

Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \leq 0$$

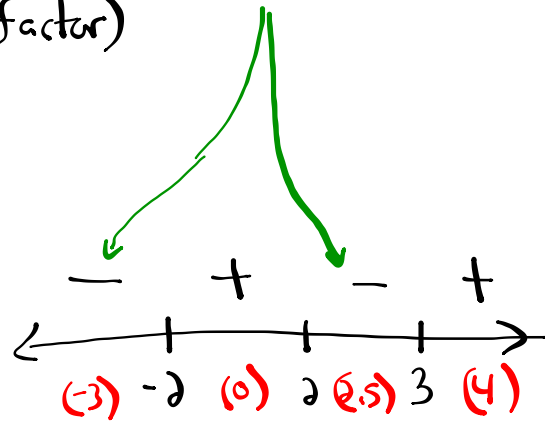
where does this function have y values that are less than or equal to zero

$$y = x^3 - 3x^2 - 4x + 12 \quad (\text{factor})$$

$$y = x^2(x-3) - 4(x-3)$$

$$y = (x-3)(x^2-4)$$

$$y = (x-3)(x-2)(x+2)$$



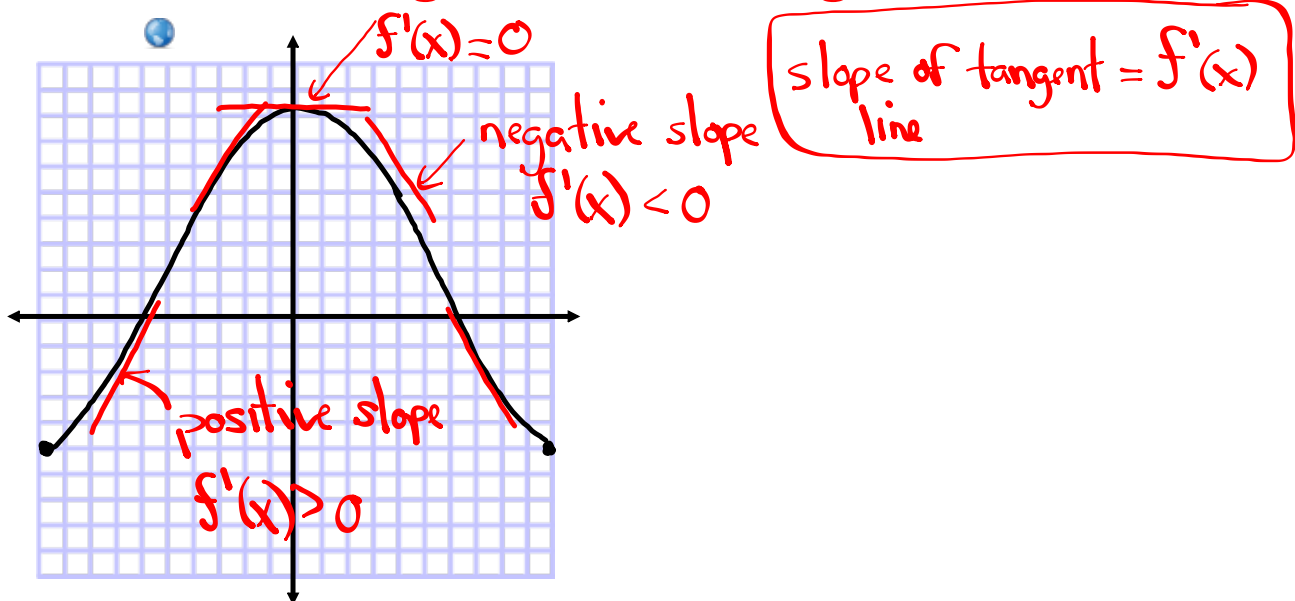
Find x int ($y=0$)

$$x \in (-\infty, -2] \cup [2, 3]$$

$$0 = (x-3)(x-2)(x+2)$$

$$\begin{array}{c|c|c} x-3=0 & x-2=0 & x+2=0 \\ \hline x=3 & x=2 & x=-2 \end{array}$$

Increasing and Decreasing Functions



Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I , then f is increasing on I .

$f'(x)$ is positive

2. If $f'(x) < 0$ for all x in an interval I , then f is decreasing on I .

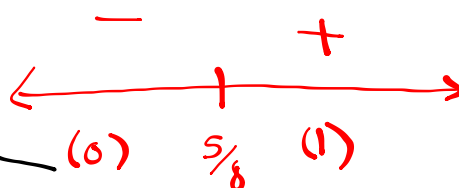
$f'(x)$ is negative

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$$f'(x) = 1 - 5x + 4x^2$$

$$f'(x) = -5 + 8x$$



CV: $f'(x) = 0$ or undefined

$$0 = -5 + 8x$$

$$5 = 8x$$

$$\frac{5}{8} = x$$

$$\text{CV: } x = \frac{5}{8} = 0.625$$

Thus f will be increasing on the interval

$$\underline{\left(\frac{5}{8}, \infty\right)}$$

Similarly,

Thus f will be decreasing on the interval

$$\underline{\left(-\infty, \frac{5}{8}\right)}$$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

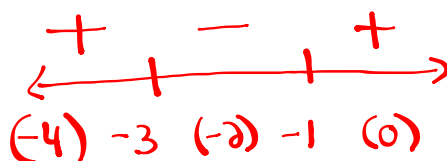
Solution

$$y = x^3 + 6x^2 + 9x + 2$$

$$y' = 3x^2 + 12x + 9$$

$$y' = 3(x^2 + 4x + 3)$$

$$y' = 3(x+3)(x+1)$$



cv: $y' = 0$ or undefined

$$0 = 3(x+3)(x+1)$$

$$\begin{array}{l|l|l} 3 \neq 0 & x+3=0 & x+1=0 \\ & x=-3 & x=-1 \end{array}$$

$$\text{w: } x = -3, -1$$

Increasing on $(-\infty, -3) \cup (-1, \infty)$

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

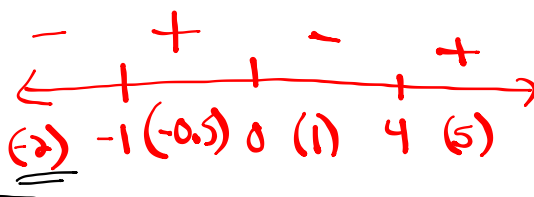
First we compute the derivative and factor it:

$$f(x) = x^4 - 4x^3 - 8x^2 - 1$$

$$f'(x) = 4x^3 - 12x^2 - 16x$$

$$f'(x) = 4x(x^2 - 3x - 4)$$

$$f'(x) = 4x(x-4)(x+1)$$



cv: $f'(x) = 0$ or undefined Increasing on $(-1, 0) + (4, \infty)$

$$0 = 4x(x-4)(x+1)$$

Decreasing on $(-\infty, -1) + (0, 4)$

$$\begin{array}{l|l|l} 4x=0 & x-4=0 & x+1=0 \\ x=0 & x=4 & x=-1 \end{array}$$

$$\text{cv: } x = -1, 0, 4$$

Homework