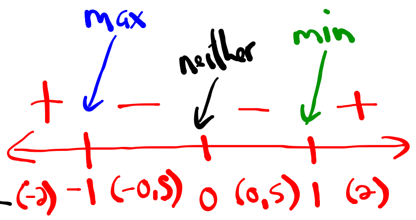
Questions from homework

②f) $h(x) = 3x^5 - 5x^3$

$h'(x) = 15x^4 - 15x^2$

$h'(x) = 15x^2(x^2 - 1)$

$h'(x) = 15x^2(x+1)(x-1)$



CV: $15x^2 = 0$ | $x+1=0$ | $x-1=0$

$x^2 = 0$ | $x = -1$ | $x = 1$

$x = 0$

Increasing on $(-\infty, -1)$ + $(1, \infty)$

Decreasing on $(-1, 0)$ + $(0, 1)$

$h(x) = 3x^5 - 5x^3$

$h(-1) = 3(-1)^5 - 5(-1)^3$

$h(-1) = 3(-1) - 5(-1)$

$h(-1) = -3 + 5$

$h(-1) = 2$

$(-1, 2)$ max

$h(x) = 3x^5 - 5x^3$

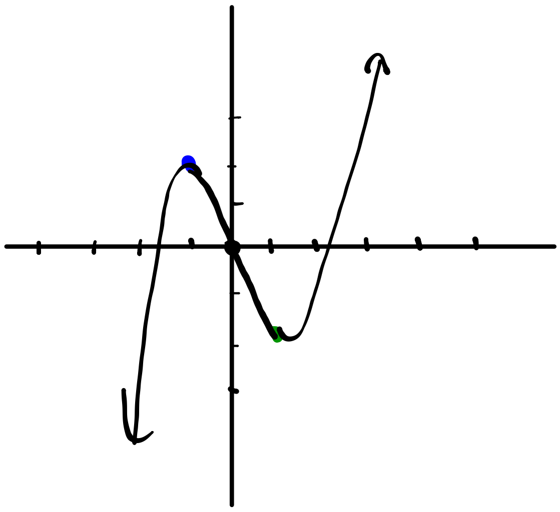
$h(1) = 3(1)^5 - 5(1)^3$

$h(1) = 3(1) - 5(1)$

$h(1) = 3 - 5$

$h(1) = -2$

$(1, -2)$ min



Questions from homework

③a) $f(x) = 2x^{2/3}(3-4x^{1/3}) = 6x^{2/3} - 8x$

$$f'(x) = 4x^{-1/3} - 8$$

$$= \frac{4}{x^{1/3}} - \frac{8}{1} \rightarrow \frac{4}{x^{1/3}} - \frac{8x^{1/3}}{x^{1/3}} \rightarrow \frac{4-8\sqrt[3]{x}}{\sqrt[3]{x}}$$

