Questions From Homework

$-\frac{3}{4}-\frac{4}{4}=\frac{-7}{4}$ $\quad -\frac{3}{4}\times 8=\frac{-24}{4}=-6$

② e) $y=\frac{1}{x^4}=1x^{-4}$

$y'=-4x^{-5}=\frac{-4}{x^5}$

d) $g(t)=8t^{-3/4}$

$g'(t)=-6t^{-7/4}=\frac{-6}{t^{7/4}}$

i) $f(x)=\sqrt[3]{x}=x^{1/3}$ $\quad \frac{1}{3}-\frac{3}{3}=\frac{-2}{3}$

$f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3x^{2/3}}=\frac{1}{3\sqrt[3]{x^2}}$

k) $y=\frac{1}{\sqrt{x}}=\frac{1}{x^{1/2}}=1x^{-1/2}$ $\quad -\frac{1}{2}-\frac{2}{2}=\frac{-3}{2}$

$y'=-\frac{1}{2}x^{-3/2}=-\frac{1}{2x^{3/2}}=\frac{-1}{2\sqrt{x^3}}$

l) $y=\frac{3}{\sqrt[4]{x}}=\frac{3}{x^{1/4}}=3x^{-1/4}$ $\quad 3\cdot -\frac{1}{4}=\frac{-3}{4}$ $\quad -\frac{1}{4}-\frac{4}{4}=\frac{-5}{4}$

$y'=\frac{-3}{4}x^{-5/4}=\frac{-3}{4x^{5/4}}$

m) $y=\sqrt{3}\,x^{\sqrt{2}}$ $\quad \sqrt{3}\cdot\sqrt{2}=\sqrt{6}$

$y'=\sqrt{6}\,x^{\sqrt{2}-1}$

③ e) $y=\sqrt{x^3}$, $x=8$

(i) Find y'

$y=x^{3/2}$

$y'=\frac{3}{2}x^{1/2}=\frac{3\sqrt{x}}{2}$

(ii) Sub in $x=8$

$y'=\frac{3\sqrt{8}}{2}=\frac{3\sqrt{2\cdot2\cdot2}}{2}$

$y'=\frac{6\sqrt{2}}{2}=\boxed{3\sqrt{2}}$ m

f) $y=\frac{6}{x}$, $x=-3$

(i) Find y'

$y=\frac{6}{x}=6x^{-1}$

$y'=-6x^{-2}=\frac{-6}{x^2}$

(ii) Sub in $x=-3$

$y'=\frac{-6}{(3)^2}=\frac{-6}{9}=\boxed{\frac{-2}{3}}$ m

⑤ $f(x)=\frac{1}{x}$ $\quad f(x+h)=\frac{1}{x+h}$

$f'(x)=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$ $\quad x(x+h)$

$f'(x)=\lim_{h\to 0}\frac{x-1(x+h)}{xh(x+h)}$

$f'(x)=\lim_{h\to 0}\frac{x-x-h}{xh(x+h)}$

$f'(x)=\lim_{h\to 0}\frac{-\cancel{h}}{x\cancel{h}(x+h)}=\frac{-1}{x^2}$

2 h) $h(y) = \left(\frac{y}{3}\right)^2 = \frac{y^2}{9} = \frac{1}{9}y^2$ $2 \times \frac{1}{9} = \frac{2}{9}$

$h'(y) = \frac{2}{9}y$

m) $y = \sqrt{3}\,x^{\sqrt{2}}$ $\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$

$y' = \sqrt{6}\,x^{\sqrt{2}-1}$

Example:

Find the slope of the tangent line to the graph of the given function at the given x value.

$$g(x) = \sqrt[5]{x} \qquad x = 32$$

① find $g'(x)$

$g(x) = \sqrt[5]{x} = x^{1/5}$

$g'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$

② sub in $x = 32$

m

$g'(32) = \frac{1}{5(32)^{4/5}} = \frac{1}{5(16)} = \frac{1}{80}$

Example:

$y - y_1 = m(x - x_1)$

Find the equation of the tangent line to the curve $f(x) = x^6$ at the point (-2, 64)

