

Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

Questions from Homework

5. Identify the following characteristics of the graph of each function.

- i) the equation of the asymptote
- ii) the domain and range
- iii) the y-intercept, to one decimal place if necessary
- iv) the x-intercept, to one decimal place if necessary

- a) $y = -5 \log_3(x+3)$
- b) $y = \log_6(4(x+9))$
- c) $y = \log_5(x+3) - 2$
- d) $y = -3 \log_2(x+1) - 6$

a) $y = -5 \log_3(x+3)$ $c = 3$ (base)

$a = -5$ $b = 1$ $h = -3$ $k = 0$

(i) VA: $x = -3$

(ii) D: $\{x | x > -3, x \in \mathbb{R}\}$ or $(-3, \infty)$

R: $\{y | y \in \mathbb{R}\}$ or $(-\infty, \infty)$

(iii) y-int ($x=0$)

$y = -5 \log_3(x+3)$

$y = -5 \log_3(0+3)$

$y = -5 \log_3(3) \rightarrow \frac{\log 3}{\log 3} = 1$

$y = -5(1)$

$y = -5$

(iv) x-int ($y=0$)

$y = -5 \log_3(x+3)$

$0 = -5 \log_3(x+3)$

$0 = \log_3(x+3)$ (log)

$3^0 = x+3$ (exp)

$1 = x+3$

$-2 = x$

b) $y = -3 \log_2(x+1) - 6$ $c = 2$ (base)

$a = -3$

(i) VA: $x = -1$

$b = 1$

(ii) D: $\{x | x > -1, x \in \mathbb{R}\}$ or $(-1, \infty)$

$h = -1$

R: $\{y | y \in \mathbb{R}\}$

$k = -6$

(iii) y-int ($x=0$)

$y = -3 \log_2(x+1) - 6$

$y = -3 \log_2(0+1) - 6$

$y = -3 \log_2(1) - 6 \rightarrow \frac{\log 1}{\log 2} = 0$

$y = -3(0) - 6$

$y = -6$

(iv) x-int ($y=0$)

$y = -3 \log_2(x+1) - 6$

$0 = -3 \log_2(x+1) - 6$

$6 = -3 \log_2(x+1)$

$-2 = \log_2(x+1)$ (log)

$2^{-2} = x+1$ (exp)

$\frac{1}{4} = x+1$

$-\frac{3}{4} = x$

or $x = -0.75$

Questions from Homework

11. Explain how the graph of $\frac{1}{3}(y+2) = \log_6(x-4)$ can be generated by transforming the graph of $y = \log_6 x$.

$$3. \quad \frac{1}{3}(y+2) = \log_6(x-4) \quad (\text{multiply a+k})$$

$$y+2 = 3\log_6(x-4)$$

$$y = \underline{3}\log_6(\underline{x-4}) - \underline{2}$$

$a=3 \rightarrow$ vertical stretch by a factor of 3

$b=1 \rightarrow$ no horizontal stretch

$h=4 \rightarrow$ translated 4 units right

$k=-2 \rightarrow$ " 2 units down.

x-int ($y=0$)

$$y = 3\log_6(x-4) - 2$$

$$0 = 3\log_6(x-4) - 2$$

$$\frac{2}{3} = \frac{3\log_6(x-4)}{3}$$

$$\frac{2}{3} = \log_6(x-4) \quad (\text{log})$$

$$6^{\frac{2}{3}} = x-4 \quad (\text{exp.})$$

$$3.3 = x-4$$

$$7.3 = x$$

y-int ($x=0$)

$$y = 3\log_6(x-4) - 2$$

$$y = 3\log_6(0-4) - 2 \quad \text{und.}$$

$$y = 3\log_6(-4) - 2$$

No y-int

General Properties of Logarithms:

If $C > 0$ and $C \neq 1$, then...

$$(i) \log_c 1 = 0$$

$$(ii) \log_c c^x = x$$

$$(iii) c^{\log_c x} = x$$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression $\log_6 1$, the argument is 1.

$$(i) \log_5 1 = 0 \quad (ii) \log_2 2^3 = 3 \quad (iii) 7^{\log_7 49} = 49$$

$$5^{\log_5 10} = 10$$

Product Law of Logarithms

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

Proof

Let $\log_c M = x$ and $\log_c N = y$, where M , N , and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$MN = (c^x)(c^y)$$

$$MN = c^{x+y}$$

$$\log_c MN = x + y$$

$$\log_c MN = \log_c M + \log_c N$$

Apply the product law of powers.

Write in logarithmic form.

