Questions from Homework

② c) $f(x) = \frac{-3}{x} + 5x^{-2}$

$F(x) = -3\ln|x| + \frac{5x^{-1}}{-1} + C$

$\boxed{F(x) = -3\ln|x| - \frac{5}{x} + C}$

② d) $f(x) = x^{-7} + x^{-5} + x^{-3} + \frac{1}{x}$

$F(x) = \frac{x^{-6}}{-6} + \frac{x^{-4}}{-4} + \frac{x^{-2}}{-2} + \ln|x| + C$

$\boxed{F(x) = -\frac{1}{6x^6} - \frac{1}{4x^4} - \frac{1}{2x^2} + \ln|x| + C}$

* ③ c) $f(x) = \sqrt{-x} = (-x)^{1/2}$

$F(x) = \frac{(-x)^{3/2}}{\frac{3}{2}}(-1) + C$

when you have a negative x under a radical sign!

$F(x) = -\frac{2}{3}(-x)^{3/2} + C$

$F'(x) = -(-x)^{1/2}(-1)$

$= \sqrt{-x}$

Warm Up

Determine the general antiderivative for the following:

- What would you differentiate that would give the function below?
- Remember add 1 to the exponent, then divide by this exponent.

Find the most general antiderivative of:

$$f'(x) = 7x^3 + 9x^2 + 8x - 1$$

$$f(x) = \frac{7x^4}{4} + \frac{9x^3}{3} + \frac{8x^2}{2} - 1x + C$$

$$f(x) = \frac{7}{4}x^4 + 3x^3 + 4x^2 - x + C$$

Antiderivatives

This operation of determining the original function from its derivative is the inverse operation of **differentiation** and we call it **antidifferentiation**.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

"$F(x)$ is an antiderivative of $f(x)$"

It should be emphasized that if $F(x)$ is an antiderivative of $f(x)$, then $F(x) + C$ **(C is any constant)** is also an antiderivative of $f(x)$.

Indefinite Integration

The process of antidifferentiation is often called **integration or indefinite integration.** To indicate that the antiderivative of $f(x) = 3x^2$ is $F(x) = x^3 + C$, we write

$$\int 3x^2\,dx = x^3 + C$$

$f(x) = 3x^2$

$F(x) = \frac{3x^3}{3} + C = x^3 + C$

We say that the **antiderivative or indefinite integral** of $3x^2$ with respect to x equals $x^3 + C$.

In general, $$\int f(x)\,dx = F(x) + C$$

Integral Sign Integrand Variable of Intergration Constant of Integration

Examples:

Determine the general antiderivative:

$$f'(x) = 8x^{1/2} + 2x^{-3} + 5x - 1$$

$$f(x) = \frac{8x^{3/2}}{3/2} + \frac{2x^{-2}}{-2} + \frac{5x^2}{2} - 1x + C$$

$$f(x) = \frac{16}{3}x^{3/2} - \frac{1}{x^2} + \frac{5}{2}x^2 - x + C$$

$$\int (x^{5/6} - 3x^{9/2} + x^{-6} - 3x^{-1/2})dx$$

$$-3 \cdot \frac{2}{11} = \frac{-6}{11}$$

$$= \frac{x^{11/6}}{11/6} - \frac{3x^{11/2}}{11/2} + \frac{x^{-5}}{-5} - \frac{3x^{1/2}}{1/2} + C$$

$$-3 \cdot 2 = -6$$

$$= \frac{6}{11}x^{11/6} - \frac{6}{11}x^{11/2} - \frac{1}{5}x^{-5} - 6x^{1/2} + C$$

Table of some of the Most General Antiderivatives

where a is a constant!

Function, f(x)	Most General Antiderivative, F(x)
a	$ax + C$
$ax^{n}\ (n \neq -1)$	$\frac{a}{n+1}x^{n+1} + C$
$\frac{a}{x}\ (x \neq 0)$	$a\ln\lvert x\rvert + C$
ae^{kx}	$\frac{a}{k}e^{kx} + C$
a^{kx}	$\frac{a^{x}}{k\ln a} + c$
$a\cos kx$	$\frac{a}{k}\sin kx + C$
$a\sin kx$	$-\frac{a}{k}\cos kx + C$
$a\sec^{2} kx$	$\frac{a}{k}\tan kx + C$
$a\sec kx\tan kx$	$\frac{a}{k}\sec kx + C$
$a\csc kx\cot kx$	$-\frac{a}{k}\csc kx + C$
$a\csc^{2} kx$	$-\frac{a}{k}\cot kx + C$
$\frac{a}{\sqrt{1-(kx)^{2}}}$	$\frac{a}{k}\sin^{-1} kx + C$
$\frac{a}{1+(kx)^{2}}$	$\frac{a}{k}\tan^{-1} kx + C$

Examples:

Determine the general antiderivative:

$$\int 5e^x dx = \frac{5}{1}e^{1x} + C$$

$$= 5e^x + C$$

Note: Constants do not change these but powers do

$$f(x) = \frac{10}{x}$$

$$F(x) = 10\ln|x| + C$$

All of these have a linear power of x (that is x is to the power of one).

Examples:

Determine the general antiderivative:

$$\int e^{10x}dx$$

If there is a constant in front of the linear x then divide by that constant (do not add one to the constant for these simple integrals).

$$= \frac{1}{10}e^{10x} + C$$

$$\int (e^{5x} - 4e^{6x} + \sin 12x - \sec^2 8x)dx$$

$$= \frac{1}{5}e^{5x} - \frac{4}{6}e^{6x} - \frac{1}{12}\cos 12x - \frac{1}{8}\tan 8x + C$$

$$= \frac{1}{5}e^{5x} - \frac{2}{3}e^{6x} - \frac{1}{12}\cos 12x - \frac{1}{8}\tan 8x + C$$

$$\int x^3 + 9x^{-5} + \frac{2}{x} + 7e^{-2x}dx$$

$$= \frac{x^4}{4} + \frac{9x^{-4}}{-4} + 2\ln x + \frac{7}{-2}e^{-2x} + C$$

$$= \frac{x^4}{4} - \frac{9}{4x^4} + 2\ln x - \frac{7}{2e^{2x}} + C$$

Practice Problems...

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Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x) = [g(x)]^n$$

$$f'(x) = n[g(x)]^{n-1} g'(x)$$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

1. $f'(x) = 2x - \cos x + 1, \quad f(0) = 3$

2. $f''(x) = 12x^2 + 6x - 4, \quad f(0) = 4 \text{ and } f(1) = 1$

$$f(x) = 2\sqrt[4]{x^5} - \frac{3}{x^2} + xe^{-8x^2} - \frac{2x}{1+x^4} + \frac{2}{5x} + 3x^3\cos 5x^2$$