Chapter 11 Permutations, Combinations, and the Binomial Theorem

Section 11.1 Permutations

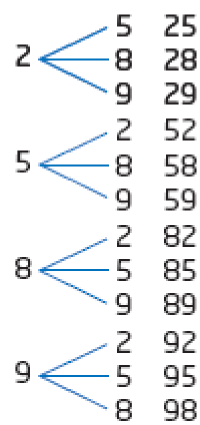
Section 11.1 Page 524 Question 1

a)

Position 1	Position 2	Position 3
Jo	Amy	Mike
Jo	Mike	Amy
Amy	Jo	Mike
Amy	Mike	Jo
Mike	Jo	Amy
Mike	Amy	Jo

There are six different arrangements.

b)



There are 12 different two-digit numbers.

c)

Starter	Main	Dessert
soup	chili	ice cream
soup	chili	fruit salad
soup	hamburger	ice cream
soup	hamburger	fruit salad
soup	chicken	ice cream
soup	chicken	fruit salad
soup	fish	ice cream
soup	fish	fruit salad
salad	chili	ice cream
salad	chili	fruit salad
salad	hamburger	ice cream
salad	hamburger	fruit salad
salad	chicken	ice cream
salad	chicken	fruit salad
salad	fish	ice cream
salad	fish	fruit salad

There are 16 different meals.

Section 11.1 Page 524 Question 2

a)

$$\begin{aligned}{}_8P_2 &= \frac{8!}{(8-2)!}\\ &= \frac{8!}{6!}\\ &= \frac{8(7)\cancel{(6!)}^{1}}{\cancel{6!}_{1}}\\ &= 56\end{aligned}$$

b)

$$\begin{aligned}{}_7P_5 &= \frac{7!}{(7-5)!}\\ &= \frac{7!}{2!}\\ &= \frac{7(6)(5)(4)(3)\cancel{(2!)}^{1}}{\cancel{2!}_{1}}\\ &= 2520\end{aligned}$$

c)

$$\begin{aligned}{}_6P_6 &= \frac{6!}{(6-6)!}\\ &= \frac{6!}{0!}\\ &= \frac{6(5)(4)(3)(2)(1)}{1}\\ &= 720\end{aligned}$$

d)

$$\begin{aligned}{}_4P_1 &= \frac{4!}{(4-1)!}\\ &= \frac{4!}{3!}\\ &= \frac{4\cancel{(3!)}^{1}}{\cancel{3!}_{1}}\\ &= 4\end{aligned}$$

Section 11.1 Page 524 Question 3

Left Side = 4! + 3!
= 4(3!) + 3!
= (4 + 1)(3!)
= 5(3!)

Right Side = (4 + 3)!
= 7!

Left Side ≠ Right Side

Section 11.1 Page 524 Question 4

a) $9! = 9(8)(7)(6)(5)(4)(3)(2)(1)$
$= 362\ 880$

b) $$\begin{aligned}\frac{9!}{5!4!} &= \frac{9(8)(7)(6)\cancel{(5!)}^{1}}{\cancel{5!}_{1}(4)(3)(2)(1)}\\ &= 126\end{aligned}$$

c) $(5!)(3!) = 5(4)(3)(2)(1)(3)(2)(1)$
$= 720$

d) $6(4!) = 6(4)(3)(2)(1)$
$= 144$

e) $\dfrac{102!}{100!2!} = \dfrac{102(101)\,(\cancel{100!})}{\cancel{100!}(2)(1)}$

$= 5151$

f) $7! - 5!$
$= 7(6)(5)(4)(3)(2)(1) - 5(4)(3)(2)(1)$
$= 5040 - 120$
$= 4920$

Section 11.1 Page 524 Question 5

a) There are six letters in *hoodie*. There are 2! ways to arrange the two *o*'s.

$\dfrac{6!}{2!} = \dfrac{6(5)(4)(3)\,(\cancel{2!})}{\cancel{2!}}$

$= 360$

The number of different six-letter arrangements is 360.

b) There are seven letters in *decided*. There are 3! ways to arrange the three *d*'s and 2! ways to arrange the two *e*'s

$\dfrac{7!}{3!2!} = \dfrac{7(6)(5)(4)\,(\cancel{3!})}{\cancel{3!}(2)(1)}$

$= 420$

The number of different seven-letter arrangements is 420.

c) There are 11 letters in *aqilluqqaaq*. There are 3! ways to arrange the three *a*'s, 4! ways to arrange the four *q*'s, and 2! ways to arrange the two *l*'s.

$\dfrac{11!}{3!4!2!} = \dfrac{11(10)(9)(8)(7)(6)(5)\,(\cancel{4!})}{3(2)(1)\cancel{4!}(2)(1)}$

$= 138\ 600$

The number of different 11-letter arrangements is 138 600.

d) There are six letters in *deeded*. There are 3! ways to arrange the three *d*'s and 3! ways to arrange the three *e*'s

$\dfrac{6!}{3!3!} = \dfrac{6(5)(4)\,(\cancel{3!})}{\cancel{3!}(3)(2)(1)}$

$= 20$

The number of different six-letter arrangements is 20.

e) There are five letters in *puppy*. There are 3! ways to arrange the three *p*'s.

$$\frac{5!}{3!} = \frac{5(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}}$$
$$= 20$$

The number of different five-letter arrangements is 20.

f) There are eight letters in *baguette*. There are 2! ways to arrange the two *e*'s and 2! ways to arrange the two *t*'s.

$$\frac{8!}{2!2!} = \frac{8(7)(6)(5)(4)(3)\cancel{(2!)}^{1}}{\cancel{2!}_{1}(2)(1)}$$
$$= 10\ 080$$

The number of different six-letter arrangements is 10 080.

Section 11.1 Page 525 Question 6

$4! = 4(3)(2)(1)$
$= 24$

The four names can be listed in 24 ways.

