Questions From Homework

③ (ii) $y = x^4 - 8x^2$

$y' = 4x^3 - 16x$

$y' = 4x(x^2 - 4)$

$y' = 4x(x-2)(x+2)$

CV: $x = -2, 0, 2$

$y'' = 12x^2 - 16$

$y'' = 4(3x^2 - 4)$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

CV: $x = +\frac{2}{\sqrt{3}} \doteq \pm 1.15$

a) Intervals of Inc/Dec.

$$\overset{\quad min\quad\ max\quad\ min}{-\ \downarrow\ +\ \downarrow\ -\ \downarrow\ +}$$

(-3) -2 (-1) 0 (1) 2 (3)

Increasing on $(-2, 0)$ & $(2, \infty)$

Decreasing on $(-\infty, -2)$ & $(0, 2)$

Max Min

b) $f(-2) = -16$ $(-2, -16)$ min
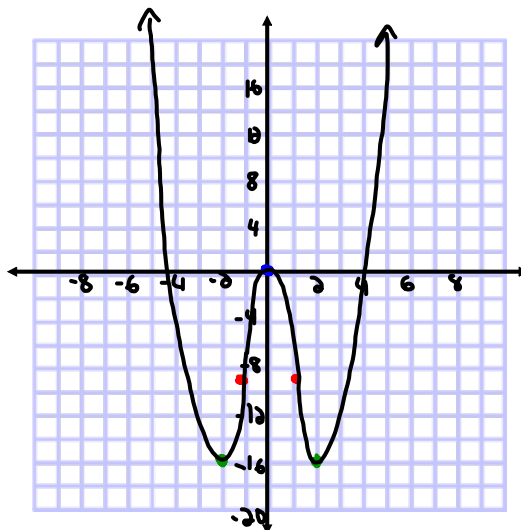
$f(0) = 0$ $(0, 0)$ max

$f(2) = -16$ $(2, -16)$ min

c) Intervals of Concavity

$$\overset{\quad IP\quad\quad IP}{+\ \downarrow\ -\ \downarrow\ +}$$

(-2) $-\frac{2}{\sqrt{3}}$ (0) $\frac{2}{\sqrt{3}}$ (2)

CU on $\left(-\infty, \frac{-2}{\sqrt{3}}\right)$ & $\left(\frac{2}{\sqrt{3}}, \infty\right)$

CD on $\left(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$

d) Inflection Points.

$f\left(\frac{2}{\sqrt{3}}\right) = -\frac{80}{9}$ $\left(\frac{-2}{\sqrt{3}}, \frac{-80}{9}\right)$

$f\left(\frac{2}{\sqrt{3}}\right) = -\frac{80}{9}$ $\left(\frac{2}{\sqrt{3}}, \frac{-80}{9}\right)$

Making a Complete Sketch

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes (Not a rational function) ← fraction
- Intervals of Increase or Decrease · $f'(x) = 4x^3 - 12x^2$
- Local Maximum and Minimum values
- Concavity and Points of Inflection $f''(x) = 12x^2 - 24x$
- Sketch the Curve

① x-int $y=0$

$f(x) = x^4 - 4x^3$

$0 = x^4 - 4x^3$ (factor)

$0 = x^3(x-4)$

$x^3 = 0 \mid x-4 = 0$

$x = 0 \mid x = 4$

$(0,0) \qquad (4,0)$

② y-int $x=0$

$f(x) = x^4 - 4x^3$

$f(0) = (0)^4 - 4(0)^3 = 0$

$(0,0)$

③ Symmetry

$f(x) = x^4 - 4x^3$

$f(-x) = (-x)^4 - 4(-x)^3$

$f(-x) = x^4 - 4(-x^3)$

$f(-x) = x^4 + 4x^3$

No symmetry

④ Asymptotes:

No asymptotes

⑤ Intervals of Inc/Dec.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^2(x-3)$

CV: $4x^2 = 0 \mid x-3 = 0$

$x^2 = 0 \mid x = 3$

$x = 0$

Increasing on $(3, \infty)$
  $x > 3$

Decreasing on $(-\infty, 3)$
  $x < 3$

⑥ Local max/min

When $x = 3$

$f(x) = x^4 - 4x^2$

$f(3) = (3)^4 - 4(3)^3$

$f(3) = 81 - 108$

$f(3) = -27$

local min @ $(3, -27)$

⑦ Intervals of concavity

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 12x(x-2)$

CV: $12x = 0 \mid x-2 = 0$

$x = 0 \mid x = 2$

Concave up on $(-\infty, 0) + (2, \infty)$
  $x < 0 \quad + \quad x > 2$

Concave down on $(0, 2)$
  $0 < x < 2$

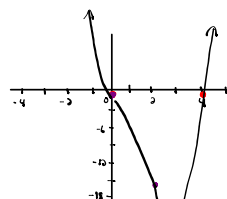⑧ Inflection Points:

when $x = 0$

$f(x) = x^4 - 4x^3$

$f(0) = (0)^4 - 4(0)^3$

$f(0) = 0$

I.P. @ $(0,0)$

when $x = 2$

$f(x) = x^4 - 4x^3$

$f(2) = (2)^4 - 4(2)^3$

$f(2) = 16 - 32 = -16$

I.P @ $(2, -16)$

FIGURE 7

$$y = x^4 - 4x^3$$

$(0, 0)$

inflection points

$(2, -16)$
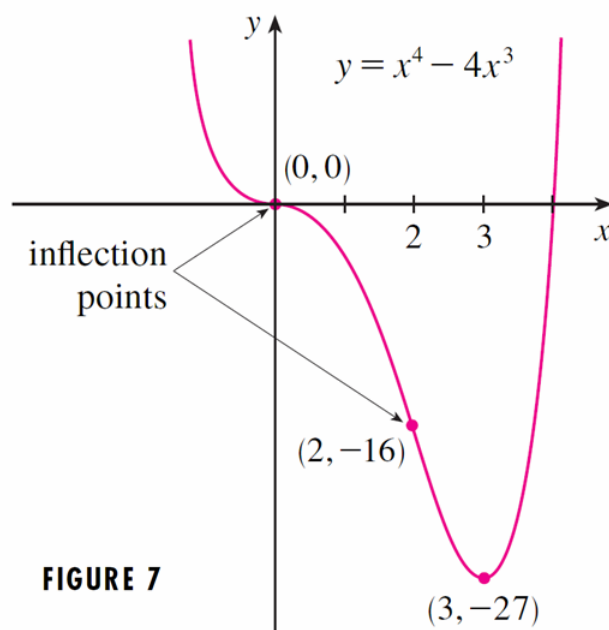
$(3, -27)$

homework

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$$f(x) = 3x^5 - 5x^3 \qquad f'(x) = 15x^4 - 15x^2 \qquad f''(x) = 60x^3 - 30x$$
$$f(x) = x^3(3x^2 - 5) \qquad f'(x) = 15x^2(x^2 - 1) \qquad f''(x) = 30x(2x^2 - 1)$$

① Intercepts:

x int (y=0)   y int (x=0)

$x = 0, \pm\sqrt{5/3}$ $y = 0$

(0,0) (1.29,0) (-1.29,0)   (0,0)
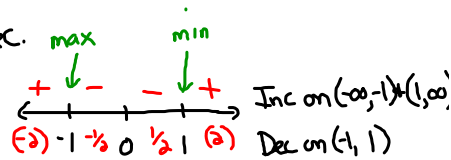
② Symmetry:

$f(-x) = 3(-x)^5 - 5(-x)^3$
$= -3x^5 + 5x^3$

$\therefore f(-x) = -f(x)$ Odd

③ Asymptotes: None

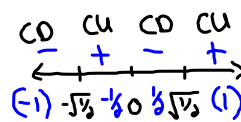**④ Intervals of Incl Dec.**

$f'(x) = 15x^2(x^2 - 1)$

     max   min

   + ↓ —   — ↓ +

(-2) -1 -½ 0 ½ 1 (2)

Inc on $(-\infty, -1) \cup (1, \infty)$

Dec on $(-1, 1)$

CV: $x = 0, \pm 1$

⑤ Max/Mins:

$f(-1) = 3(-1)^5 - 5(-1)^3$ (-1, 2) max
$= -3 + 5$
$= 2$

$f(1) = 3(1)^5 - 5(1)^3$ (1, -2) min
$= 3 - 5$
$= -2$

⑥ Concavity:

$f''(x) = 30x(2x^2 - 1)$

   CD   CU   CD   CU
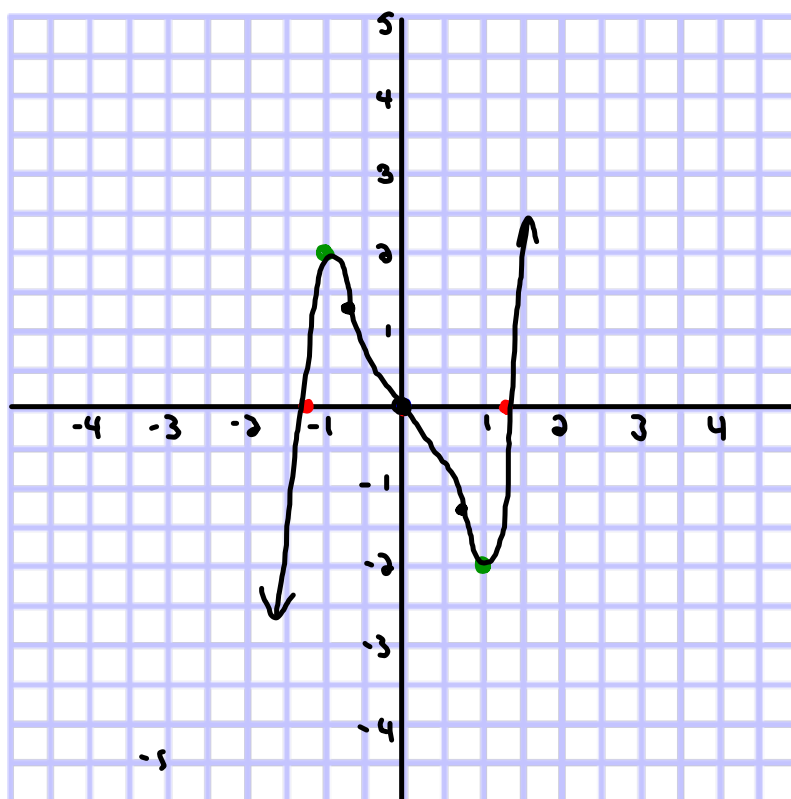
   —   +   —   +

(-1) $-\sqrt{½}$ -½ 0 ½ $\sqrt{½}$ (1)

CV: $x = 0, \pm\sqrt{½}$

⑦ Inflection Points:

$f(-\sqrt{½}) \approx -0.53 + 1.767 \approx 1.238$ (-0.707, 1.238)

$f(0) = 0$ (0,0)

$f(\sqrt{½}) \approx 0.53 - 1.767 \approx -1.238$ (0.707, -1.238)

homework

Examine the function $f(x) = \dfrac{x^2}{1 - x^2}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

homework

Examine the function $f(x) = \dfrac{x^2}{x - 7}$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve