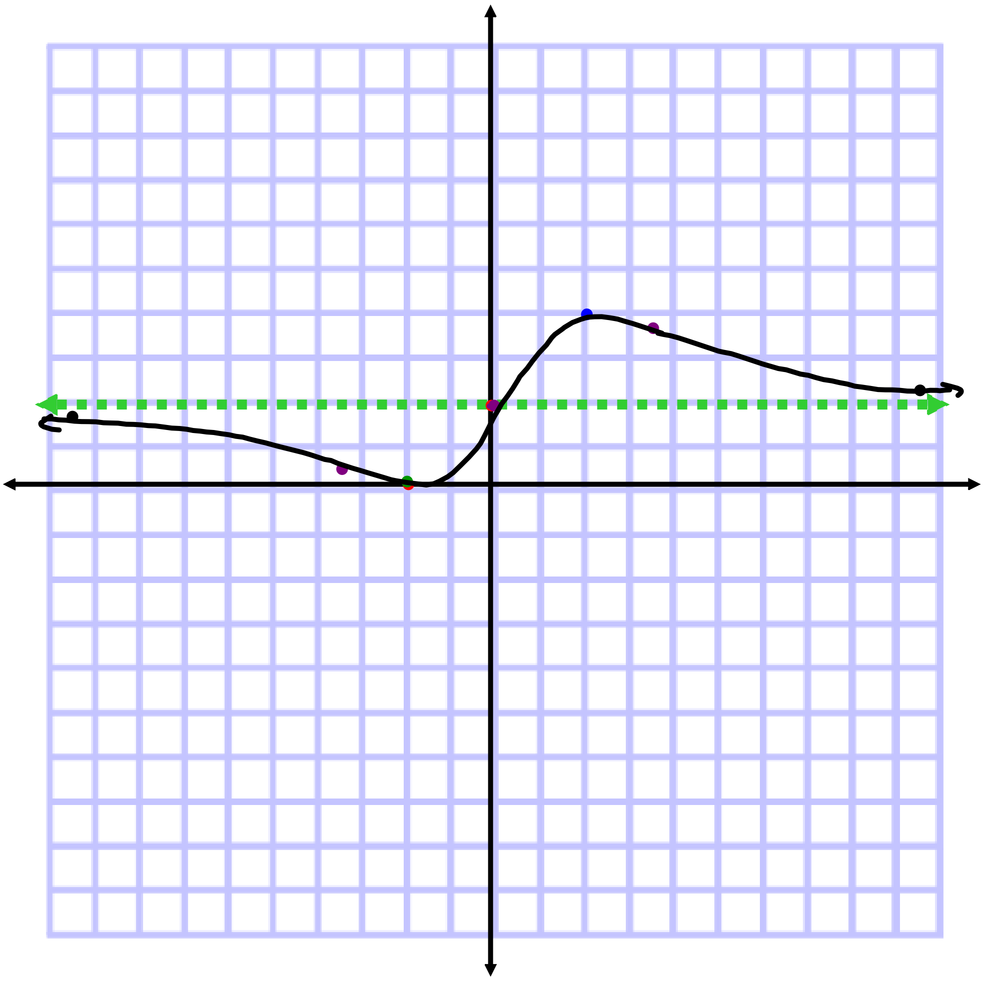
CD on $(-\infty,-\sqrt{12})$ and $(0,\sqrt{12})$

(viii) IP.

$f(-3.46)=\frac{(-3.46+2)^2}{(-3.46)^2+4}=\frac{2.1316}{15.9716}\doteq 0.13$ $(-3.46, 0.13)$

$f(0)=\frac{(0+2)^2}{(0)^2+4}=\frac{4}{4}=1$ $(0,1)$ Same as y-int

$f(3.46)=\frac{(3.46+2)^2}{(3.46)^2+4}=\frac{29.8116}{15.9716}\doteq 1.87$ $(3.46, 1.87)$



$f(x) = 3x^5 - 5x^3$

$f(x) = x^3(3x^2 - 5)$

(i) x-int $(y=0)$

$0 = x^3(3x^2-5)$

$x^3 = 0$ | $3x^2 - 5 = 0$

$x = 0$ | $3x^2 = 5$

$x = \pm\sqrt{5/3}$

$(0,0)$

$(\pm 1.3, 0)$

y-int $(x=0)$

$f(0) = 3(0)^5 - 5(0)^3 = 0$

$(0,0)$

(ii) Intervals of Inc/Dec:

$f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$f'(x) = 15x^2(x^2-1)$

$f'(x) = 15x^2(x-1)(x+1)$

CV: $15x^2 = 0$, $x = 0$ | $x - 1 = 0$, $x = 1$ | $x + 1 = 0$, $x = -1$

max, min

$+ \quad - \quad - \quad +$

$(-2) \quad -1 \quad (-0.5) \quad 0 \quad (0.5) \quad 1 \quad (2)$

Increasing on $(-\infty, -1)$ + $(1, \infty)$

Decreasing on $(-1, 1)$

(iii) Local min $(x=1)$

$f(1) = 3(1)^5 - 5(1)^3$

$f(1) = 3 - 5$

$f(1) = -2$

$(1, -2)$

(iv) Local max $(x=-1)$

$f(-1) = 3(-1)^5 - 5(-1)^3$

$f(-1) = -3 + 5$

$f(-1) = 2$

$(-1, 2)$

(v) Intervals of Concavity:

$f(x) = 3x^5 - 5x^3$

$f'(x) = 15x^4 - 15x^2$

$f''(x) = 60x^3 - 30x$

$f''(x) = 30x(2x^2 - 1)$

CV: $30x = 0$, $x = 0$ | $2x^2 - 1 = 0$, $x^2 = \frac{1}{2}$, $x = \pm\sqrt{\frac{1}{2}}$, $x \simeq \pm 0.707$

IP IP IP

$- \quad + \quad - \quad +$

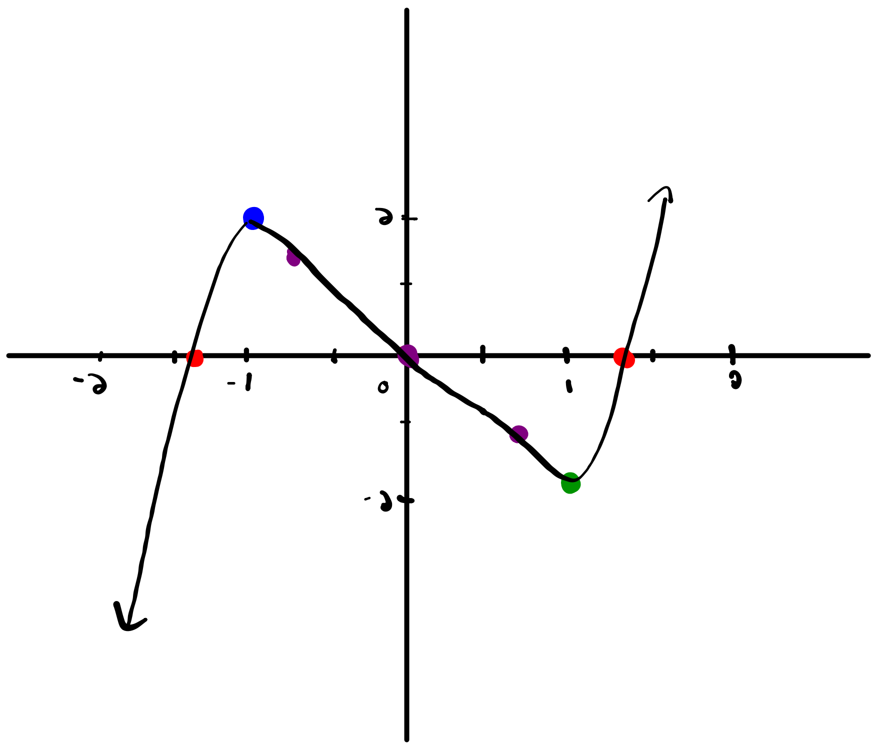
$(-1) \quad -\sqrt{1/2} \quad (-0.5) \quad 0 \quad (0.5) \quad \sqrt{1/2} \quad (1)$

(vi) Inflection Points:

$f(-0.71) = 3(-0.71)^5 - 5(-0.71)^3 = 1.24 \quad (-0.71, 1.24)$

$f(0) = 3(0)^5 - 5(0)^3 = 0 \quad (0,0)$

$f(0.71) = 3(0.71)^5 - 5(0.71)^3 = -1.24 \quad (0.71, -1.24)$


2
-2
-1
0
1
2
-2

1. Consider the function : $f(x) = \dfrac{2 + x - x^2}{(x-1)^2}$

$f'(x) = \dfrac{x-5}{(x-1)^3}$ and $f''(x) = \dfrac{2(7-x)}{(x-1)^4}$

1. Consider the function: $f(x) = \dfrac{2+x-x^2}{(x-1)^2}$

CV: x = 1, 5

$f'(x) = \dfrac{x-5}{(x-1)^3}$ and $f''(x) = \dfrac{2(7-x)}{(x-1)^4}$ CV: x = 1, 7

x-int: (y = 0)

