

Warm Up

Differentiate:

$$\sec^{-1}(\tan^2 \sqrt{x})$$

$$(1) f(x) = e^{\log \sqrt{x}} - 3^{\cos^{-1}(\ln x^5)} + \text{Arc sec}(\tan^2 \sqrt{x})$$

$$f'(x) = e^{\log \sqrt{x}} \left(\frac{1}{\sqrt{x} \ln 10} \right) \left(\frac{1}{2\sqrt{x}} \right) - 3^{\cos^{-1}(\ln x^5)} (\ln 3) \left(\frac{-1}{\sqrt{1-(\ln x^5)^2}} \right) \left(\frac{5x^4}{x^5} \right) + \left(\frac{1}{\tan^2 \sqrt{x} \sqrt{(\tan^2 \sqrt{x})^2 - 1}} \right) \left(\frac{2(\tan \sqrt{x}) \sec^2 \sqrt{x}}{2\sqrt{x}} \right)$$

Problems with homework?

Antiderivatives

Up to this point in our study of calculus, we have been concerned primarily with the problem:

Given a function, find its derivative.

Many important applications of calculus involve the inverse problem:

Given the derivative, find the original function.

"The basic problem of differentiation is: given the path of a moving point, to calculate its velocity, or given a curve, to calculate its slope. The basic problem of integration is the inverse: given the velocity of a moving point at every instant, to calculate its path, or given the slope of a curve at each of its points, to calculate the curve."

For example, suppose we are given the following

derivatives: $f'(x) = 2$, $g'(x) = 3x^2$, $h'(t) = 4t$
 $f(x) = 2x + C$, $g(x) = x^3 + C$, $h(t) = 2t^2 + C$

Our goal is to determine $f(x)$, $g(x)$, and $h(x)$ that have the respective derivatives given above. If we make some educated guesses, what would these functions be????

This operation of determining the original function from its derivative is the inverse operation of differentiation and we call it antidifferentiation.

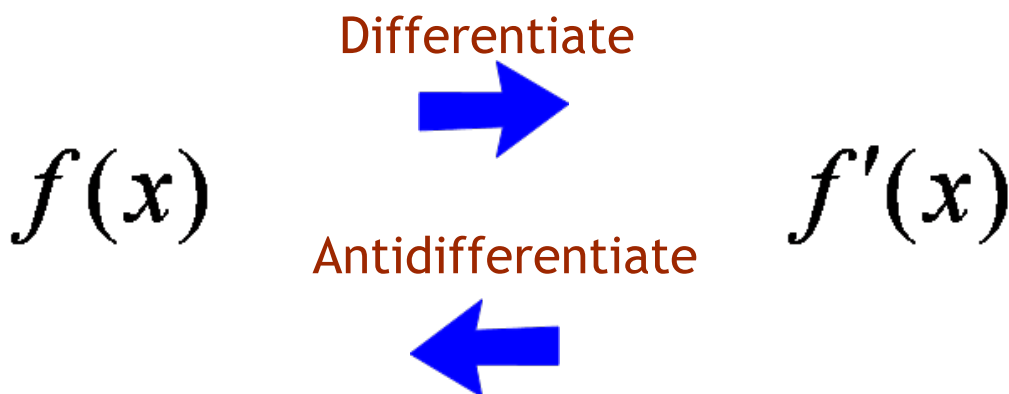
Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I .

" $F(x)$ is an antiderivative of $f(x)$ "

It should be emphasized that if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$ (C is any constant) is also an antiderivative of $f(x)$.

$$\begin{array}{l|l} f(x) = -5 & f'(x) = 3 \\ F(x) = -5x + C & f(x) = 3x + C \end{array}$$

Antiderivatives



Definition A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I .

Example:

$$F(x) = \frac{x^2}{2} \quad \begin{array}{c} \dashrightarrow \\ \dashleftarrow \end{array} \quad F'(x) = x$$

What about $F(x) = \frac{x^2}{2} + 2$?

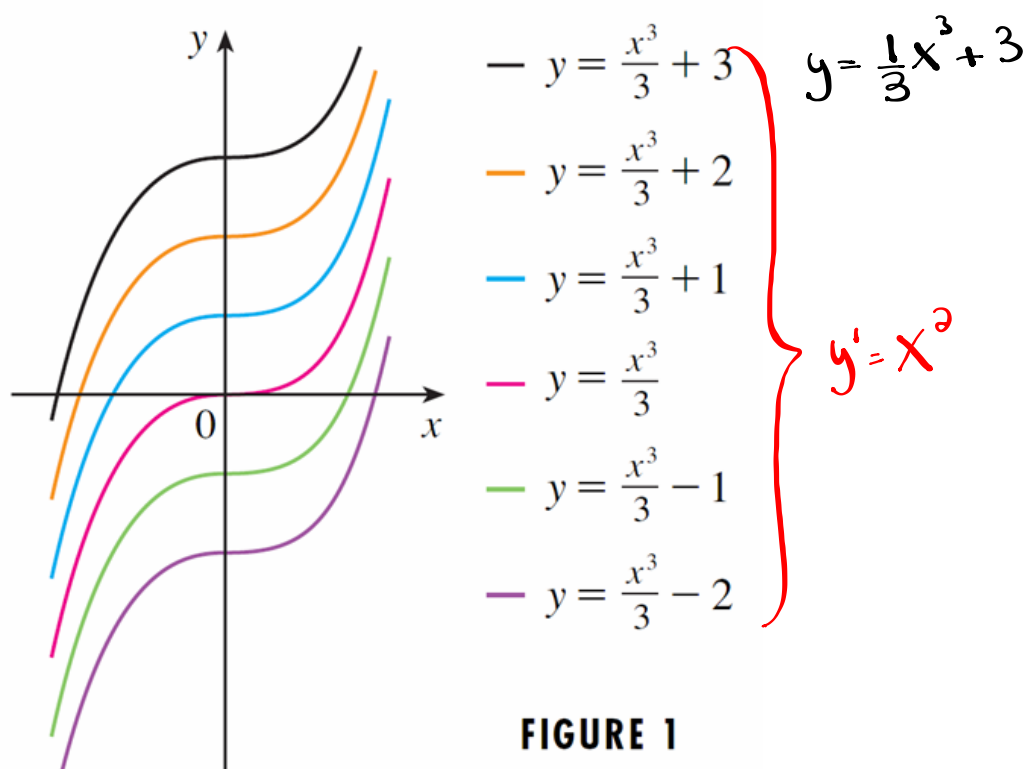
1 Theorem If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

