

Slant Asymptotes:

$$\textcircled{d)} \quad y = \frac{(x-1)^3}{x^2} = \frac{(x-1)(x-1)(x-1)}{x^2} = \frac{x^3 - 3x^2 + 3x - 1}{x^2}$$

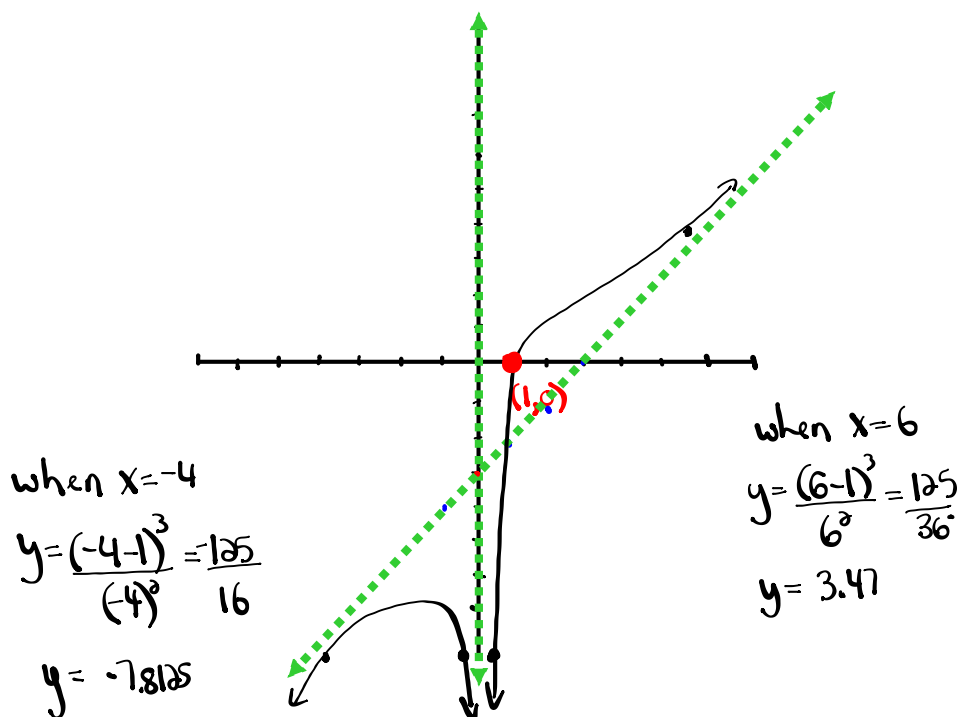
<p>i) x-int (y=0)</p> $x^2 \cdot 0 = \frac{(x-1)^3}{x^2} \cdot x^2$ $0 = (x-1)^3$ $0 = x-1$ $1 = x$ <p>(1,0)</p>	<p>ii) y-int (x=0)</p> $y = \frac{(0-1)^3}{0^2} = \frac{-1}{0}$ <p>undefined</p> <p>No y-int</p>	<p>iii) VA: $x^2 = 0$</p> <div style="border: 1px solid green; padding: 2px; display: inline-block;"> $x = 0$ </div> <p>$\lim_{x \rightarrow 0^-} \frac{(-)}{(+)} = -\infty$ (x=-0.1)</p> <p>$\lim_{x \rightarrow 0^+} \frac{(-)}{(+)} = -\infty$ (x=0.1)</p>
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iv) SA:
$$\begin{array}{r} x^2 \overline{) \frac{x^3 - 3x^2 + 3x - 1}{x^3}} \\ \underline{-(x^3)} \\ -3x^2 + 3x - 1 \\ \underline{-(-3x^2)} \\ 3x - 1 \quad R \end{array}$$

$y = x - 3$

$m = \frac{1}{1}$ rise
run

$b = -3$ y-int



Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \leq 0$$

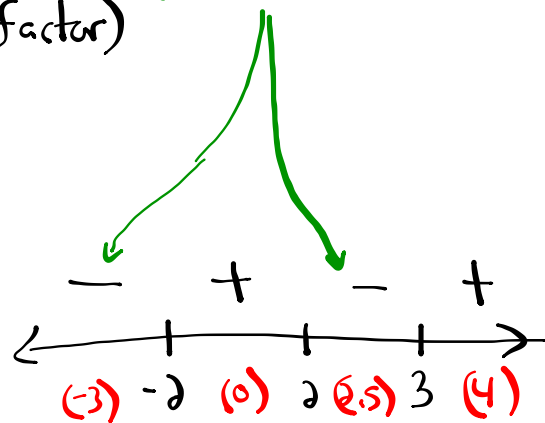
where does this function have y values that are less than or equal to zero

$$y = x^3 - 3x^2 - 4x + 12 \quad (\text{factor})$$

$$y = x^2(x-3) - 4(x-3)$$

$$y = (x-3)(x^2-4)$$

$$y = (x-3)(x-2)(x+2)$$



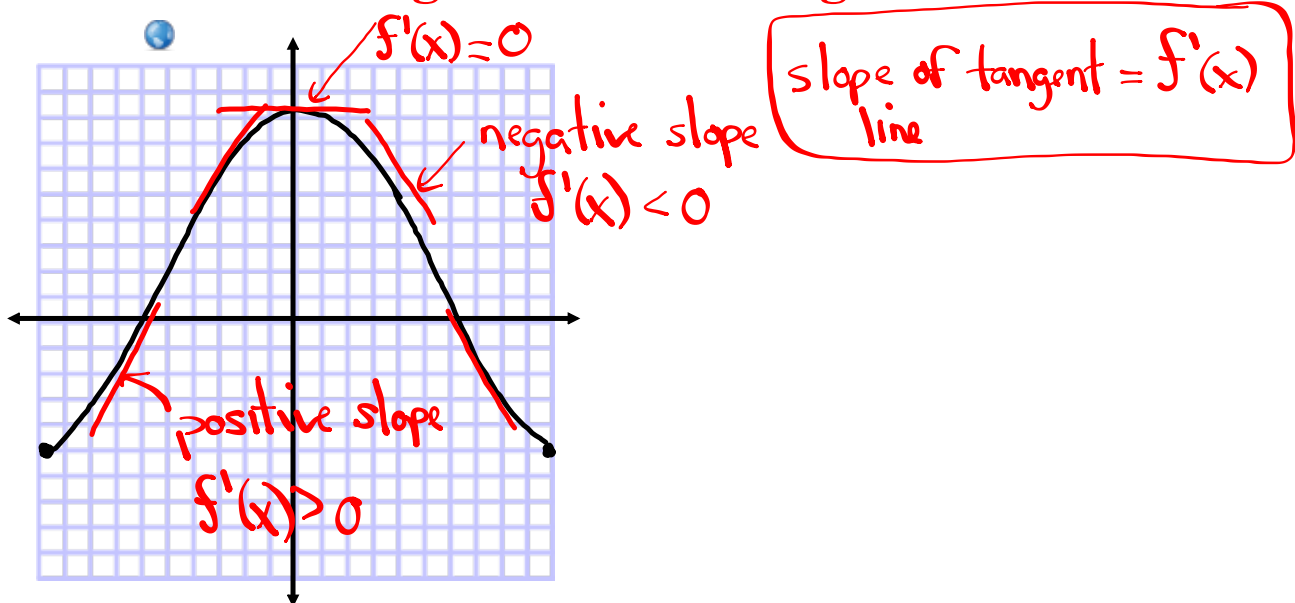
Find x int ($y=0$)

$$x \in (-\infty, -2] \cup [2, 3]$$

$$0 = (x-3)(x-2)(x+2)$$

$$\begin{array}{l|l|l} x-3=0 & x-2=0 & x+2=0 \\ x=3 & x=2 & x=-2 \end{array}$$

Increasing and Decreasing Functions



Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I , then f is increasing on I .

$f'(x)$ is positive

2. If $f'(x) < 0$ for all x in an interval I , then f is decreasing on I .

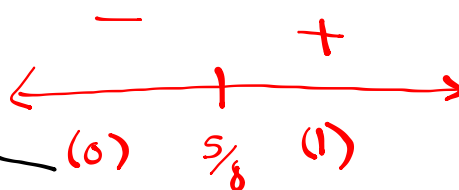
$f'(x)$ is negative

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

$$f'(x) = 1 - 5x + 4x^2$$

$$f'(x) = -5 + 8x$$



CV: $f'(x) = 0$ or undefined

$$0 = -5 + 8x$$

$$5 = 8x$$

$$\frac{5}{8} = x$$

$$\text{CV: } x = \frac{5}{8} = 0.625$$

Thus f will be increasing on the interval

$$\underline{\left(\frac{5}{8}, \infty\right)}$$

Similarly,

Thus f will be decreasing on the interval

$$\underline{\left(-\infty, \frac{5}{8}\right)}$$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