Section 11.1 Page 525 Question 7

a)

$$_nP_2 = 30$$
$$\frac{n!}{(n-2)!} = 30$$
$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$
$$n(n-1) = 6(5)$$

Using reasoning, the solution to $_nP_2$ is $n = 6$.

b)

$$_nP_3 = 990$$
$$\frac{n!}{(n-3)!} = 990$$
$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 990$$
$$n(n-1)(n-2) = 11(10)(9)$$

Using reasoning, the solution to $_nP_3$ is $n = 11$.

c) $$_6P_r = 30$$

$$\frac{6!}{(6-r)!} = 30$$

$$\frac{6!}{30} = (6-r)!$$

$$\frac{\cancel{6}\,\cancel{(5)}(4!)}{\cancel{30}} = (6-r)!$$

$$4! = (6-r)!$$

$$4 = 6 - r$$

$$r = 2$$

Using reasoning, the solution to $_6P_r$ is $r = 2$.

d) $$2(_nP_2) = 60$$

$$_nP_2 = \frac{60}{2}$$

$$_nP_2 = 30$$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 30$$

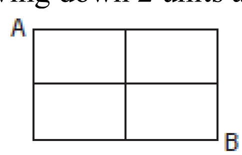
$$n(n-1) = 6(5)$$

Using reasoning, the solution to $2(_nP_2)$ is $n = 6$.

Section 11.1 Page 525 Question 8

a) Determine the number of arrangements of moving down 2 units and right 2 units.

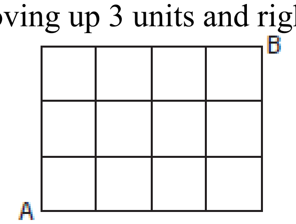
$$\frac{4!}{2!2!} = \frac{4(3)(2!)}{2(1)(2!)}$$
$$= 6$$



There are 6 pathways from A to B.

b) Determine the number of arrangements of moving up 3 units and right 4 units.

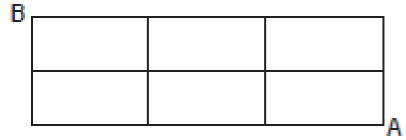
$$\frac{7!}{3!4!} = \frac{7(6)(5)(4!)}{3(2)(1)(4!)}$$
$$= 35$$



There are 35 pathways from A to B.

c) Determine the number of arrangements of moving up 2 units and left 3 units.

$$\frac{5!}{2!3!} = \frac{5(4)(3!)}{2(1)(3!)}$$
$$= 10$$



There are 10 pathways from A to B.

Section 11.1 Page 525 Question 9

a) Case 1: first digit is 3 or 5; Case 2: first digit is 2 or 4

b) Case 1: first letter is B; Case 2: first letter is E

Section 11.1 Page 525 Question 10

a) The boys can be arranged on the ends in 2! = 2 ways. The girls can be arranged in the middle in 4! = 24 ways.

$2!4! = 2(24)$
$\quad = 48$

The two boys and four girls can be arranged in 48 ways.

b) There are 2! = 2 ways to arrange the boys together. Considering the boys as one object means there are five objects to arrange in 5! = 120 ways. Since the boys must be seated together, there are 5!2! = 240 ways to arrange the two boys and four girls.

c) There are 2! = 2 ways to arrange the boys in the middle of the row. There are 4! = 24 ways to arrange the girls. Since the boys must be seated together, there are 4!2! = 48 ways to arrange the two boys and four girls.

Section 11.1 Page 525 Question 11

a) $7! = 7(6)(5)(4)(3)(2)(1)$
$\quad = 5040$

Seven books can be arranged in 7! = 5040 ways.

b) $$\frac{7!}{2!} = \frac{7(6)(5)(4)(3)(2!)}{(2!)}$$
$$= 2520$$

If two of the books are identical, the seven books can be arranged in 2520 ways.

c) Case 1: mathematics book is on the left end
There are 6! = 720 ways to arrange the other six books.
Case 2: mathematics book is not on the left end, so it must be on the right end
There are 6! = 720 ways to arrange the other six books.

The books can be arranged in 720 + 720 = 1440 ways.

d) There are 4! = 24 ways to arrange the four particular books together. Considering those four books as one object means there are 3 + 1 = 4 objects to arrange in 4! = 24 ways. Since four of the books must be together, there are 4!4! = 576 ways to arrange the books.

Section 11.1 Page 525 Question 12

The letters can be arranged in 6! = 720 ways.

First letter	Middle letters	Last letter
4 choices	4! choices	4 – 1 = 3 choices

Total number of arrangements that begin and end with a consonant: 4(4!)(3) = 288

Section 11.1 Page 525 Question 13

First character	Last 3 characters
26 – 1 = 25 choices	10(9)(8) = 720 choices

total number of different codes: 25(720) = 18 000

Since 18 000 < 25 300, the organization will not have enough different ID codes for its members.

Section 11.1 Page 525 Question 14

Iblauk has five colour choices for the body of the mitt and four choices for the wrist. She can make 5(4) = 20 different colour combinations of mitts.

Section 11.1 Page 525 Question 15

There are 40 choices for each of the three numbers in the sequence.

40(40)(40) = 64 000

Total time to test all combinations: 64 000(15 s) = 960 000 s

Convert 960 000 s to hours:

$$960\ 000 \text{ s}\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right) = 266\frac{2}{3} \text{ h}$$

It would take $266\frac{2}{3}$ h to test all possible combinations.

Section 11.1 Page 526 Question 16

a) Jodi can park the vehicles in 7! = 5040 ways.

b) There are 2! = 2 ways to arrange the truck and hybrid together. Considering those two vehicles as one object means there are six objects to arrange in 6! = 720 ways. Since the truck and hybrid must be beside each other, there are 6!2! = 1440 ways to arrange the seven vehicles.

c) Number of ways the vehicles can be parked so the convertible is not next to the subcompact
= total arrangements of the vehicles – number of ways two of the vehicles can be parked beside each other
= 5040 – 1440 (from parts a) and b))
= 3600
There are 3600 ways for the vehicles to be parked without the convertible next to the subcompact.

