③ $y = \dfrac{(2x+5)^2}{3x}$ $f(x)$ $g(x)$

$$y' = \frac{2(2x+5)^1(2)(3x) - 3(2x+5)^2}{(3x)^2}$$

$$y' = \frac{12x(2x+5) - 3(2x+5)^2}{9x^2}$$

$$y' = \frac{3(2x+5)[4x - (2x+5)]}{9x^2}$$ Common Factor

$4x - 2x - 5$

$$y' = \frac{\cancel{3}(2x+5)(2x-5)}{\cancel{9}x^2} = \frac{(2x+5)(2x-5)}{3x^2}$$

④ $f(x) = \dfrac{x+2}{(x-3)^3}$ $f(x)$ $g(x)$

$$f'(x) = \frac{1(x-3)^3 - (x+2)(3)(x-3)^2(1)}{[(x-3)^3]^2}$$

$$f'(x) = \frac{(x-3)^3 - 3(x+2)(x-3)^2}{(x-3)^6}$$

$$f'(x) = \frac{(x-3)^2[(x-3) - 3(x+2)]}{(x-3)^6}$$

$x - 3 - 3x - 6$

$$f'(x) = \frac{\cancel{(x-3)^2}(-2x-9)}{(x-3)^{\cancel{6}\,4}} = \frac{-2x-9}{(x-3)^4} = -\frac{2x+9}{(x-3)^4}$$

6 $y = \left(\frac{x^2+1}{x+1}\right)^9$

$2x^2+2x-x^2-1$

$$y' = 9\left(\frac{x^2+1}{x+1}\right)^8\left[\frac{2x(x+1)-1(x^2+1)}{(x+1)^2}\right]$$

$$y' = 9\,\frac{(x^2+1)^8}{(x+1)^8}\,\frac{(x^2+2x-1)}{(x+1)^2}$$

$$y' = \frac{9(x^2+1)^8(x^2+2x-1)}{(x+1)^{10}}$$

Chain:

$y = (x^2+1)^9$

$y' = 9(x^2+1)^8(2x)$

Quotient:

$y = \dfrac{x^2+1}{x+1}$

$y' = \dfrac{2x(x+1) - 1(x^2+1)}{(x+1)^2}$

5. $$g(x) = \frac{6x^2}{(2x+1)^2}$$

$$g'(x) = \frac{12x(2x+1)^2 - 6x^2(2)(2x+1)^1(2)}{\left[(2x+1)^2\right]^2}$$

$$g'(x) = \frac{12x(2x+1)^2 - 24x^2(2x+1)}{(2x+1)^4}$$

$$g'(x) = \frac{12x\cancel{(2x+1)}\overset{2x+1-2x}{\left[(2x+1)-2x\right]}}{(2x+1)^{\cancel{4}\,3}} = \frac{12x}{(2x+1)^3}$$

Review Sheet.

① a) $f(x) = \boxed{\sqrt{x-5}}$ $\quad f(x+h) = \boxed{\sqrt{x+h-5}}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\left(\sqrt{x+h-5} - \sqrt{x-5}\right)}{h} \frac{\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}{\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

$-x + 5$

$$f'(x) = \lim_{h\to 0} \frac{x+h-5 - (x-5)}{h\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-5} + \sqrt{x-5}\right)} = \frac{1}{\sqrt{x-5} + \sqrt{x-5}} = \frac{1}{2\sqrt{x-5}}$$

Review Sheet

①b) $f(x) = \boxed{\dfrac{2x-2}{x+3}}$

$f(x+h) = \dfrac{2(x+h)-2}{(x+h)+3}$

$f(x+h) = \boxed{\dfrac{2x+2h-2}{x+h+3}}$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h\to 0} \frac{\boxed{\dfrac{2x+2h-2}{x+h+3}} - \boxed{\dfrac{2x-2}{x+3}}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$ ← Expand

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + 6h - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - 2h - \cancel{6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

$$\boxed{f'(x) = \frac{8}{(x+3)^2}}$$

Review Sheet

② b) $y = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$

$3 \cdot -\frac{1}{2} = \frac{-3}{2}$

$-\frac{1}{2} - \frac{2}{2} = \frac{-3}{2}$

$y' = \frac{-3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}} = \frac{-3}{2\sqrt{x^3}}$

$y = \frac{3}{x^{1/2}}$ (Quotient Rule)

$y' = \frac{0(\cancel{x^{1/2}}) - 3\left(\frac{1}{2}x^{-1/2}\right)}{(x^{1/2})^2}$

$y' = \frac{-\frac{3}{2}x^{-1/2}}{x}$

$y' = \frac{-\frac{3}{2\cancel{x^{1/2}}} \cancel{2x^{1/2}}}{x \cdot 2x^{1/2}}$

CD: $2x^{1/2}$

$x^1 \cdot x^{1/2} = x^{2/2 + 1/2}$

$y' = \frac{-3}{2x^{3/2}}$

Review Sheet

③ a) $y = \underbrace{(3x^2-2)}_{f(x)}\underbrace{(4x+5)}_{g(x)}$

$y' = 6x(4x+5) + 4(3x^2-2)$

$y' = 24x^2 + 30x + 12x^2 - 8$

$\boxed{y' = 36x^2 + 30x - 8}$

Review Sheet

4 b) $y = \dfrac{\sqrt{x}}{3+x^2} = \dfrac{x^{1/2}}{3+x^2}$ $\quad \dfrac{f(x)}{g(x)}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x(x^{1/2})}{(3+x^2)^2}$$

$x^1 \cdot x^{1/2}$
$= x^{3/2}$

$$y' = \frac{\frac{1}{2x^{1/2}}(3+x^2) - 2x^{3/2}}{(3+x^2)^2}$$

$$y' = \frac{\cancel{2x^{1/2}}\,\frac{3+x^2}{\cancel{2x^{1/2}}} - \frac{2x^{3/2}}{1}\; 2x^{1/2}}{2x^{1/2}(3+x^2)^2}$$

CD: $2x^{1/2}$

$$y' = \frac{3+x^2-4x^2}{2x^{1/2}(3+x^2)^2} = \boxed{\frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}}$$

Review Sheet

⑥ b) $y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$

$y' = -8(x-1)^{-3/2}(1)$

$y' = -8(x-1)^{-3/2}$

$y' = \frac{-8}{(x-1)^{3/2}} = \frac{-8}{\sqrt{(x-1)^3}}$

$y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}}$ $\frac{f(x)}{g(x)}$

$\left(\frac{1}{2} - \frac{2}{2} = -\frac{1}{2}\right.$

$y' = \frac{\cancel{0(x-1)^{1/2}} - 16\left(\frac{1}{2}\right)(x-1)^{-1/2}(1)}{\left[(x-1)^{1/2}\right]^2}$

$y' = \frac{-8(x-1)^{-1/2}}{(x-1)}$

$1 + \frac{1}{2} = \frac{3}{2}$

$y' = \frac{-8}{(x-1)(x-1)^{1/2}} = \frac{-8}{(x-1)^{3/2}}$

Chain Rule:

④ $G(x) = \sqrt{x^4 - x + 1} = (x^4 - x + 1)^{1/2}$

$G'(x) = \frac{1}{2}(x^4 - x + 1)^{-1/2}(4x^3 - 1)$

$G'(x) = \frac{1}{2(x^4 - x + 1)^{1/2}} \cdot (4x^3 - 1)$

$G'(x) = \frac{4x^3 - 1}{2\sqrt{x^4 - x + 1}}$

⑧ $y = \frac{4}{\sqrt{9 - x^2}} = \frac{4}{(9 - x^2)^{1/2}} = 4(9 - x^2)^{-1/2}$

$y' = -2(9 - x^2)^{-3/2}(-2x)$

$y' = 4x(9 - x^2)^{-3/2}$

$y' = \frac{4x}{(9 - x^2)^{3/2}} = \frac{4x}{\sqrt{(9 - x^2)^3}}$

⑩ $y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$

$y' = \left[\frac{1}{2(x + \sqrt{x})^{1/2}}\right]\left[\frac{1}{1} + \frac{1}{2\sqrt{x}}\right]$ Add by finding common denom

$y' = \left[\frac{1}{2\sqrt{x + \sqrt{x}}}\right]\left[\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right]$

$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$ or $\frac{2\sqrt{x} + 1}{4\sqrt{x^2 + x^{3/2}}}$

Review:

4. $g(x) = (x^2-3x+4)(2x^2+4x)$

$g'(x) = (x^2-3x+4)(4x+4) + (2x-3)(2x^2+4x)$

$g'(x) = 4x^3+4x^2-12x^2-12x+16x+16 + 4x^3+8x^2-6x^2-12x$

$g'(x) = 8x^3-6x^2-8x+16$

Differentiation Rules:

⑥ $y = (1+x)^{10}$ $x = \underline{0}$ $y = \underline{\underline{1}}$ $(0,1)$

* $y = (1+0)^{10} = \underline{\underline{1}}$

① Find Derivative

$y' = 10(1+x)^9(1)$

$y' = 10(1+x)^9$

② Find Slope $(x=0)$

$y'(0) = 10(1+0)^9$

$= 10(1)^9$

$\boxed{= 10}$

m

③ Equation

$y - y_1 = m(x - x_1)$

$y - 1 = 10(x - 0)$

$y - 1 = 10x$

$\boxed{0 = 10x - y + 1}$

③ Sum + Difference Rule:

a) $y = x^{10} + 20x^5 - 12\sqrt[4]{x^3} + 30$

$= x^{10} + 20x^5 - 12x^{3/4} + 30$

$y' = 10x^9 + 100x^4 - 9x^{-1/4} + 0$

$= 10x^9 + 100x^4 - \dfrac{9}{x^{1/4}}$

8 $y = \dfrac{4}{\sqrt{9-x^2}} = \dfrac{4}{(9-x^2)^{1/2}} = 4(9-x^2)^{-1/2}$

$y' = -2(9-x^2)^{-3/2}(-2x)$

$= 4x(9-x^2)^{-3/2}$

$= \dfrac{4x}{(9-x^2)^{3/2}}$ or $\dfrac{4x}{\sqrt{(9-x^2)^3}}$

④ $$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

b) $f(x) = \boxed{\dfrac{2x-1}{4x}}$

$$f(x+h) = \frac{2(x+h)-1}{4(x+h)} = \boxed{\frac{2x+2h-1}{4x+4h}}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2x+2h-1}{(4x+4h)} - \frac{(2x-1)}{(4x)}}{h}$$

$$= \lim_{h\to 0} \frac{4x(2x+2h-1) - (2x-1)(4x+4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{8x^2+8xh-4x - (8x^2+8xh-4x-4h)}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{\cancel{8x^2}+\cancel{8xh}-\cancel{4x}-\cancel{8x^2}-\cancel{8xh}+\cancel{4x}+4h}{h(4x)(4x+4h)}$$

$$= \lim_{h\to 0} \frac{4\cancel{h}}{\cancel{h}(4x)(4x+4h)} = \frac{4}{(4x)^2} = \frac{4}{16x^2} = \boxed{\frac{1}{4x^2}}$$

Chain Rule:

(18) $y = \sqrt{x + \sqrt{x}} = (x + \sqrt{x})^{1/2} = (x + x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x + x^{1/2})^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$

$y' = \left[\frac{1}{2(x + x^{1/2})^{1/2}}\right]\left[\frac{1}{1} + \frac{1}{2x^{1/2}}\right]$

$y' = \left[\frac{1}{2\sqrt{x + \sqrt{x}}}\right]\left[\frac{2\sqrt{x} + 1}{2\sqrt{x}}\right]$

$y' = \frac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$ or $\frac{2x^{1/2} + 1}{4x^{1/2}(x + x^{1/2})^{1/2}}$

Ex: 2.5

6) $y = \dfrac{x^2}{2x+5}$

$$y' = \frac{(2x+5)(2x) - x^2(2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2+10x}{(2x+5)^2} = \frac{2x(x+5)}{(2x+5)^2}$$

$$\frac{0}{1} = \frac{2x(x+5)}{(2x+5)^2}$$

$$(2x)(x+5) = 0$$

$2x = 0$, $x = 0$ | $x+5 = 0$, $x = -5$

$x = 0$

$$y = \frac{(0)^2}{2(0)+5} = \frac{0}{5} = 0$$

$(0,0)$

$x = -5$

$$y = \frac{(-5)^2}{2(-5)+5} = \frac{25}{-5} = -5$$

$(-5,-5)$