$$= \frac{4-8x^{1/3}}{x^{1/3}}.$$

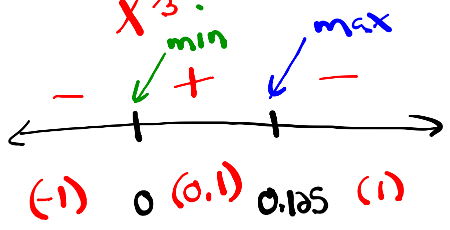
CV: $4-8x^{1/3}=0$ | $x^{1/3}=0$

$4=8x^{1/3}$ | $x=0$

$\frac{1}{2} = x^{1/3}$

$\frac{1}{8} = x$

$0.125 = x$



Decreasing on $(-\infty, 0)$ and $(\frac{1}{8}, \infty)$

Increasing on $(0, \frac{1}{8})$

min $(x=\underline{\underline{0}})$

$h(x) = 6x^{2/3} - 8x$

$h(0) = 6(0)^{2/3} - 8(0)$

$h(0) = 0$

$(0,0)$ min

max $(x = 1/8)$

$h(x) = 6x^{2/3} - 8x$

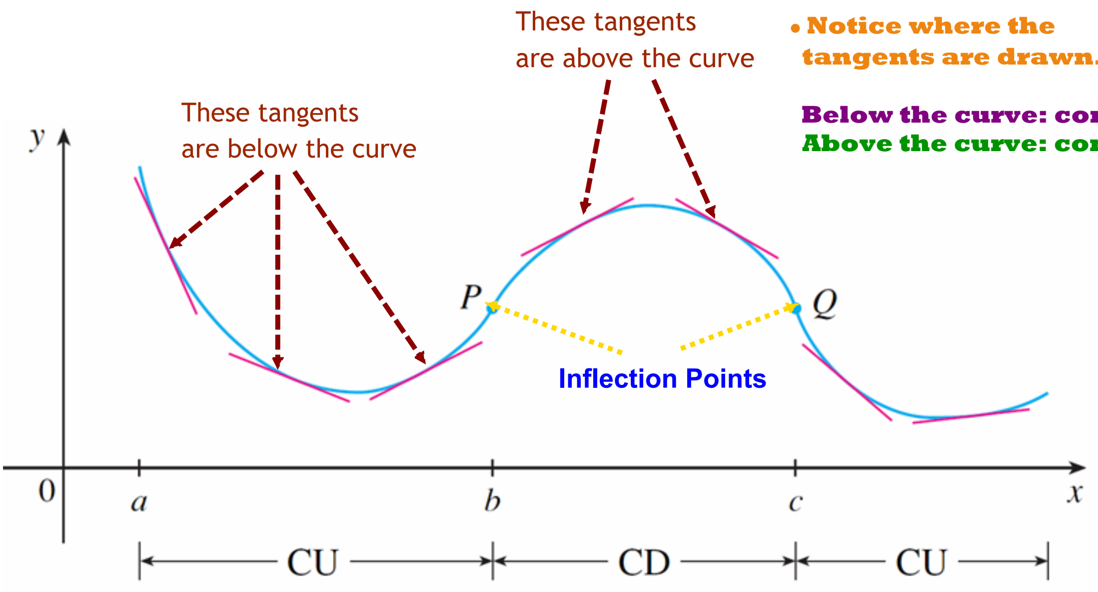
$h(\frac{1}{8}) = 6(\frac{1}{8})^{2/3} - 8(\frac{1}{8})$

$h(\frac{1}{8}) = \frac{3}{2} \quad -1$

$h(\frac{1}{8}) = \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$

$(\frac{1}{8}, \frac{1}{2})$ max

Concavity



- **Notice where the tangents are drawn...**

Below the curve: concave up
Above the curve: concave down

- In general, the graph of *f* is called ***concave upward*** on an interval *I* if it lies above all its tangents. $f''(x)>0$
- It is called ***concave downward*** on *I* if it lies below all of these tangents. $f''(x)<0$
- A point where a curve changes its direction of concavity is called an ***inflection point***.

If $f'(x) > 0$ then $f(x)$ is increasing,
so if $f''(x) > 0$ then $f'(x)$ is increasing.

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Thus there is a point of inflection at any point where the second derivative changes sign.

Determine where the curve $y = x^3 - 3x^2 + 4x - 5$ is concave upward and concave downward

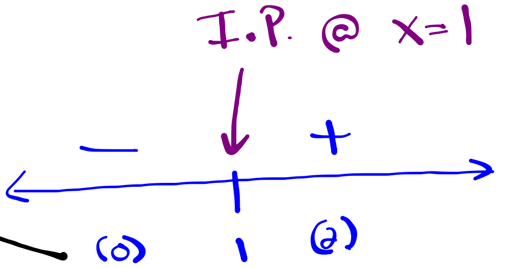
Find the points of inflection

$y = x^3 - 3x^2 + 4x - 5$

$y' = 3x^2 - 6x + 4$

$y'' = 6x - 6$

$y'' = 6(x-1)$



CV: $6 \neq 0$ | $x - 1 = 0$

$x = 1$

CV: $x = 1$

Concave Up on $(1, \infty)$

$x > 1$

Concave Down on $(-\infty, 1)$

$x < 1$

$y = x^3 - 3x^2 + 4x - 5$

$y = (1)^3 - 3(1)^2 + 4(1) - 5$

$y = 1 - 3 + 4 - 5$

$y = -3$

$(1, -3)$ Inflection Point

Determine where the curve $y=\frac{x}{x^2+1}$ is concave upward and concave downward

Find the points of inflection

$$y=\frac{x}{x^2+1}$$

$$y'=\frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=\frac{x^2+1-2x^2}{(x^2+1)^2}=\boxed{\frac{-x^2+1}{(x^2+1)^2}}$$

$$y''=\frac{(x^2+1)^2(-2x)-(-x^2+1)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$y''=\frac{-2x(x^2+1)^2+4x(x^2-1)(x^2+1)}{(x^2+1)^4}$$

$$-x^2-1\quad+2x^2-2$$

$$y''=\frac{2x\cancel{(x^2+1)}\left[-(x^2+1)+2(x^2-1)\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$y''=\frac{2x(x^2-3)}{(x^2+1)^3}$$

← Always positive

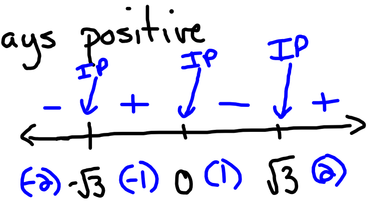
CV: $2x=0$

$x=0$

$x^2-3=0$

$x^2=3$

$x=\pm\sqrt{3}$



CU on $(-\sqrt{3},0)$ and $(\sqrt{3},\infty)$

CD on $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$

Inflection Points: $y=\frac{x}{x^2+1}$

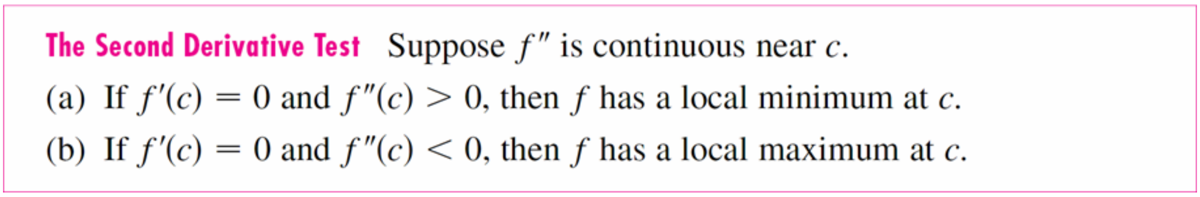
$f(-\sqrt{3})=\frac{-\sqrt{3}}{4}$ $\left(-\sqrt{3},\frac{-\sqrt{3}}{4}\right)$

$F(0)=\frac{0}{1}=0$ $(0,0)$

$f(\sqrt{3})=\frac{\sqrt{3}}{4}$ $\left(\sqrt{3},\frac{\sqrt{3}}{4}\right)$

homework

Second Derivative Test for Local Extrema



The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

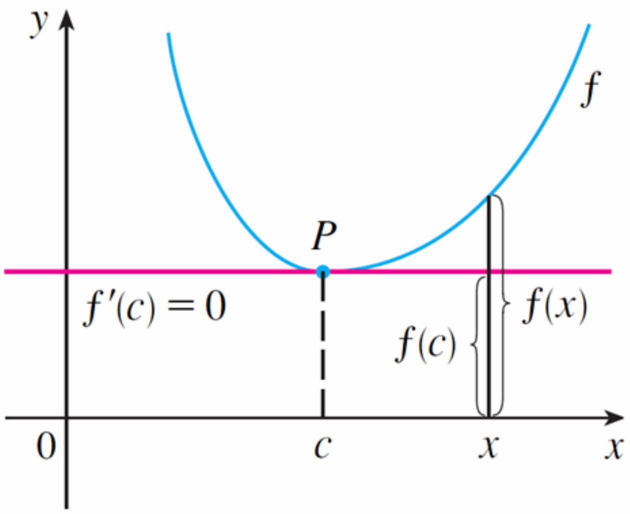


FIGURE 6

$f''(c) > 0$, f is concave upward

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

Solution

Example:

Using the function: $f(x) = \dfrac{x^2}{x-7}$

Determine each of the following...

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values