Warm Up

Evaluate the following limits:

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

$$\lim_{x\to\pi/4} \frac{\cos x + \sin x}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \frac{2\sqrt{2}}{2}$$

$$= \sqrt{2}$$

$$\lim_{x\to 0} \frac{Arc\sin x}{x}$$

$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1}$$

$$= 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{\sin 2x}$$

$$\lim_{x\to 0} \frac{e^x}{2\cos 2x}$$

$$= \frac{1}{2}$$

$$\lim_{x\to\infty} \frac{10x^2}{e^x}$$

$$\lim_{x\to\infty} \frac{20x}{e^x}$$

$$\lim_{x\to\infty} \frac{20}{e^x} = 0$$

Sigma Notation

A series is the sum of a sequence. We can write a series using sigma notation.

$$\boxed{\sum_{i=1}^{n} t_i = t_1 + t_2 + t_3 + \ldots + t_n}$$

$$1+2+4+\ldots+64 = \sum_{i=1}^{7} 2^{n-1}$$

2 2

the terms form a geometric sequence with a = 1, r = 2, $t_n = 1(2)^{n-1}$

This symbol is read as "the sum of the terms of the sequence given by $t_n=2^{n-1}$ from n = 1 to n = 7"

$t_n = ar^{n-1}$ (geometric)

$t_n = (1)(2)^{n-1}$

Example 1

Express the series 1 + 3 + 5 + 7 + 9 in sigma notation. (arithmetic)

$d = 3-1 = 5-3 = 7-5$

$d = 2$

$a = 1$

$t_n = a + (n-1)d$

$t_n = 1 + (n-1)(2)$

$t_n = 1 + 2n - 2$

$t_n = 2n - 1$

$$\boxed{\sum_{i=1}^{5} 2n-1}$$

The properties of Sigma Notation that we use in this section are summarized below:

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c = nc \quad \Big| \quad \sum_{i=1}^{5} 4 = 5(4) = 20$$

$$\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i \text{, } c \text{ is a constant} \quad \Big| \quad \sum_{i=1}^{4} 3t_i = 3\sum_{i=1}^{4} t_i$$

$$\sum_{i=1}^{n} (t_i + s_i) = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} s_i \quad \Big| \quad \sum_{i=1}^{n} (4i + 7i) = \sum_{i=1}^{n} 4i + \sum_{i=1}^{n} 7i$$

$$= 4\sum_{i=1}^{n} i + 7\sum_{i=1}^{n} i$$

Example 2

Use the basic properties of sigma notation to express $\sum_{i=1}^{n}(3i-2)^2$

in terms of monomial summations.

$$\sum_{i=1}^{n}(3i-2)(3i-2)$$

$$\sum_{i=1}^{n}(9i^2-12i+4)$$

$$\sum_{i=1}^{n}9i^2+\sum_{i=1}^{n}-12i+\sum_{i=1}^{n}4$$

$$\boxed{9\boxed{\sum_{i=1}^{n}i^2}-12\boxed{\sum_{i=1}^{n}i}+4n}$$

$$9\left[\frac{n(n+1)(2n+1)}{6}\right]-12\left[\frac{n(n+1)}{2}\right]+4n$$

The following sigma formulas will be extremely useful in the next few days when we are faced with the challenge of calculating the area under a curve.

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \ldots n = \boxed{\frac{n(n+1)}{2}}$$ Linear

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$$ Quadratic

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots n^3 = \boxed{\left[\frac{n(n+1)}{2}\right]^2}$$ Cubic

Example 3

$$\sum_{i=1}^{n} \left(3i^2 - 2i\right)$$

$$= \sum_{i=1}^{n} 3i^2 + \sum_{i=1}^{n} -2i$$

$$= 3\boxed{\sum_{i=1}^{n} i^2} - 2\boxed{\sum_{i=1}^{n} i}$$

$$= \cancel{3}^{1}\left[\frac{n(n+1)(2n+1)}{\cancel{6}\ 2}\right] - {}^{1}\cancel{2}\left[\frac{n(n+1)}{\cancel{2}\ 1}\right]$$

$$= \frac{n(n+1)(2n+1)}{2} - \frac{n(n+1)}{1}$$

$$= \frac{n(n+1)(2n+1)}{2} - \frac{2n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1) - 2n(n+1)}{2}$$ Factor

$$= \frac{n(n+1)\left[(2n+1) - 2\right]}{2}$$

$$= \boxed{\frac{n(n+1)(2n-1)}{2}}$$

Example 4

$$\sum_{i=11}^{20}(2i^2-3i) = \sum_{i=1}^{20}(2i^2-3i) - \sum_{i=1}^{10}(2i^2-3i)$$

$$= \sum_{i=1}^{20}2i^2 + \sum_{i=1}^{20}-3i - \left(\sum_{i=1}^{10}2i^2 + \sum_{i=1}^{10}-3i\right)$$

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$$= \underbrace{2\sum_{i=1}^{20}i^2 - 3\sum_{i=1}^{20}i}_{n=20} - \underbrace{2\sum_{i=1}^{10}i^2 + 3\sum_{i=1}^{10}i}_{n=10}$$

$$= \cancel{2}^{1}\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right] - 3\left[\frac{n(n+1)}{2}\right] - \cancel{2}^{1}\left[\frac{n(n+1)(2n+1)}{\cancel{6}\,3}\right] + 3\left[\frac{n(n+1)}{2}\right]$$

$$= \frac{20(21)(41)}{3} - \frac{3(20)(21)}{2} - \frac{(10)(11)(21)}{3} + \frac{3(10)(11)}{2}$$

$$= 5740 - 630 - 770 + 165$$

$$= 4505$$

$$\sum_{i=11}^{20}(2i^2-3i) = [2(11)^2-3(11)]+[2(12)^2-3(12)]+\ldots+[2(20)^2-3(20)]$$

$$= 209+252+299+350+405+464+527+594+665+740$$

$$= 4505$$

Homework

Page 447 #1-3 omit ~~3 d, e~~ a, d, e
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