General antiderivatives are considered a family of curves...

Here is an example of a family of general antiderivatives:



Notice the slopes of the tangents on each curve at the same x-coordinate.

Antidifferentiation Rules...

Constants:

Determine the antiderivative of any constant

$$f(x) = 17$$

$$F(x) = 17x + C$$

Rule:

$$f(x) = k \Rightarrow F(x) = kx + C$$

constant

Power Law:

How will we put the power rule in reverse?

ie. If $f'(x) = 6x^2$ what is $f(x)$?

$$f(x) = 2x^3 + C$$

Flip your brain into reverse...what will be the rule used to antidifferentiate power rules?

Rule:

$$f(x) = kx^n \Rightarrow F(x) = \frac{k}{n+1} x^{n+1} + C$$

Add one to the exponent and divide by this NEW exponent

$$f(x) = 4x^7$$

$$F(x) = \frac{4}{8} x^8 + C$$

$$F(x) = \frac{1}{2} x^8 + C$$

$$f'(x) = 23x^{-1/4}$$

$$f(x) = \frac{23x^{3/4}}{3/4} + C$$

$$f(x) = \frac{92}{3} x^{3/4} + C$$

$$23 \cdot \frac{4}{3} = \frac{92}{3}$$

Determine the general antiderivative of each of the following...

$$(1) f(x) = x^5 - 2x^4 - 3x^3 + \frac{2}{x^2} + 5x^{-3} + 5$$

$$f(x) = x^5 - 2x^4 - 3x^3 + 2x^{-2} + 5x^{-3} + 5$$

$$F(x) = \frac{x^6}{6} - \frac{2x^5}{5} - \frac{3x^4}{4} + \frac{2x^{-1}}{-1} + \frac{5x^{-2}}{-2} + 5x + C$$

$$F(x) = \frac{1}{6}x^6 - \frac{2}{5}x^5 - \frac{3}{4}x^4 - \frac{2}{x} - \frac{5}{2}x^{-2} + 5x + C$$

$$(2) f(x) = 3\sqrt{x} - \frac{2}{5x^4} + \sqrt[5]{x^7} - \frac{6\sqrt{x}}{x^2} + e^2$$

$\frac{x^{1/2}}{x^2} = x^{-3/2}$
constant

$$f(x) = 3x^{1/2} - \frac{2}{5}x^{-4} + x^{7/5} - 6x^{-3/2} + e^2$$

$$F(x) = \frac{3x^{3/2}}{3/2} - \frac{2x^{-3}}{-3} + \frac{1x^{12/5}}{12/5} - \frac{6x^{-1/2}}{-1/2} + e^2x + C$$

$$F(x) = 2x^{3/2} + \frac{2}{3}x^{-3} + \frac{5}{12}x^{12/5} + 12x^{-1/2} + e^2x + C$$

$$3 \cdot \frac{2}{3} = \frac{6}{3} = 2 \quad \left| \quad \frac{-2}{5} \cdot \frac{-1}{3} = \frac{2}{15} \quad \left| \quad 1 \cdot \frac{5}{12} = \frac{5}{12} \quad \left| \quad -6 \cdot -2 = 12 \right.$$

- ✓ constants
- ✓ power rules
- logarithmic functions
- trigonometric functions
- exponential functions
- inverse trigonometric functions
- chain rules

Table of some of the Most General Antiderivatives

where a is a constant!

Function, $f(x)$	Most General Antiderivative, $F(x)$
a	$ax + C$
ax^n ($n \neq -1$)	$\frac{a}{n+1} x^{n+1} + C$
$\frac{a}{x}$ ($x \neq 0$)	$a \ln x + C$
ae^{kx}	$\frac{a}{k} e^{kx} + C$
a^{kx}	$\frac{a^x}{k \ln a} + C$
$a \cos kx$	$\frac{a}{k} \sin kx + C$
$a \sin kx$	$-\frac{a}{k} \cos kx + C$
$a \sec^2 kx$	$\frac{a}{k} \tan kx + C$
$a \sec kx \tan kx$	$\frac{a}{k} \sec kx + C$
$a \csc kx \cot kx$	$-\frac{a}{k} \csc kx + C$
$a \csc^2 kx$	$-\frac{a}{k} \cot kx + C$
$\frac{a}{\sqrt{1 - (kx)^2}}$	$\frac{a}{k} \sin^{-1} kx + C$
$\frac{a}{1 + (kx)^2}$	$\frac{a}{k} \tan^{-1} kx + C$

Practice:

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