

## Questions From Homework

③ (ii)  $y = x^4 - 8x^2$

$$y' = 4x^3 - 16x$$

$$y' = 4x(x^2 - 4)$$

$$y' = 4x(x-2)(x+2)$$

CV:  $x = -2, 0, 2$

$$y'' = 12x^2 - 16$$

$$y'' = 4(3x^2 - 4)$$

$$3x^2 - 4 = 0$$

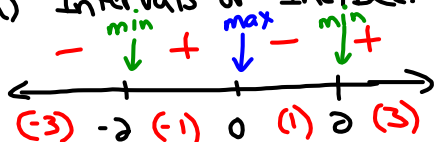
$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \pm \frac{2}{\sqrt{3}}$$

CV:  $x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.15$

a) Intervals of Inc/Dec.



Increasing on  $(-2, 0) + (2, \infty)$

Decreasing on  $(-\infty, -2) + (0, 2)$

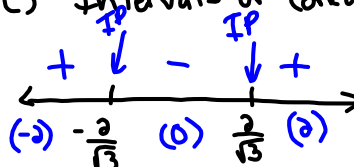
Max Min

b)  $f(-2) = -16$   $(-2, -16)$  min

$f(0) = 0$   $(0, 0)$  max

$f(2) = -16$   $(2, -16)$  min

c) Intervals of Concavity



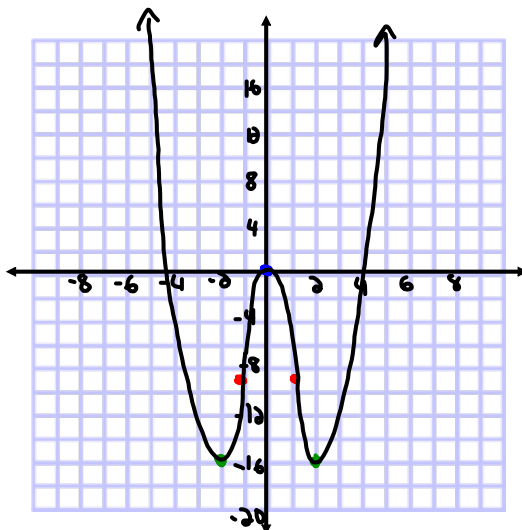
CU on  $(-\infty, \frac{2}{\sqrt{3}}) + (\frac{2}{\sqrt{3}}, \infty)$

CO on  $(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

d) Inflection Points:

$f(\frac{2}{\sqrt{3}}) = -\frac{80}{9}$   $(\frac{2}{\sqrt{3}}, \frac{80}{9})$

$f(\frac{-2}{\sqrt{3}}) = -\frac{80}{9}$   $(\frac{-2}{\sqrt{3}}, \frac{-80}{9})$



# Making a Complete Sketch

**Example:**

Examine the function  $f(x) = x^4 - 4x^3$  with respect to...

- ✓ Intercepts
- ✓ Symmetry
- ✓ Asymptotes (Not a rational function) fraction
- Intervals of Increase or Decrease  $f'(x) = 4x^3 - 12x^2$
- Local Maximum and Minimum values
- Concavity and Points of Inflection  $f''(x) = 12x^2 - 24x$
- Sketch the Curve

<p>① x-int <math>y=0</math></p> $f(x) = x^4 - 4x^3$ $0 = x^4 - 4x^3 \text{ (factor)}$ $0 = x^3(x-4)$ $x^3=0 \quad   \quad x-4=0$ $x=0 \quad   \quad x=4$ <p style="color: red;">(0,0)    (4,0)</p>	<p>② y-int <math>x=0</math></p> $f(x) = x^4 - 4x^3$ $f(0) = 0^4 - 4(0)^3 = 0$ <p style="color: red;">(0,0)</p>
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<p>③ Symmetry</p> $f(x) = x^4 - 4x^3$ $f(-x) = (-x)^4 - 4(-x)^3$ $f(-x) = x^4 - 4(-x^3)$ $f(-x) = x^4 + 4x^3$ <p style="color: red;">No symmetry</p>	<p>④ Asymptotes:</p> <p style="color: red;">No asymptotes</p>
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⑤ Intervals of Inc/Dec.

$$f(x) = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2$$

$$f'(x) = 4x^2(x-3)$$

$4x^2=0$ $x^2=0$ $x=0$	$x-3=0$ $x=3$	<p style="color: red;">Increasing on (3, ∞) <math>x &gt; 3</math></p> <p style="color: red;">Decreasing on (-∞, 3) <math>x &lt; 3</math></p>
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⑥ Local max/min

When  $x=3$

$$f(x) = x^4 - 4x^3$$

$$f(3) = (3)^4 - 4(3)^3$$

$$f(3) = 81 - 108$$

$$f(3) = -27$$

local min @ (3, -27)

⑦ Intervals of concavity

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

$$f''(x) = 12x(x-2)$$

$12x=0$ $x=0$	$x-2=0$ $x=2$	<p style="color: blue;">Concave up on (-∞, 0) + (2, ∞) <math>x &lt; 0</math>    <math>x &gt; 2</math></p> <p style="color: blue;">Concave down on (0, 2) <math>0 &lt; x &lt; 2</math></p>
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⑧ Inflection Points:

<p>When <math>x=0</math></p> $f(x) = x^4 - 4x^3$ $f(0) = 0^4 - 4(0)^3$ $f(0) = 0$ <p style="color: blue;">I.P. @ (0,0)</p>	<p>When <math>x=2</math></p> $f(x) = x^4 - 4x^3$ $f(2) = (2)^4 - 4(2)^3$ $f(2) = 16 - 32 = -16$ <p style="color: blue;">I.P. @ (2, -16)</p>
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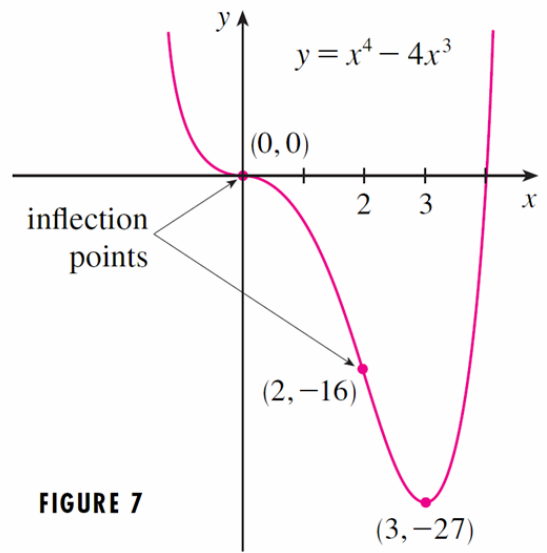
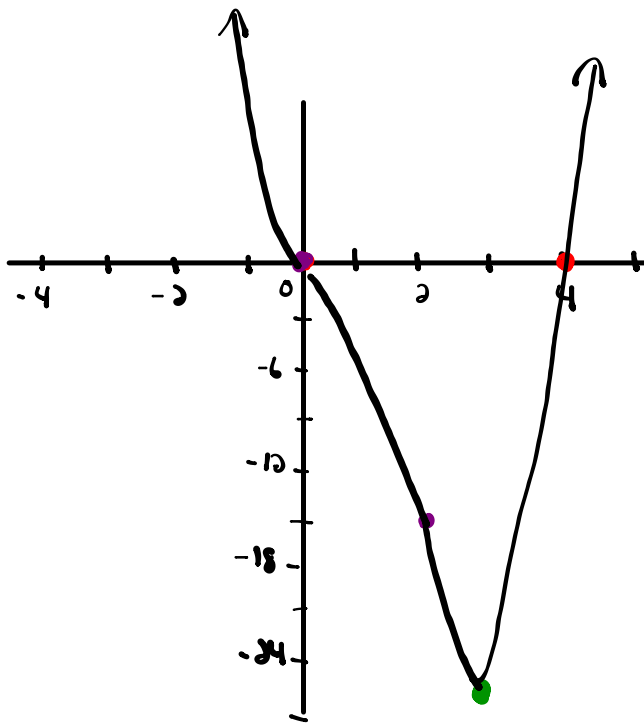


FIGURE 7

Examine the function  $f(x) = 3x^5 - 5x^3$  with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$$f(x) = 3x^5 - 5x^3 \quad \left| \begin{array}{l} f'(x) = 15x^4 - 15x^2 \\ f'(x) = 15x^2(x^2 - 1) \end{array} \right. \quad \left| \begin{array}{l} f''(x) = 60x^3 - 30x \\ f''(x) = 30x(x^2 - 1) \end{array} \right.$$

① Intercepts:

x int ( $y=0$ )  
 $x = 0, \pm\sqrt[5]{3}$

$(0,0)$   $(1.29,0)$   $(-1.29,0)$

y int ( $x=0$ )  
 $y = 0$   
 $(0,0)$

② Symmetry:

$$f(-x) = 3(-x)^5 - 5(-x)^3 = -3x^5 + 5x^3$$

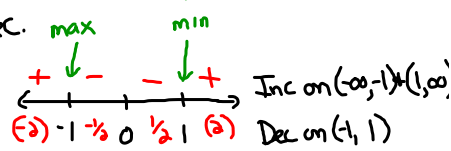
$\therefore f(-x) = -f(x)$  Odd

③ Asymptotes: None

④ Intervals of Inc/Dec.

$$f'(x) = 15x^2(x^2 - 1)$$

$$f'(x) = 15x^2(x-1)(x+1)$$



CV:  $x = 0, \pm 1$

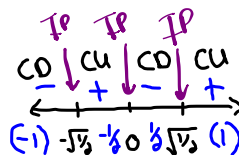
⑤ Max/Min:

$$f(-1) = 3(-1)^5 - 5(-1)^3 = -3 + 5 = 2 \quad (-1, 2) \text{ max}$$

$$f(1) = 3(1)^5 - 5(1)^3 = 3 - 5 = -2 \quad (1, -2) \text{ min}$$

⑥ Concavity:

$$f''(x) = 30x(x^2 - 1)$$



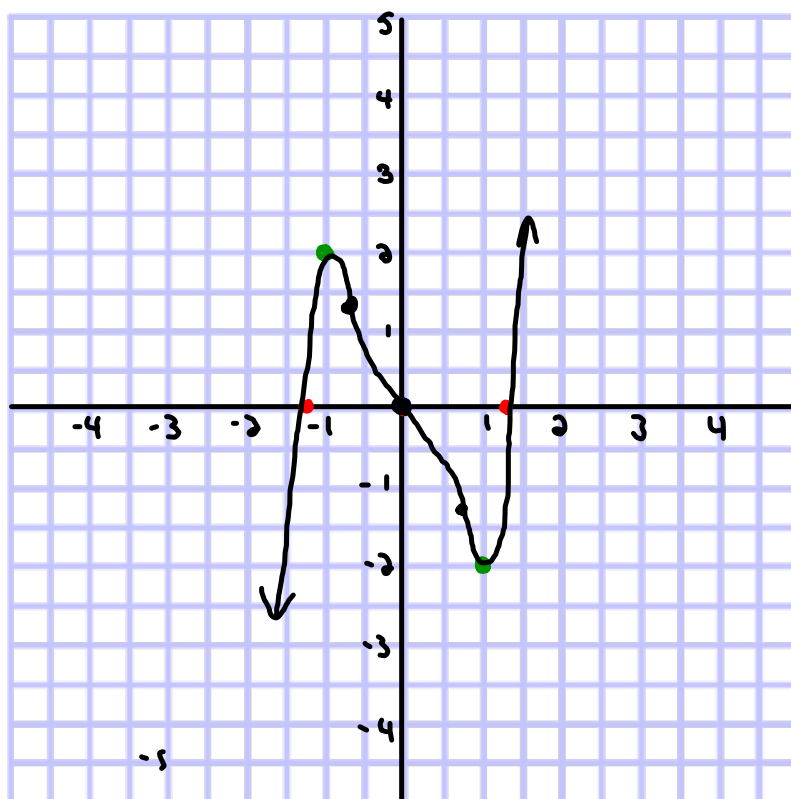
CV:  $x = 0, \pm\sqrt[3]{1/2}$

⑦ Inflection Points:

$$f(\sqrt[3]{1/2}) \approx -0.53 + 1.767 \approx 1.238 \quad (-0.707, 1.238)$$

$$f(0) = 0 \quad (0, 0)$$

$$f(-\sqrt[3]{1/2}) \approx 0.53 - 1.767 \approx -1.238 \quad (0.707, -1.238)$$



homework

Examine the function  $f(x) = \frac{x^2}{x-7}$  with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

<p>① x-int (y=0)</p> $f(x) = \frac{x^2}{x-7}$ $(x-7) \cdot 0 = \frac{x^2}{x-7} \cdot (x-7)$ $0 = \frac{x^2}{x-7}$ $0 = x^2$ $0 = x$ <p>(0,0)</p>	<p>② y-int (x=0)</p> $f(x) = \frac{x^2}{x-7}$ $f(0) = \frac{0^2}{0-7} = \frac{0}{-7} = 0$ <p>(0,0)</p>	<p>③ Symmetry:</p> $f(x) = \frac{x^2}{x-7}$ $f(-x) = \frac{(-x)^2}{(-x)-7}$ $f(-x) = \frac{x^2}{-x-7}$ <p>No symmetry</p>
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④ VA: (denom=0)

$$x-7=0$$

$$x=7$$

$\lim_{x \rightarrow 7^-} \frac{x^2}{x-7} = \frac{49}{(-)} = -\infty$ 
 $\lim_{x \rightarrow 7^+} \frac{x^2}{x-7} = \frac{49}{(+)} = +\infty$

⑤ SA:  $\frac{x+7}{-(x^2-7x)}$

$y = x+7$      $m = \frac{1}{1}$  rise / run

$b = 7$  y-int

$-\frac{7x}{(x-7)(x-0)}$

⑥ Intervals of Inc/Dec:

$f'(x) = \frac{x(x-14)}{(x-7)^2}$

$\leftarrow \begin{array}{ccccccc} & + & & - & & - & + \\ & \downarrow & & \downarrow & & \downarrow & \\ \leftarrow & (-) & | & 0 & | & 7 & | & 14 & | & (+) & \rightarrow \end{array}$

CV:  $x=0$  |  $x-14=0$  |  $(x-7)^2=0$   
 $x=14$  |  $x-7=0$  |  $x=7$

Increasing on  $(-\infty, 0) \cup (14, \infty)$   
 $x < 0$  +  $x > 14$   
 Decreasing on  $(0, 14)$   
 $0 < x < 14$

⑦ Local max/min

$f(x) = \frac{x^2}{x-7}$

When  $x=0$     When  $x=14$

$f(0) = \frac{0^2}{0-7} = \frac{0}{-7} = 0$      $f(14) = \frac{14^2}{14-7} = \frac{196}{7} = 28$

(0,0)    (14,28)  
 local max @ (0,0)    local min @ (14,28)

⑧ Intervals of Concavity:

$f''(x) = \frac{98}{(x-7)^3}$

$\leftarrow \begin{array}{ccc} & - & + \\ & \downarrow & \\ \leftarrow & (-) & | & 7 & | & (+) & \rightarrow \end{array}$

CV:  $98 \neq 0$  |  $(x-7)^3=0$   
 $x-7=0$  |  $x=7$

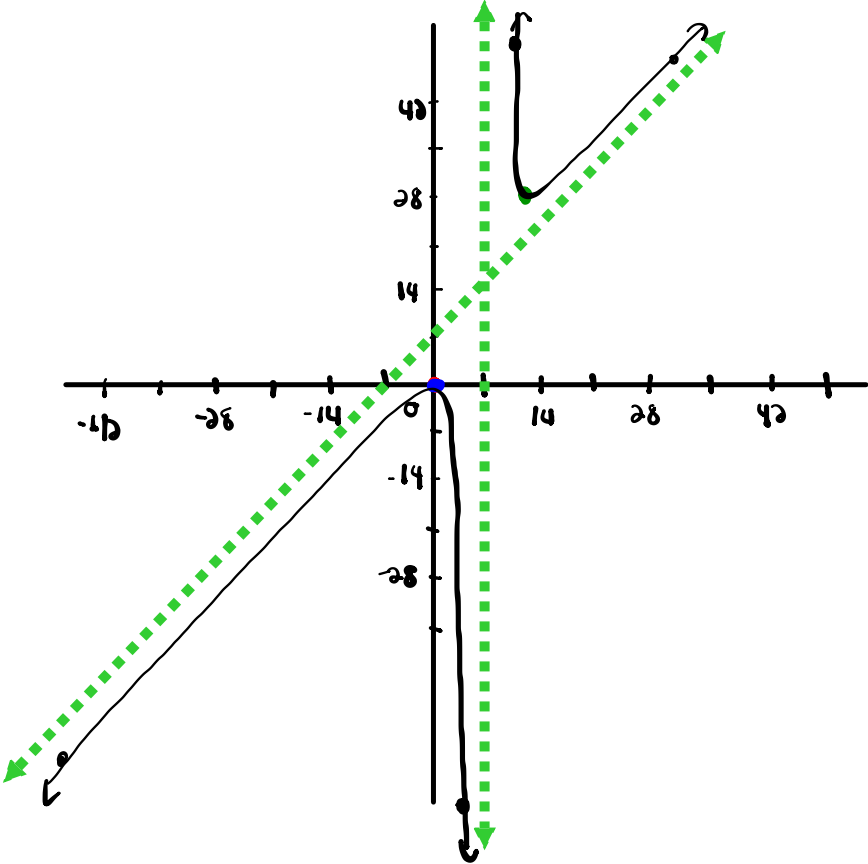
Concave down on  $(-\infty, 7)$   
 $x < 7$   
 Concave up on  $(7, \infty)$   
 $x > 7$