Remember that the equation of a line is found by using the point-slope formula... $y - y_1 = m(x - x_1)$

The curve is the graph of the function f(x) = x^6 and we know that the slope of the tangent line at (-2, 64) is the derivative f '(-2)

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

$x_1 = -2$ $m = -192$

$y_1 = 64$

i) find $f'(x)$

$f(x) = x^6$

$f'(x) = 6x^5$

(ii) sub in $x = -2$

$f'(-2) = 6(-2)^5$

$= 6(-32)$

$= -192$

m

(iii) $y - y_1 = m(x - x_1)$

$y - 64 = -192(x - (-2))$

$y - 64 = -192(x + 2)$

$y - 64 = -192x - 384$

$\boxed{y = -192x - 320}$

or

$192x + y + 320 = 0$

Sums and Differences

- These next rules say that <u>the derivative of a sum (difference) of functions is the sum (difference) of the derivatives:</u>

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x) = 2x^4 + \sqrt{x} = 2x^4 + x^{1/2}$

$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2} = 8x^3 + \frac{1}{2x^{1/2}}$

2. $f(x) = 6x^4 - 5x^3 - 2x + 17$

$f'(x) = 24x^3 - 15x^2 - 2$

3. $f(x) = (2x^3 - 5)^2 = (2x^3-5)(2x^3-5)$

$f(x) = 4x^6 - 10x^3 - 10x^3 + 25$

$f(x) = 4x^6 - 20x^3 + 25$

$f'(x) = 24x^5 - 60x^2$

Homework

① g) $y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2}(x+1) = x^{1/2} + x^{-1/2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

④ c) $\frac{x\cancel{y}}{\cancel{x}} = \frac{1}{x}$, $(\underset{x_1}{5}, \underset{y_1}{\tfrac{1}{5}})$

$y = \frac{1}{x} = 1x^{-1}$

① Find y'

$y' = -1x^{-2} = \frac{-1}{x^2}$

② Plug in $x=5$ into y'

$y'(5) = \frac{-1}{(5)^2} = \frac{-1}{25}$

$m = \frac{-1}{25}$

③ $y - y_1 = m(x - x_1)$

$y - \frac{1}{5} = \frac{-1}{25}(x - 5)$

$y - \frac{1}{5} = \frac{-1x}{25} + \frac{5}{25}$

$y - \frac{1}{5} = \frac{-1}{25}x + \frac{1}{5}$

$25 \quad \boxed{y = {}^{25}\frac{-1}{25}x + {}^{25}\frac{2}{5}}$

$25y = -1x + 10$

$\boxed{x + 25y - 10 = 0}$

⑤ limit definition of the derivative

$f(x) = \boxed{\frac{1}{x}}$ $\quad f(x+h) = \boxed{\frac{1}{x+h}}$

$$f'(x) = \lim_{h \to 0} \frac{\underline{\underline{f(x+h)}} - \underline{f(x)}}{h}$$

CD: $x(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x - 1(x+h)}{hx(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - h}{hx(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)} = \frac{-1}{x(x)} = \frac{-1}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

		Parallel	Perpendicular
① $y = 7x + 2$	$m = 7$	$m = 7$	$m = -\frac{1}{7}$
② $y = \frac{3}{2}x - 4$	$m = 3/2$	$m = 3/2$	$m = -\frac{2}{3}$
③ $3y + 6 = 4x$ $3y = 4x - 6$ $y = \frac{4}{3}x - 2$	$m = 4/3$	$m = 4/3$	$m = -\frac{3}{4}$

8) $y = x\sqrt{x}$

$y = x(x^{1/2})$

$y = x^{1+1/2}$

$y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2}$

$6 = \frac{3}{2}x^{1/2}$

$12 = 3x^{1/2}$

$4 = x^{1/2}$

$16 = x$

$6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6 \leftarrow y'$

$y = x\sqrt{x}$

$y = (16)\sqrt{16}$

$y = 16(4)$

$y = 64$

$(16, 64)$ is the point