② $f(x)=x^4-3x^3+3x^2-x$

$f'(x)=4x^3-9x^2+6x-1$

$f''(x)=12x^2-18x+6$

③ y int: $(x=0)$

$f(0)=(0)^4-3(0)^3+3(0)^2-0=0$

$(0,0)$

① x int: $(y=0)$

$0=x^4-3x^3+3x^2-x$ ← synthetic sub.

$0=x(x^3-3x^2+3x-1)$

1	1	-3	3	-1
		1	-2	1
	1	-2	1	

$(x-1)(x^2-2x+1)$ ← simple trinom

$0=x(x-1)(x^3-2x+1)$

$(x-1)(x-1)(x-1)$

$0=x(x-1)^3$

$x=0,1$

$(0,0)$ & $(1,0)$

③ No asymptotes

④ Intervals of Inc/Dec.

$f'(x)=4x^3-9x^2+6x-1$ ← syn. sub.

1	4	-9	6	-1
		4	-5	1
	4	-5	1	

$(x-1)(4x^2-5x+1)$ ← Decom

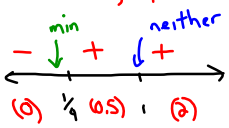
$f'(x)=(x-1)^2(4x-1)$

$(x-1)[(4x^2-4x)(x+1)]$

$(x-1)[4x(x-1)-1(x-1)]$

$(x-1)(x-1)(4x-1)$

CV: $x=1, \frac{1}{4}$



Increasing on $(\frac{1}{4},\infty)$

Decreasing on $(-\infty,\frac{1}{4})$

⑤ Local Max/Min:

$f(\frac{1}{4})=(\frac{1}{4})^4-3(\frac{1}{4})^3+3(\frac{1}{4})^2-(\frac{1}{4})=-0.1$ $(0.25,-0.1)$ ← min

No max!

⑥ Intervals of concavity:

$f''(x)=12x^2-18x+6$

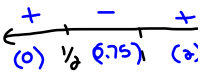
$f''(x)=6(2x^2-3x+1)$ ← Decomp.

$f''(x)=6[2x^2-2x-x+1]$

$f''(x)=6[2x(x-1)-1(x-1)]$

$f''(x)=6(2x-1)(x-1)$

CV: $x=1,\frac{1}{2}$



CU on $(-\infty,\frac{1}{2})$ & (1

CD on $(\frac{1}{2},1)$

⑦ Inflection Points:

$f(1)=(1)^4-3(1)^3+3(1)^2-(1)$ ... $(1,0)$

$f(x) = \frac{8(x-2)}{x^2}$ $f'(x) = \frac{-8(x-4)}{x^3}$ $f''(x) = \frac{16(x-6)}{x^4}$

① Intercepts:

x-int ($y=0$)

$\frac{0}{1} = \frac{8(x-2)}{x^2}$

$8(x-2) = 0$

$x-2=0$

$x=2$

(2,0)

y-int ($x=0$)

$f(0) = \frac{8(0-2)}{(0)^2}$

$f(0) = \frac{-16}{0}$

$f(0)$ = undefined

No y-int

② Symmetry:

$f(x) = \frac{8(x-2)}{x^2} = \frac{8x-16}{x^2}$

$f(-x) = \frac{8(-x-2)}{(-x)^2}$

$f(-x) = \frac{8(-x-2)}{x^2}$

$f(-x) = -\frac{8(x+2)}{x^2} = \frac{-8x-16}{x^2}$

No symmetry

③ VA: (zeros of denom)

$f(x) = \frac{8(x-2)}{x^2}$

$x^2 = 0$

$x = 0$

Check behaviour of $f(x)$ near VA:

$\lim_{x\to 0^-} f(x) = \frac{(-)}{(+)} = -\infty$

($x = -0.01$)

$\lim_{x\to 0^+} f(x) = \frac{(-)}{(+)} = -\infty$

($x = 0.01$)

④ HA:

$f(x) = \frac{8(x-2)}{x^2} = \frac{8x-16}{x^2}$

$\lim_{x\to\infty} \frac{8x-16}{x^2} = 0$

$y = 0$

⑤ Intervals of Inc/Dec

$f'(x) = \frac{-8(x-4)}{x^3}$

CV:

$-8(x-4)=0$ | $x^3=0$

$x-4=0$ | $x=0$

$x=4$

min, max

(-1) 0 (1) 4 (5)

Increasing on (0,4)

Decreasing on $(-\infty,0) \cup (4,\infty)$

⑥ Max/Min:

$f(x) = \frac{8(x-2)}{x^2}$

$f(0) = \frac{8(0-2)}{(0)^2}$ = undefine

no min

$x=0$ is the VA

$f(4) = \frac{8(4-2)}{(4)^2} = \frac{16}{16} = 1$

max at (4,1)

⑦ Concavity:

$f''(x) = \frac{16(x-6)}{x^4}$

CV:

$16(x-6)=0$ | $x^4=0$

$x-6=0$ | $x=0$

$x=6$

IP.

(-1) 0 (1) 6 (7)

CU on $(6,\infty)$

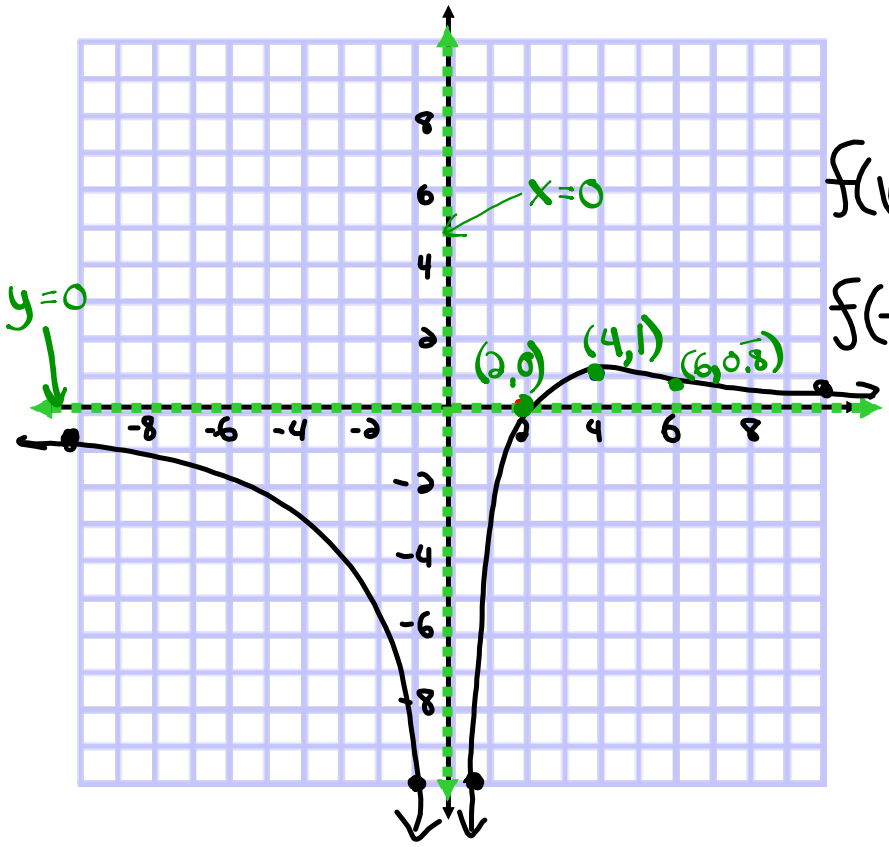
CD on $(-\infty,0) \cup (0,6)$

⑧ Inflection Point

$f(x) = \frac{8(x-2)}{x^2}$

$f(6) = \frac{8(6-2)}{6^2} = \frac{32}{36}$

$(6, 0.\overline{8})$


x=0
y=0
(2,0)
(4,1)
(6,0.8)
f(10)=0.64
f(-10)=-0.64

① Plot all points: x-int, y-int, max, min, I.P., behaviour near VA.

② Plot all asymptotes

③ Use intervals of inc/dec and concavity to connect everything

② $f(x) = x^4 - 3x^3 + 3x^2 - x$

x-int ($y=0$)

$0 = x^4 - 3x^3 + 3x^2 - x$

$0 = x(x^3 - 3x^2 + 3x - 1)$

synthetic sub.

$x(x-1)(x-1)(x-1)$

$x = 0$ | $x-1=0$

$x = 0$ | $x = 1$