Warm Up

Prove the following identity:

$$\frac{\sin x}{1-\cos x} - \frac{\sin x \cos x}{1+\cos x} = \csc x\left(1+\cos^2 x\right)$$

$$\frac{\sin x(1+\cos x)}{(1-\cos x)(1+\cos x)} - \frac{\sin x\cos x(1-\cos x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{\sin x + \sin x\cos x}{(1-\cos x)(1+\cos x)} - \frac{(\sin x\cos x - \sin x\cos^2 x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{\sin x + \cancel{\sin x\cos x} - \cancel{\sin x\cos x} + \sin x\cos^2 x}{1+\cancel{\cos x}-\cancel{\cos x}-\cos^2 x}$$

$$\frac{\sin x + \sin x\cos^2 x}{1-\cos^2 x}$$

$$\frac{\cancel{\sin x}(1+\cos^2 x)}{\sin^{\cancel{2}} x}$$

$$\frac{1+\cos^2 x}{\sin x}$$

$$\frac{1}{\sin x}\left(1+\cos^2 x\right)$$

$$\frac{1+\cos^2 x}{\sin x}$$

⑤ $\frac{\sin^2 x}{\cos^2 x} = \boxed{\sec^2 x - 1}$

$\tan^2 x$

$\frac{\sin^2 x}{\cos^2 x}$

⑨ $\sin x + \cos x \cot x = \csc x$

$\sin x + \cos x\left(\frac{\cos x}{\sin x}\right)$ | $\frac{1}{\sin x}$

$\frac{\sin x}{1} + \frac{\cos^2 x}{\sin x}$

$\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$

$\frac{\boxed{\sin^2 x + \cos^2 x}}{\sin x}$

$\frac{1}{\sin x}$

⑬ $\tan x\left(\sin x + \cot x \cos x\right) = \sec x$

$\frac{\sin x}{\cos x}\left(\sin x + \frac{\cos x}{\sin x} \cdot \cos x\right)$ | $\frac{1}{\cos x}$

$\frac{\sin x}{\cos x}\left(\frac{\sin x}{1} + \frac{\cos^2 x}{\sin x}\right)$

$\frac{\sin x}{\cos x}\left(\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}\right)$

$\frac{\cancel{\sin x}}{\cos x}\left(\frac{1}{\cancel{\sin x}}\right)$

$\frac{1}{\cos x}$

⑧ $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

$(\sin x - \cos x)(\sin x - \cos x) + (\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x \cancel{- \sin x \cos x} \cancel{- \sin x \cos x} + \cos^2 x + \sin^2 x \cancel{+ \sin x \cos x} \cancel{+ \sin x \cos x} + \cos^2 x$

$2\sin^2 x + 2\cos^2 x$

$2\boxed{(\sin^2 x + \cos^2 x)}$

$2(1)$

2

⑮ $\tan^3 x \sec^2 x - \tan^3 x = \tan^5 x$

$\tan^3 x(\sec^2 x - 1)$

$\tan^3 x(\tan^2 x)$

$\tan^5 x$

⑯ $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = 2\csc^2 x$

$\dfrac{1-\cos x}{(1+\cos x)(1-\cos x)} + \dfrac{1+\cos x}{(1+\cos x)(1-\cos x)}$

$\dfrac{1 \cancel{- \cos x} + 1 \cancel{+ \cos x}}{(1+\cos x)(1-\cos x)}$

$\dfrac{2}{1 - \cos x + \cos x - \cos^2 x}$

$\dfrac{2}{\boxed{1-\cos^2 x}}$

$\dfrac{2}{\sin^2 x}$

$2\left(\dfrac{1}{\sin^2 x}\right)$

$\dfrac{2}{\sin^2 x}$

Bonus

Prove the following identity:

$$\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$$

$$\frac{(1+\sin x)(1+\sin x)}{\cos x(1+\sin x)}+\frac{\cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{1+2\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{1+2\sin x+1}{\cos x(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}}$$

$$\frac{2}{\cos x}$$

$$2\left(\frac{1}{\cos x}\right)$$

$$\frac{2}{\cos x}$$

Quiz & Homework