⑨ I.P. (x=7)

$f(x) = \frac{x^2}{x-7}$

$x=7$  is the vertical asymptote

$f(7) = \frac{7^2}{7-7} = \frac{49}{0} = \text{DNE}$





# homework

Examine the function  $f(x) = \frac{x^2}{1-x^2}$  with respect to...

- Intercepts ✓
- Symmetry ✓
- Asymptotes ✓
- Intervals of Increase or Decrease ✓
- Local Maximum and Minimum values ✓
- Concavity and Points of Inflection ✓
- Sketch the Curve

$$f(x) = \frac{x^2}{1-x^2} \quad \left| \quad f'(x) = \frac{2x}{(1-x^2)^2} \quad \left| \quad f''(x) = \frac{2+6x^2}{(1-x^2)^3} = \frac{2(3x^2+1)}{(1-x^2)^3}$$

① Intercepts:  $x$  int ( $y=0$ )  $y$  int ( $x=0$ )      ② Symmetry:  $f(x) = \frac{(-x)^2}{1-(-x)^2} = \frac{x^2}{1-x^2}$

$x^2 = 0$        $y = \frac{0}{1} = 0$        $\therefore f(-x) = f(x)$  Even

$x = 0$        $(0,0)$

③ Asymptotes:

HA:  $y = -1$       VA:  $1-x^2 = 0$   
 $1 = x^2$   
 $\boxed{+1 = x}$   
 $\boxed{-1 = x}$

$\lim_{x \rightarrow 1^-} \frac{(+)}{(-)} = -\infty$   
 $x = 1.01$   
 $\lim_{x \rightarrow 1^+} \frac{(+)}{(+)} = +\infty$   
 $x = 0.99$   
 $\lim_{x \rightarrow 1^-} \frac{(+)}{(+)} = +\infty$   
 $\lim_{x \rightarrow 1^+} \frac{(+)}{(-)} = -\infty$

④ Intervals of Inc/Dec:  $\min$

$f'(x) = \frac{2x}{(1-x^2)^2}$        $\leftarrow - \quad - \quad + \quad + \rightarrow$        $\leftarrow (-) \quad - \quad 1/2 \quad 0 \quad 1/2 \quad (+) \quad (+) \rightarrow$

$\boxed{\text{Inc on } (0, \infty)}$   
 $\boxed{\text{Dec on } (-\infty, 0)}$

CV:  $x = 0, \pm 1$

⑤ Max/Mins:

$f(0) = 0$        $(0,0)$   $\min$

⑥ Concavity:

$f''(x) = \frac{2(3x^2+1)}{(1-x^2)^3}$        $\leftarrow \text{Always } (+) \quad - \quad + \quad - \rightarrow$        $\leftarrow (-) \quad - \quad 1 \quad (0) \quad (+) \quad (-) \rightarrow$

$\boxed{\text{CU: on } (-1, 1)}$   
 $\boxed{\text{CO: on } (-\infty, -1) \cup (1, \infty)}$

CV:  $x = \pm 1$

⑦ Inflection Points

$f(-1) = \text{undefined}$   
 $f(1) = \text{undefined}$

No Inflection Points  $x = \pm 1$  are V.A.