Section 11.1 Page 526 Question 17

a) There are eight letters in *parallel*. There are 3! ways to arrange the three *l*'s and 2! ways to arrange the two *a*'s

$$\frac{8!}{3!2!} = \frac{8(7)(6)(5)(4)\,\cancel{(3!)}^{1}}{\cancel{3!}_{1}(2)(1)}$$
$$= 3360$$

The number of eight-letter arrangements possible is 3360.

b) Considering the *l*'s as one object means there are 5 + 1 = 6 objects to arrange in 6! = 720 ways. Since there are 2! ways to arrange the two *a*'s, there are

$$\frac{6!}{2!} = \frac{6(5)(4)(3)\,\cancel{(2!)}}{\cancel{2!}}$$
$$= 360$$

ways to arrange the letters.

Section 11.1 Page 526 Question 18

a) The word has five letters, with two pairs of identical letters. Example: AABBS

b) Examples: teeth, radar, sells

Section 11.1 Page 526 Question 19

First digit	Second digit	Third digit	Last digit
5 possibilities	9 possibilities	9 possibilities	9 possibilities
(3, 4, 5, 6, or 8)			

Number of integers from 3000 to 8999 that do not contain 7s: 5(9)(9)(9) = 3645

Section 11.1 Page 526 Question 20

a) 26(10)(26)(10)(26)(10)
There are 17 576 000 different postal codes.

b) Example: Yes, Canada will eventually exceed 15.5 million postal communities.

Section 11.1 Page 526 Question 21

a) For each of the 14 lines, there are 10 pages of lines that could be substituted. There are 10^{14} arrangements of the lines

b) Yes, the title is reasonable because 10^{14} = 100 000 000 000 000, which is 100 million million.

Section 11.1 Page 526 Question 22

a)

$$\begin{aligned} {}_3P_r &= 3! \\ \frac{3!}{(3-r)!} &= 3! \\ 1 &= (3-r)! \\ 0! &= (3-r)! \\ 0 &= 3-r \\ r &= 3 \end{aligned}$$

b)

$$\begin{aligned} {}_7P_r &= 7! \\ \frac{7!}{(7-r)!} &= 7! \\ 1 &= (7-r)! \\ 0! &= (7-r)! \\ 0 &= 7-r \\ r &= 7 \end{aligned}$$

c)

$$\begin{aligned} {}_nP_3 &= 4({}_{n-1}P_2) \\ \frac{n!}{(n-3)!} &= 4\left(\frac{(n-1)!}{(n-1-2)!}\right) \\ \frac{n!}{(n-3)!} &= \frac{4(n-1)!}{(n-3)!} \\ n! &= 4(n-1)! \\ n(n-1)! &= 4(n-1)! \\ n &= 4 \end{aligned}$$

d)

$$\begin{aligned} n({}_5P_3) &= {}_7P_5 \\ n\left(\frac{5!}{(5-3)!}\right) &= \frac{7!}{(7-5)!} \\ \frac{n(5!)}{2!} &= \frac{7!}{2!} \\ n &= \frac{7!}{5!} \\ n &= \frac{7(6)(5!)}{5!} \\ n &= 42 \end{aligned}$$

Section 11.1 Page 526 Question 23

$$\begin{aligned} {}_nP_n &= \frac{n!}{(n-n)!} \\ &= \frac{n!}{0!} \end{aligned}$$

And ${}_nP_n = n!$; so, 0! = 1.

Section 11.1 Page 526 Question 24

${}_3P_5$ gives an error message on a calculator because the number of items taken from the set is greater than the number of items in the set.

Section 11.1 Page 526 Question 25

Case 1: One-digit number
3 possibilities for a single digit odd number

Case 2: Two-digit number
4 possibilities for the first digit (1, 2, 3, 4, 5 minus one digit, since the last digit must be odd)
3 possibilities for the last digit (1, 3, 5)
4(3) = 12 ways two-digit numbers to be formed

Case 3: Three-digit number
4 possibilities for the first digit (1, 2, 3, 4, 5 minus one digit, since the last digit must be odd)
4 possibilities for the middle digit (0, 1, 2, 3, 4, 5 minus two, for the first and last digits)
3 possibilities for the last digit (1, 3, 5)
4(4)(3) = 48 ways for three-digit numbers to be formed

The odd numbers can be formed in 3 + 12 + 48 = 63 ways.

Section 11.1 Page 526 Question 26

Case 1a: Four-digit number starting with 2

First digit	Second digit	Third digit	Last digit
1 choice	3 choices	2 choices	1 choice

Four digit numbers can be formed in 1(3)(2)(1) = 6 ways.

Case 1b: Four-digit number starting with 1, 3, or 5

First digit	Second digit	Third digit	Last digit
3 choices	3 choices	2 choices	2 choices

Four digit numbers can be formed in 3(3)(2)(2) = 36 ways.

Case 2a: Five-digit number starting with 2

First digit	Second digit	Third digit	Fourth Digit	Last digit
1 choice	3 choices	2 choices	1 choice	1 choice

Five digit numbers can be formed in 1(3)(2)(1)(1) = 6 ways.

Case 2b: Five-digit number starting with 1, 3, or 5

First digit	Second digit	Third digit	Fourth Digit	Last digit
3 choices	3 choices	2 choices	1 choice	2 choices

Five digit numbers can be formed in 3(3)(2)(1)(2) = 36 ways.

Total number of even numbers: 6 + 36 + 6 + 36 = 84

Section 11.1 Page 526 Question 27

Case 1: One-digit number
8 ways

Case 2: Two-digit number

First digit	Second digit
9 choices	9 choices

Two-digit numbers can be formed in 9(9) = 81 ways

Case 3: Three-digit number

First digit	Second digit	Last digit
9 choices	9 choices	8 choices

Three-digit numbers can be formed in 9(9)(8) = 648 ways

Total number of integers: 8 + 81 + 648 = 737

Section 11.1 Page 527 Question 28

There are six possible locations for the first yellow cube and five possible locations for the other yellow cube.

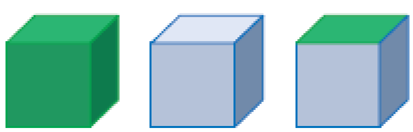
Total number of arrangements: $\frac{6(5)}{2} = 15$ (Divide by 2 because the cubes are identical.)

Total number of permutations with two objects the same and four objects the same:

$$\begin{aligned}\frac{6!}{2!4!} &= \frac{6(5)(4!)}{2(1)4!}\\ &= \frac{6(5)}{2}\\ &= 15\end{aligned}$$

Section 11.1 Page 527 Question 29

Consider the cases for each number of sides painted green and other sides painted grey.



Zero sides painted green: 1 way
One side painted green: 1 way
Two sides painted green: 2 ways
Three sides painted green: 2 ways
Four sides painted green: 2 ways
Five sides painted green: 1 way
Six sides painted green: 1 way
Total ways of painting the cube using two colours: 1 + 1 + 2 + 2 + 2 + 1 + 1 = 10

Section 11.1 Page 527 Question 30

Example: Use the numbers 1 to 9 to represent the different students.

Day 1	Day 2	Day 3	Day 4
1 2 3	1 4 7	1 4 9	1 6 8
4 5 6	2 5 8	2 6 7	2 4 9
7 8 9	3 6 9	3 5 8	3 5 7

Section 11.1 Page 527 Question 31

Determine how many factors of 5 there are in 100!. Each multiple of 5 has one factor of 5 except 25, 50, 75, and 100, which have two factors of 5. So, there are 20 + 4 = 24 factors of 5 in 100!. There are more than enough factors of 2 to match up with the 5s to make factors of 10, so there are 24 zeros.

Section 11.1 Page 527 Question 32

a) EDACB or BCADE

b) 2

c) None. Since F only knows A, then F must stand next to A. However, in both arrangements from part a), A must stand between C and D. Therefore, no possible arrangement satisfies the conditions.

Section 11.1 Page 527 Question C1

a) ${}_aP_b$ represents the number of permutations or arrangements in order of b objects taken from a group of a objects. For example, ${}_{20}P_3$ represents the number of arrangements of three council positions taken from a team of 20 members.

b) $b \leq a$ because the number of objects selected must be less than or equal to the number of objects in the set

Section 11.1 Page 527 Question C2

If all of the objects are different, they can be arranged in n! ways. If some of the objects are identical, you need to divide by the number of ways of arranging the objects in each group of identical objects.

Section 11.1 Page 527 Question C3

a) $$\frac{3!(n+2)!}{4!(n-1)!}=\frac{3!(n+2)(n+1)(n)(n-1)!}{4!(n-1)!}$$
$$=\frac{3!(n+2)(n+1)n}{4!}$$
$$=\frac{3!(n+2)(n+1)n}{4(3!)}$$
$$=\frac{(n+2)(n+1)n}{4}$$

b) $$\frac{7!(r-1)!}{6!(r+1)!}+\frac{5!r!}{3!(r+1)!}=\frac{7(6)(5)(4)\,\overset{1}{\cancel{(3!)}}\,(r-1)!}{6(5)(4)\,\underset{1}{\cancel{3!}}(r+1)!}+\frac{5(4)\,\overset{1}{\cancel{(3!)}}\,r!}{\underset{1}{\cancel{3!}}(r+1)!}$$
$$=\frac{7(6)(5)(4)(r-1)!}{6(5)(4)(r+1)!}+\frac{5(4)r!}{(r+1)!}\times\frac{6(5)(4)}{6(5)(4)}$$
$$=\frac{7(6)(5)(4)(r-1)!\ +\ 5(4)r!(6)(5)(4)}{6(5)(4)(r+1)!}$$
$$=\frac{7(r-1)!\ +\ 5(4)r!}{(r+1)!}$$
$$=\frac{7(r-1)!\ +\ 5(4)(r)(r-1)!}{(r+1)(r)(r-1)!}$$
$$=\frac{7\ +\ 5(4)(r)}{(r+1)(r)}$$
$$=\frac{7+20r}{r(r+1)}$$

Section 11.1 Page 527 Question C5

a) 9! = 9(8)(7)(6)(5)(4)(3)(2)(1)
= 362 880

b) log (9!) = 5.559 763 …

c) log (10!) = 6.559 763 …

d) The answer to part c) is 1 greater than the answer to part b).
This is because 10! = 10(9!) and log (10!) = log 10 + log (9!) = 1 + log (9!).

Section 11.2 Combinations

Section 11.2 Page 534 Question 1

a) Combination, because the order that members shake hands is not important

b) Permutation, because the order of the digits is important

c) Combination, because the order that the cars are purchased is not important

d) Combination, because the order that players are selected to ride in the van is not important

Section 11.2 Page 534 Question 2

${}_5P_3$ is a permutation representing the number of ways of arranging 3 objects taken from a group of 5 objects. ${}_5C_3$ is a combination representing the number of ways of choosing any 3 objects from a group of 5 objects.

$$\begin{aligned} {}_5P_3 &= \frac{5!}{2!} \\ &= \frac{5(4)(3)(2!)}{2!} \\ &= 60 \end{aligned}$$

$$\begin{aligned} {}_5C_3 &= \frac{5!}{2!3!} \\ &= \frac{5(4)(3!)}{2(1)3!} \\ &= 10 \end{aligned}$$

Section 11.2 Page 534 Question 3

a) $$\begin{aligned} {}_6P_4 &= \frac{6!}{2!} \\ &= \frac{6(5)(4)(3)(2!)}{2!} \\ &= 360 \end{aligned}$$

b) $$\begin{aligned} {}_7C_3 &= \frac{7!}{4!3!} \\ &= \frac{7(6)(5)\cancel{(4!)}^{1}}{\cancel{4!}_{1}(3)(2)(1)} \\ &= 35 \end{aligned}$$

c) $$\begin{aligned} {}_5C_2 &= \frac{5!}{3!2!} \\ &= \frac{5(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}(2)(1)} \\ &= 10 \end{aligned}$$

d) $$\begin{aligned} {}_{10}C_7 &= \frac{10!}{3!7!} \\ &= \frac{10(9)(8)\cancel{(7!)}^{1}}{3(2)(1)\cancel{(7!)}_{1}} \\ &= 120 \end{aligned}$$

