

# Warm Up

Evaluate the following limits, if they exist:

$$1. \lim_{x \rightarrow 2} \frac{x-2}{x^3-8}$$

$$\lim_{x \rightarrow 2} \frac{\cancel{x-2}}{\cancel{x-2}(x^2+2x+4)} = \frac{1}{12}$$

$$2. \lim_{x \rightarrow 7} \frac{(\sqrt{x+2}-3)(\sqrt{x+2}+3)}{x-7}$$

$$\lim_{x \rightarrow 7} \frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$\lim_{x \rightarrow 7} \frac{\cancel{x-7}}{\cancel{x-7}(\sqrt{x+2}+3)} = \frac{1}{6}$$

$$3. \lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{[(a+h)+a][(a+h)-a]}{h}$$

$$\lim_{h \rightarrow 0} \frac{(a+h+a)(a+h-a)}{h}$$

$$\lim_{h \rightarrow 0} \frac{(2a+h)\cancel{h}}{\cancel{h}} = 2a$$

$$\lim_{h \rightarrow 0} \frac{(a+h)^2 - a^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{(a+h)(a+h) - a^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{a^2 + 2ah + h^2 - a^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{2ah + h^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{h}(2a+h)}{\cancel{h}} = 2a$$

## Questions from Homework

$$\textcircled{5} \text{ a) } \lim_{h \rightarrow 0} \frac{(4+h)^3 - 64}{h}$$

$$\lim_{h \rightarrow 0} \frac{[(4+h) - 4] [(4+h)^2 + 4(4+h) + 16]}{h}$$

$$\lim_{h \rightarrow 0} \frac{\cancel{h} [(4+h)^2 + 4(4+h) + 16]}{\cancel{h}} = 48$$

$$\textcircled{5} \text{ f) } \lim_{h \rightarrow 0} \frac{4(2+h)^2 \cdot \frac{1}{(2+h)^2} - \frac{1}{4} \cdot 4(2+h)^2}{\frac{h}{1} \cdot 4(2+h)^2} \quad \text{CD: } 4(2+h)^2$$

$$\lim_{h \rightarrow 0} \frac{4 - (2+h)^2}{4h(2+h)^2} \quad \leftarrow \text{factor}$$

$$\lim_{h \rightarrow 0} \frac{[2+(2+h)][2-(2+h)]}{4h(2+h)^2}$$

$$\lim_{h \rightarrow 0} \frac{[2+2+h][2-2-h]}{4h(2+h)^2}$$

$$\lim_{h \rightarrow 0} \frac{(4+h)\cancel{h}^{-1}}{4h(2+h)^2} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

**The common sense definition of a limit...**

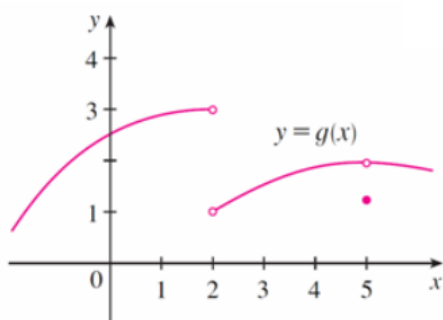


When does a limit exist?



# One-sided limits

Use the graph shown below to evaluate the following limits:



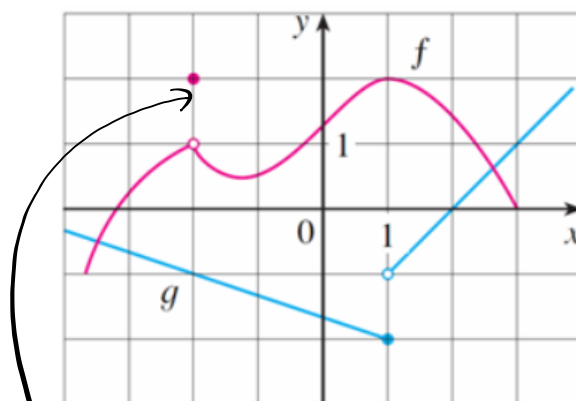
1.  $\lim_{x \rightarrow 2^-} g(x) = \boxed{3}$     2.  $\lim_{x \rightarrow 2^+} g(x) = \boxed{1}$     3.  $\lim_{x \rightarrow 2} g(x) = \boxed{\text{DNE}}$

"as x approaches 2 from the left"      "as x approaches 2 from the right"

4.  $\lim_{x \rightarrow 5^-} g(x) = \boxed{2}$     5.  $\lim_{x \rightarrow 5^+} g(x) = \boxed{2}$     6.  $\lim_{x \rightarrow 5} g(x) = \boxed{2}$

Notice...  $g(5) = 1.2$  (pick closed dot)  
 $g(2) = \text{undefined}$

Example:



Evaluate each of the following:

$f(-2) = 2$        $\lim_{x \rightarrow 1^-} g(x) = -2$        $g(1) = -2$

$\lim_{x \rightarrow 1^+} g(x) = -1$        $\lim_{x \rightarrow 1} g(x) = \text{DNE}$        $\lim_{x \rightarrow 1} f(x) = 2$

$\lim_{x \rightarrow -2} f(x) =$

# Homework

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