Substitute for x and y .

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Proof

Let $\log_c M = x$ and $\log_c N = y$, where M , N , and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y}$$

Apply the quotient law of powers.

$$\log_c \frac{M}{N} = x - y$$

Write in logarithmic form.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Substitute for x and y .

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^P = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Proof

Let $\log_c M = x$, where M and c are positive real numbers with $c \neq 1$.

Write the equation in exponential form as $M = c^x$.

Let P be a real number.

$$M = c^x$$

$$M^P = (c^x)^P$$

$$M^P = c^{xP}$$

Simplify the exponents.

$$\log_c M^P = xP$$

Write in logarithmic form.

$$\log_c M^P = (\log_c M)P$$

Substitute for x .

$$\log_c M^P = P \log_c M$$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.

Product Law of Logarithms

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

$$\begin{aligned} \text{Ex: } \log_2 100 &= \log_2 (50 \cdot 2) \\ &= \log_2 100 \\ &= 2 \end{aligned}$$

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

$$\begin{aligned} \text{Ex: } \log_2 64 - \log_2 16 &= \log_2 \left(\frac{64}{16} \right) \\ &= \log_2 4 \\ &= 2 \end{aligned}$$

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^P = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

$$\begin{aligned} \text{Ex: } \log_2 \sqrt{8} &= \log_2 8^{1/2} \\ &= \frac{1}{2} \log_2 8 \\ &= \frac{1}{2} (3) \\ &= \frac{3}{2} \text{ or } 1.5 \end{aligned}$$

Homework

Finish Exercise 2

Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x , y , and z .

a) $\log_5 \frac{xy}{z}$

b) $\log_7 \sqrt[3]{x}$

c) $\log_6 \frac{1}{x^2}$

d) $\log \frac{x^3}{y\sqrt{z}}$

$$\text{a) } \log_5 \frac{xy}{z} = \boxed{\log_5 x + \log_5 y - \log_5 z}$$

$$\text{b) } \log_7 \sqrt[3]{x} = \log_7 x^{\frac{1}{3}} = \boxed{\frac{1}{3} \log_7 x}$$

$$\text{c) } \log_6 \frac{1}{x^2} = \log_6 1 - \log_6 x^2$$

$$= 0 - 2 \log_6 x$$

$$= \boxed{-2 \log_6 x}$$

$$\text{d) } \log \frac{x^3}{y\sqrt{z}} = \log x^3 - (\log y + \log \sqrt{z})$$

$$= \log x^3 - \log y - \log z^{\frac{1}{2}}$$

$$= \boxed{3 \log x - \log y - \frac{1}{2} \log z}$$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_6 8 + \log_6 9 - \log_6 2$

b) $\log_7 7\sqrt{7}$

c) $2 \log_2 12 - (\log_2 6 + \frac{1}{3} \log_2 27)$

a) $\log_6 8 + \log_6 9 - \log_6 2$

$$= \log_6 \left(\frac{8 \cdot 9}{2} \right)$$

$$= \log_6 36$$

$$= 2$$

b) $\log_7 7\sqrt{7}$

$$= \log_7 7 + \log_7 7^{\frac{1}{2}}$$

$$= \boxed{\log_7 7} + \frac{1}{2} \boxed{\log_7 7}$$

$$= 1 + \frac{1}{2}(1)$$

$$= 1 + \frac{1}{2}$$

$$= \frac{2}{2} + \frac{1}{2}$$

$$= \frac{3}{2} \text{ or } 1.5$$

c) $2 \log_2 12 - (\log_2 6 + \frac{1}{3} \log_2 27)$

$$2 \log_2 12 - \log_2 6 - \frac{1}{3} \log_2 27$$

$$\log_2 12^2 - \log_2 6 - \log_2 27^{\frac{1}{3}}$$

$$\log_2 144 - \log_2 6 - \log_2 3$$

$$\log_2 \left(\frac{144}{6 \cdot 3} \right)$$

$$\log_2 8$$

$$3$$

Example 3

Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $\log_7 x^2 + \log_7 x - \frac{5 \log_7 x}{2}$

b) $\log_5 (2x - 2) - \log_5 (x^2 + 2x - 3)$

Key Ideas

- Let P be any real number, and M , N , and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

Homework

Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

c) Express the following as a single logarithm... $\frac{1}{2}[(\log_5 a + 2\log_5 b) - 3\log_5 c]$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4} \left[12(\log_8 x^2 - 2\log_8 x) + 8\log_8 \sqrt{x} - 4\log_8 \frac{1}{x^7} \right]$$