Section 11.2 Page 534 Question 4

a) $_{10}C_4 = \frac{10!}{6!4!}$

$= \frac{10(9)(8)(7)\ \cancel{(6!)}^{1}}{\cancel{6!}_{1}(4)(3)(2)(1)}$

$= 210$

b) $_{10}P_4 = \frac{10!}{6!}$

$= \frac{10(9)(8)(7)\ \cancel{(6!)}^{1}}{\cancel{6!}_{1}}$

$= 5040$

Section 11.2 Page 534 Question 5

a) AB, AC, AD, BC, BD, CD

b) AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC

c) The number of permutations is 2! times the number of combinations.

Section 11.2 Page 534 Question 6

a)

$$
\begin{aligned}
{}_nC_1 &= 10 \\
\frac{n!}{(n-1)!1!} &= 10 \\
\frac{n!}{(n-1)!} &= 10 \\
\frac{n\ \cancel{(n-1)!}^{1}}{\cancel{(n-1)!}_{1}} &= 10 \\
n &= 10
\end{aligned}
$$

b)

$$
\begin{aligned}
{}_nC_2 &= 21 \\
\frac{n!}{(n-2)!2!} &= 21 \\
\frac{n!}{(n-2)!} &= 21(2!) \\
\frac{n(n-1)\ \cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}} &= 21(2)(1) \\
n(n-1) &= 42 \\
n^2 - n - 42 &= 0 \\
(n-7)(n+6) &= 0
\end{aligned}
$$

$n - 7 = 0$ or $n + 6 = 0$

$n = 7$ $\quad$ $n = -6$

Since $n \geq 2$, the solution is $n = 7$.

c)

$$
\begin{aligned}
{}_{n}C_{n-2} &= 6\\
\frac{n!}{(n-(n-2))!(n-2)!} &= 6\\
\frac{n!}{(n-n+2)!(n-2)!} &= 6\\
\frac{n!}{2!(n-2)!} &= 6\\
\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{2!\,\cancel{(n-2)!}_{1}} &= 6\\
\frac{n(n-1)}{2!} &= 6\\
n(n-1) &= 12\\
n(n-1) &= 4(3)\\
n &= 4
\end{aligned}
$$

d)

$$
\begin{aligned}
{}_{n+1}C_{n-1} &= 15\\
\frac{(n+1)!}{(n+1-(n-1))!(n-1)!} &= 15\\
\frac{(n+1)!}{(n+1-n+1)!(n-1)!} &= 15\\
\frac{(n+1)!}{2!(n-1)!} &= 15\\
\frac{(n+1)(n)\,\cancel{(n-1)!}^{1}}{2!\,\cancel{(n-1)!}_{1}} &= 15\\
\frac{(n+1)(n)}{2!} &= 15\\
n(n+1) &= 15(2)(1)\\
n(n+1) &= 30\\
n(n+1) &= 5(6)\\
n &= 5
\end{aligned}
$$

Section 11.2 Page 534 Question 7

a) Case 1: one-digit numbers, Case 2: two-digit numbers, Case 3: three-digit numbers
b) Cases involve grouping the 4 members from either grade.
Case 1: four grade 11s, Case 2: three grade 11s and one grade 12, Case 3: two grade 11s and two grade 12s, Case 4: one grade 11 and three grade 12s, Case 5: four grade 12s

Section 11.2 Page 534 Question 8

Left Side $= {}_{11}C_3$

$$
\begin{aligned}
&= \frac{11!}{(11-3)!3!}\\
&= \frac{11!}{8!3!}
\end{aligned}
$$

Right Side $= {}_{11}C_8$

$$
\begin{aligned}
&= \frac{11!}{(11-8)!8!}\\
&= \frac{11!}{3!8!}
\end{aligned}
$$

Left Side = Right Side

Section 11.2 Page 534 Question 9

a)

$$
\begin{aligned}
{}_{5}C_{5} &= \frac{5!}{(5-5)!5!}\\
&= \frac{5!}{0!5!}\\
&= 1
\end{aligned}
$$

b) $_5C_0 = \frac{5!}{(5-0)!0!}$
$= \frac{5!}{5!0!}$
$= 1$

There is only one way to choose 5 objects from a group of 5 objects, and only one way to choose 0 objects from a group of 5 objects.

Section 11.2 Page 534 Question 10

a) $_4C_3 = \frac{4!}{(4-3)!3!}$
$= \frac{4!}{1!3!}$
$= \frac{4(\cancel{3!})^{1}}{\cancel{3!}_{1}}$
$= 4$

b) Case 1: one coin

$_4C_1 = \frac{4!}{(4-1)!1!}$
$= \frac{4!}{3!}$
$= \frac{4(\cancel{3!})^{1}}{\cancel{3!}_{1}}$
$= 4$

Case 2: two coins

$_4C_2 = \frac{4!}{(4-2)!2!}$
$= \frac{4!}{2!2!}$
$= \frac{4(3)(\cancel{2!})^{1}}{\cancel{2!}_{1}(2!)}$
$= \frac{12}{2(1)}$
$= 6$

Total number of sums: 4 + 6 = 10

Section 11.2 Page 534 Question 11

a) $_6C_4 = \frac{6!}{(6-4)!4!}$
$= \frac{6(5)(\cancel{4!})^{1}}{2!\cancel{4!}_{1}}$
$= 15$

b) Case 1: four females

${}_6C_4$ = 15 from part a)

Case 2: five females

$$\begin{aligned}{}_6C_5 &= \frac{6!}{(6-5)!5!}\\ &= \frac{6!}{1!5!}\\ &= \frac{6\,(\cancel{5!})}{\cancel{5!}}\\ &= 6\end{aligned}$$

Case 3: six females

$$\begin{aligned}{}_6C_6 &= \frac{6!}{(6-6)!6!}\\ &= \frac{6!}{0!6!}\\ &= 1\end{aligned}$$

Total number of ways of selecting at least four females: 15 + 6 + 1 = 22

Section 11.2 Page 534 Question 12

a) Left Side = ${}_nC_{r-1} + {}_nC_r$

$$\begin{aligned}&= \frac{n!}{(n-(r-1))!(r-1)!} + \frac{n!}{(n-r)!r!}\\ &= \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!r!}\\ &= \frac{n!(n-r)!r! \;+ n!(n-r+1)!(r-1)!}{(n-r+1)!(r-1)!(n-r)!r!}\\ &= \frac{n!(n-r)!r(r-1)! \;+ n!(n-r+1)(n-r)!(r-1)!}{(n-r+1)!(r-1)!(n-r)!r!}\\ &= \frac{n!\,\cancel{(n-r)!}\,\cancel{(r-1)!}[r+(n-r+1)]}{(n-r+1)!\,\cancel{(r-1)!}\,\cancel{(n-r)!}\,r!}\\ &= \frac{n!(r+n-r+1)}{(n-r+1)!r!}\\ &= \frac{n!(n+1)}{(n-r+1)!r!}\\ &= \frac{(n+1)!}{(n-r+1)!r!}\end{aligned}$$

Right Side = ${}_{n+1}C_r$

$$= \frac{(n+1)!}{(n+1-r)!r!}$$

Left Side = Right Side

Section 11.2 Page 534 Question 13

$$\begin{aligned}{}_6C_3 &= \frac{6!}{(6-3)!3!} \\ &= \frac{6!}{3!3!} \\ &= \frac{6(5)(4)\,\cancel{(3!)}^{1}}{3(2)(1)\,\cancel{(3!)}_{1}} \\ &= 20\end{aligned}$$

You can order 20 different burgers. This is a combination because the order of the ingredients is not important.

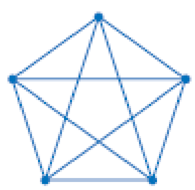
Section 11.2 Page 535 Question 14

a) $$\begin{aligned}{}_{10}C_4 &= \frac{10!}{(10-4)!4!} \\ &= \frac{10!}{6!4!} \\ &= \frac{10(9)(8)(7)\,\cancel{(6!)}^{1}}{\cancel{(6!)}_{1}\,(4)(3)(2)(1)} \\ &= 210\end{aligned}$$

b) This question involves a combination because the order of toppings is not important.

Section 11.2 Page 535 Question 15

a) Method 1: Use a diagram.



Method 2: Use combinations.

$$\begin{aligned}{}_5C_2 &= \frac{5!}{(5-2)!2!} \\ &= \frac{5!}{3!2!} \\ &= \frac{5(4)\,\cancel{(3!)}^{1}}{\cancel{(3!)}_{1}\,(2)(1)} \\ &= 10\end{aligned}$$

You can draw 10 line segments connecting any two of the five points.

b) $_5C_3 = \frac{5!}{(5-3)!3!}$

$= \frac{5!}{2!3!}$

$= \frac{5(4)\cancel{(3!)}^{1}}{(2)(1)\cancel{(3!)}_{1}}$

$= 10$

You can draw 10 triangles from the five given points.

c) $_{10}C_3 = \frac{10!}{(10-3)!3!}$

$= \frac{10!}{7!3!}$

You can draw $\frac{10!}{7!3!}$ triangles from ten non-collinear points.

Number of line segments: $_{10}C_2 = \frac{10!}{(10-2)!2!}$

$= \frac{10!}{8!2!}$

The number of triangles is determined by the number of selections of choosing three points, whereas the number of line segments is determined by the number of selections of choosing two points.

Section 11.2 Page 535 Question 16

Left Side = $_nC_r$

$= \frac{n!}{(n-r)!r!}$

Right Side = $_nC_{n-r}$

$= \frac{n!}{(n-(n-r))!(n-r)!}$

$= \frac{n!}{(n-n+r)!(n-r)!}$

$= \frac{n!}{r!(n-r)!}$

Left Side = Right Side

Section 11.2 Page 535 Question 17

a)
$$
\begin{aligned}
{}_{20}C_{12} &= \frac{20!}{(20-12)!12!} \\
&= \frac{20!}{8!12!} \\
&= \frac{20(19)(18)(17)(16)(15)(14)(13)\,(\cancel{12!})^{1}}{8(7)(6)(5)(4)(3)(2)(1)\,(\cancel{12!})_{1}} \\
&= 125\ 970
\end{aligned}
$$
There can be 125 970 12-person juries selected.

b)
$$
\begin{aligned}
{}_{12}C_7({}_8C_5) &= \frac{12!}{(12-7)!7!}\left(\frac{8!}{(8-5)!5!}\right) \\
&= \frac{12!8!}{5!7!3!5!} \\
&= \frac{12(11)(10)(9)(8)\,(\cancel{7!})^{1}(8)(7)(6)\,(\cancel{5!})^{1}}{5(4)(3)(2)(1)\,(\cancel{7!})_{1}(3)(2)(1)\,(\cancel{5!})_{1}} \\
&= 44\ 352
\end{aligned}
$$
There can be 44 352 juries containing seven women and five men.

c) Case 1: 10 women
$$
\begin{aligned}
{}_{12}C_{10}({}_8C_2) &= \frac{12!}{(12-10)!10!}\left(\frac{8!}{(8-2)!2!}\right) \\
&= \frac{12!}{2!10!}\left(\frac{8!}{6!2!}\right) \\
&= \frac{12(11)(\cancel{10!})^{1}}{2(1)\,\cancel{10!}_{1}}\left(\frac{8(7)(\cancel{6!})^{1}}{\cancel{6!}_{1}(2)(1)}\right) \\
&= 66(28) \\
&= 1848
\end{aligned}
$$

Case 2: 11 women
$$
\begin{aligned}
{}_{12}C_{11}({}_8C_1) &= \frac{12!}{(12-11)!11!}\left(\frac{8!}{(8-1)!1!}\right) \\
&= \frac{12!}{1!11!}\left(\frac{8!}{7!1!}\right) \\
&= \frac{12(\cancel{11!})^{1}}{\cancel{11!}_{1}}\left(\frac{8(\cancel{7!})^{1}}{\cancel{7!}_{1}1!}\right) \\
&= 12(8) \\
&= 96
\end{aligned}
$$

Case 3: 12 women and 0 men
$$
\begin{aligned}
{}_{12}C_{12}({}_8C_0) &= \frac{12!}{(12-12)!12!}\left(\frac{8!}{(8-0)!0!}\right) \\
&= \frac{\cancel{12!}^{1}}{0!\,\cancel{12!}_{1}}\left(\frac{\cancel{8!}^{1}}{\cancel{8!}_{1}\,0!}\right) \\
&= 1
\end{aligned}
$$

There are 1848 + 96 + 1 = 1945 juries containing at least ten women.

Section 11.2 Page 535 Question 18

a)
$$\begin{aligned} {}_{52}C_5 &= \frac{52!}{(52-5)!5!} \\ &= \frac{52!}{47!5!} \\ &= \frac{52(51)(50)(49)(48)\cancel{(47!)}^{1}}{\cancel{(47!)}_{1}(5)(4)(3)(2)(1)} \\ &= 2\ 598\ 960 \end{aligned}$$

You can select five cards from a deck of 52 cards in 2 598 96 ways.

b) Determine the number of ways to select three of 13 hearts and 2 cards from the rest of the deck.

$$\begin{aligned} {}_{13}C_3({}_{39}C_2) &= \frac{13!}{(13-3)!3!}\left(\frac{39!}{(39-2)!2!}\right) \\ &= \frac{13!}{10!3!}\left(\frac{39!}{37!2!}\right) \\ &= \frac{13(12)(11)(\cancel{10!})^{1}}{\cancel{109!}_{1}(3)(2)(1)}\left(\frac{39(38)(\cancel{37!})^{1}}{\cancel{37!}_{1}(2)(1)}\right) \\ &= 286(741) \\ &= 211\ 926 \end{aligned}$$

There are 211 926 ways to select three hearts from a standard deck of cards.

c) Determine the number of ways to select one black card and four red cards.

$$\begin{aligned} {}_{26}C_1({}_{26}C_4) &= \frac{26!}{(26-1)!1!}\left(\frac{26!}{(26-4)!4!}\right) \\ &= \frac{26!}{25!}\left(\frac{26!}{22!4!}\right) \\ &= \frac{26(\cancel{25!})^{1}}{\cancel{25!}_{1}}\left(\frac{26(25)(24)(23)(\cancel{22!})^{1}}{\cancel{22!}_{1}(4)(3)(2)(1)}\right) \\ &= 388\ 700 \end{aligned}$$

There are 388 700 ways to select one black card and four red cards from a standard deck.

Section 11.2 Page 535 Question 19

a) $$\begin{aligned}{}_6C_4({}_7C_3) &= \frac{6!}{(6-4)!4!}\left(\frac{7!}{(7-3)!3!}\right)\\ &= \frac{6!}{2!4!}\left(\frac{7!}{4!3!}\right)\\ &= \frac{6(5)\cancel{(4!)}}{2(1)\cancel{(4!)}}\left(\frac{7(6)(5)\cancel{(4!)}}{\cancel{(4!)}(3)(2)(1)}\right)\\ &= 525\end{aligned}$$

You can select the books in 525 ways.

b) Considering the four science books as one object means there are four objects to arrange in 4! = 24 ways. Since the science books must be together, there are 4!4! = 576 ways to arrange the books.

Section 11.2 Page 535 Question 20

a) $$\begin{aligned}{}_{40}C_{20} &= \frac{40!}{(40-20)!20!}\\ &= \frac{40!}{20!20!}\end{aligned}$$

The number of selections possible is $\frac{40!}{20!20!}$.

b) $$\begin{aligned}{}_{20}P_4 &= \frac{20!}{(20-4)!}\\ &= \frac{20!}{16!}\\ &= \frac{20(19)(18)(17)\cancel{(16!)}}{\cancel{16!}}\\ &= 116\ 280\end{aligned}$$

Four of the paintings can be displayed in a row in 116 280 ways.

Section 11.2 Page 536 Question 21

a)

$$\begin{aligned}{}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} &= \frac{52!}{(52-13)!13!} \times \frac{39!}{(39-13)!13!} \times \frac{26!}{(26-13)!13!} \times \frac{13!}{(13-13)!13!}\\ &= \frac{52!}{39!13!} \times \frac{39!}{26!13!} \times \frac{26!}{13!13!} \times \frac{13!}{0!13!}\end{aligned}$$

b) $_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} = \frac{52!}{39!13!} \times \frac{39!}{26!13!} \times \frac{26!}{13!13!} \times \frac{13!}{0!13!}$

$$= \frac{52!}{\cancel{39!}13!} \times \frac{\cancel{39!}}{\cancel{26!}13!} \times \frac{\cancel{26!}}{13!13!} \times \frac{\cancel{13!}}{0!\cancel{13!}}$$

$$= \frac{52!}{13!13!13!13!}$$

$$= \frac{52!}{(13!)^4}$$

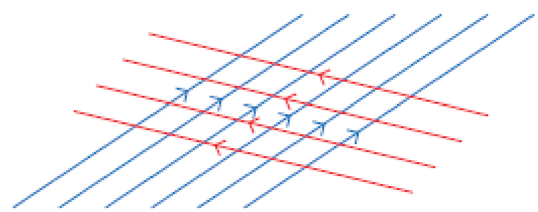
c) $_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13} = \frac{52!}{(13!)^4}$

$$= \frac{52(51)(50)...(15)(14)\ \cancel{(13!)}}{\cancel{13!}\left[(13)(12)(11)...(3)(2)(1)\right]^3}$$

$$= 5.364... \times 10^{28}$$

Section 11.2 Page 536 Question 22

Using the given diagram, count the number of possible parallelograms of each dimension.