$2+x-x^2=0$

$-x^2+x+2=0$

$-(x^2-x-2)=0$

$-(x+1)(x-2)=0$

$x+1=0 \mid x-2=0$

$x=-1 \mid x=2$

$(-1,0)$ $(2,0)$

y int (x = 0)

$f(0) = \dfrac{2+(0)-(0)^2}{(0-1)^2}$

$f(0) = \dfrac{2}{1} = 2$

$(0,2)$

VA: (set denom = 0)

$f(x) = \dfrac{2+x-x^2}{(x-1)^2} = -\dfrac{(x-2)(x+1)}{(x-1)^2}$

$(x-1)^2=0$

$x-1=0$

$x=1$

Test behaviour near VA

$\lim_{x\to 1^-} f(x) = \dfrac{-(-)(+)}{(+)} = +\infty$

$x = 0.99$

$\lim_{x\to 1^+} f(x) = \dfrac{-(-)(+)}{+} = +\infty$

$x = 1.01$

HA: (Compare the degree of the num. and denom.)

$f(x) = \dfrac{2+x-x^2}{(x-1)^2} = \dfrac{2+x-x^2}{x^2-2x+1}$

$\lim_{x\to\infty} \dfrac{2+x-x^2}{x^2-2x+1} = -1$ $y=-1$

$f'(x) = \dfrac{x-5}{(x-1)^3}$

CV: x = 1, 5

+	−	+
(0) 1	(2) 5	(6)

no max x = 1 is VA; min

$f(x) = \dfrac{2+x-x^2}{(x-1)^2}$

$f(5) = \dfrac{2+(5)-(5)^2}{(5-1)^2}$

$f(5) = \dfrac{-18}{16} = \dfrac{-9}{8}$

$(5, -\tfrac{9}{8})$ or $(5, -1.125)$

$f''(x) = \dfrac{2(7-x)}{(x-1)^4}$

CV: x = 1, 7

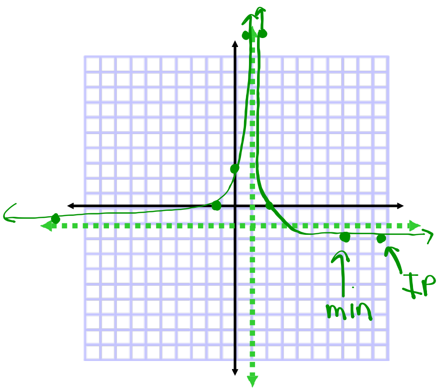
+	+	−
(0) 1	(2) 7	(8)

IP

$f(7) = \dfrac{2+(7)-(7)^2}{(7-1)^2}$

$f(7) = \dfrac{-40}{36} = -\dfrac{10}{9}$

$(7, -\tfrac{10}{9})$ or $(7, -1.\overline{1})$



$f(x) = x^3 - 9x^2 + 24x - 10$

① y-intercept $(x=0)$

$f(0) = (0)^3 - 9(0)^2 + 24(0) - 10$

$= -10$

$(0,-10)$

② Asymptotes: None

(Polynomial Function)

③ $f(x) = x^3 - 9x^2 + 24x - 10$

$f'(x) = 3x^2 - 18x + 24$

$f'(x) = 3(x^2 - 6x + 8)$

$f'(x) = 3(x-4)(x-2)$

CV: $x = 2, 4$

max min

+ − +

(1) 2 (3) 4 (5)

Increasing on $(-\infty,2) + (4,\infty)$

Decreasing on $(2,4)$

Max/min

$f(x) = x^3 - 9x^2 + 24x - 10$

$f(2) = (2)^3 - 9(2)^2 + 24(2) - 10 = 10$ $(2,10)$ max

$f(4) = (4)^3 - 9(4)^2 + 24(4) - 10 = 6$ $(4,6)$ min

Concavity:

$f(x) = x^3 - 9x^2 + 24x - 10$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18$

$f''(x) = 6(x-3)$

CV: $x = 3$

IP

− +

(0) 3 (4)

CD on $(-\infty,3)$

CU on $(3,\infty)$

IP

$f(x) = x^3 - 9x^2 + 24x - 10$

$f(3) = (3)^3 - 9(3)^2 + 24(3) - 10 = 8$ $(3,8)$ IP