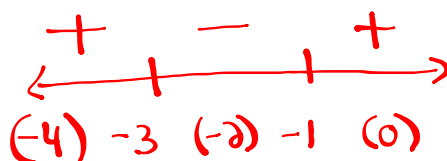
Solution

$$y = x^3 + 6x^2 + 9x + 2$$

$$y' = 3x^2 + 12x + 9$$

$$y' = 3(x^2 + 4x + 3)$$

$$y' = 3(x+3)(x+1)$$



cv: $y' = 0$ or undefined

$$0 = 3(x+3)(x+1)$$

$$\begin{array}{l|l|l} 3 \neq 0 & x+3=0 & x+1=0 \\ & x=-3 & x=-1 \end{array}$$

$$w: x = -3, -1$$

Increasing on $(-\infty, -3) \cup (-1, \infty)$

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

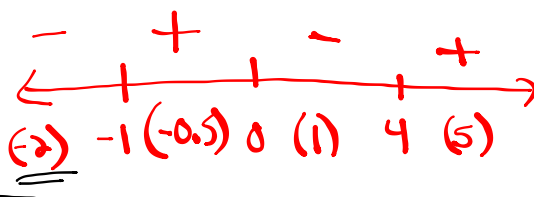
First we compute the derivative and factor it:

$$f(x) = x^4 - 4x^3 - 8x^2 - 1$$

$$f'(x) = 4x^3 - 12x^2 - 16x$$

$$f'(x) = 4x(x^2 - 3x - 4)$$

$$f'(x) = 4x(x-4)(x+1)$$



cv: $f'(x) = 0$ or undefined Increasing on $(-1, 0) + (4, \infty)$

$0 = 4x(x-4)(x+1)$ Decreasing on $(-\infty, -1) + (0, 4)$

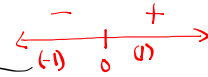
$$\begin{array}{l|l|l} 4x=0 & x-4=0 & x+1=0 \\ x=0 & x=4 & x=-1 \end{array}$$

$$\text{cv: } x = -1, 0, 4$$

Homework $f'(x) > 0 \rightarrow$ increasing
 $f'(x) < 0 \rightarrow$ decreasing

Ⓐ b) $f(x) = x^4$

$f'(x) = 4x^3$



CV: $f'(x) = 0$ or undefined

$0 = 4x^3$

$0 = x^3$

$0 = x$

CV: $x = 0$

decreasing on $(-\infty, 0)$
 increasing on $(0, \infty)$

Ⓒ g) $y = x\sqrt{4-x} = \underbrace{x}_{f(x)} \cdot \underbrace{(4-x)^{1/2}}_{g(x)}$ $f'(x)g(x) + f(x)g'(x)$

$y' = 1(4-x)^{1/2} + x \left(\frac{1}{2}\right)(4-x)^{-1/2}(-1)$

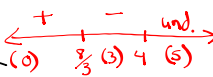
$y' = (4-x)^{1/2} - \frac{x}{2}(4-x)^{-1/2}$ $\frac{(4-x)^{1/2}}{(4-x)^{1/2}} = (4-x)^{1/2 - (1/2)}$

$y' = (4-x)^{-1/2} \left[(4-x) - \frac{x}{2} \right]$

$y' = (4-x)^{-1/2} \left(\frac{8}{2} - \frac{2x}{2} - \frac{x}{2} \right)$

$y' = (4-x)^{-1/2} \left(\frac{8-3x}{2} \right)$

$y' = \frac{8-3x}{2(4-x)^{1/2}}$



CV: $y' = 0$ or undefined

$8-3x=0$ $2(4-x)^{1/2}=0$ Increasing on $(-\infty, 8/3)$

$8=3x$ $(4-x)^{1/2}=0$ Decreasing on $(8/3, 4)$

$8/3 = x$ $4-x=0$

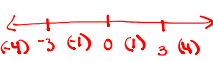
$x = 2.\bar{6}$ $4 = x$

CV: $x = 8/3, 4$

Ⓓ h) $y = (x^2-9)^{3/3}$

$y' = \frac{2}{3}(x^2-9)^{1/3}(2x)$

$y' = \frac{4x}{3(x^2-9)^{1/3}}$



CV: $4x=0$ $3(x^2-9)^{1/3}=0$

$x=0$ $(x^2-9)^{1/3}=0$

$x^2-9=0$

$x^2=9$

$x = \pm 3$