1 × 1 parallelogram: 15 ways
1 × 2 parallelogram: 22 ways
1 × 3 parallelogram: 14 ways
1 × 4 parallelogram: 6 ways
1 × 5 parallelogram: 3 ways
2 × 2 parallelogram: 8 ways
2 × 3 parallelogram: 10 ways
2 × 4 parallelogram: 4 ways
2 × 5 parallelogram: 2 ways
3 × 3 parallelogram: 3 ways
3 × 4 parallelogram: 2 ways
3 × 5 parallelogram: 1 way

Total number of possibilities: 15 + 22 + 14 + 6 + 3 + 8 + 10 + 4 + 2 + 3 + 2 + 1 = 90
There are 90 possible parallelograms formed.

Section 11.2 Page 536 Question 23

a) Let x represent the number of flavours available.

$$\begin{aligned}
{}_xC_2 &= 630 \\
\frac{x!}{(x-2)!2!} &= 630 \\
\frac{x(x-1)\,\cancel{(x-2)!}}{\cancel{(x-2)!}\,2!} &= 630 \\
\frac{x(x-1)}{(2)(1)} &= 630 \\
x(x-1) &= 1260 \\
x(x-1) &= 36(35)
\end{aligned}$$

Using reasoning, $x = 36$.
There are 36 flavours of ice cream available in the store.

b) If you can duplicate flavours, you can choose from 36 flavours for each scoop.
36(36) = 1296
There are 1296 possible two-scoop bowls of ice cream.

Section 11.2 Page 536 Question 24

a)
$$\begin{aligned}
{}_5C_2 &= \frac{5!}{(5-2)!2!} \\
&= \frac{5!}{3!2!} \\
&= \frac{5(4)\,\cancel{(3)!}}{\cancel{(3)!}(2)(1)} \\
&= 10
\end{aligned}$$

$$\begin{aligned}
{}_{3(5)}C_{3(2)} &= {}_{15}C_6 \\
&= \frac{15!}{(15-6)!6!} \\
&= \frac{15!}{9!6!} \\
&= \frac{15(14)(13)(12)(11)(10)\,\cancel{(9)!}}{\cancel{(9)!}(6)(5)(4)(3)(2)(1)} \\
&= 5005
\end{aligned}$$

$$\frac{{}_5C_2}{3} = \frac{10}{3} = 3 \text{ remainder } 1$$

$$\frac{{}_{15}C_6}{3} = \frac{5005}{3} = 1668 \text{ remainder } 1$$

Both expressions have the same remainder, so, the statement is true.

b) ${}_{7(5)}C_{7(2)} = {}_{35}C_{14}$

$$= \frac{35!}{(35-14)!14!}$$

$$= \frac{35!}{21!14!}$$

$$= \frac{35(34)(33)(32)(31)(30)(29)(28)(27)(26)(25)(24)(23)(22)\cancel{(21)}!}{\cancel{(21)}!(14)(13)(12)(11)(10)(9)(8)(7)(6)(5)(4)(3)(2)(1)}$$

$$= 2\ 319\ 959\ 400$$

$$\frac{{}_{35}C_{14}}{7} = \frac{2\ 319\ 959\ 400}{7} = 331\ 422\ 771 \text{ remainder } 3$$

$$\frac{{}_{5}C_{2}}{7} = \frac{10}{7} = 1 \text{ remainder } 3$$

Both expressions have the same remainder, so, the statement is true.

c) Seven remainders are possible when dividing by 7. They are 0, 1, 2, 3, 4, 5, and 6.

d) Example: First, I would try a few more cases to try to find a counterexample. Since the statement seems to be true, I would write a computer program to test many cases in an organized way.

Section 11.2 Page 536 Question C1

No, the order of the numbers on the lock matters, so a combination lock would be better called a permutations lock.

Section 11.2 Page 536 Question C2

a) ${}_{a}C_{b} = \frac{a!}{(a-b)!b!}$ is the formula for calculating the number of ways that *b* objects can be selected from a group of *a* objects, if order is not important. For example, if you have a group of 20 students and you want to choose a team of any three people.

b) $a \geq b$

c) $b \geq 0$

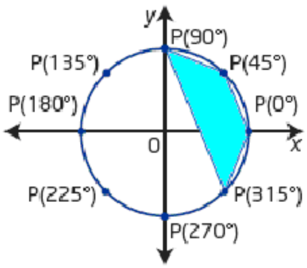
Section 11.2 Page 536 Question C3

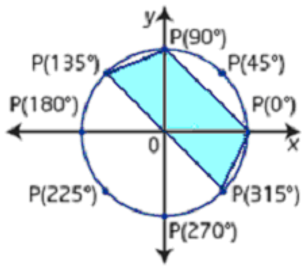
Example: Assuming that the rooms are the same and so any patient can be assigned to any of the six rooms, this is a combination situation. Beth is correct.

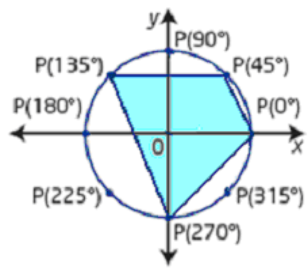
Section 11.2 Page 536 Question C4

Step 1

Examples:







Step 2

Shape	Number of Quadrilaterals of Each Shape
square	2
rectangle	4
parallelogram	0
isosceles trapezoid	24

Step 3

In general, there are ${}_8C_4 = 70$ possible quadrilaterals. Because of the symmetry of the points on the circle, some of these quadrilaterals are the same.

Section 11.3 The Binomial Theorem

Section 11.3 Page 542 Question 1

a) 1 4 6 4 1

b) 1 8 28 56 70 56 28 8 1

c) 1 11 55 165 330 462 462 330 165 55 11 1

Section 11.3 Page 542 Question 2

a) ${}_2C_0$ ${}_2C_1$ ${}_2C_2$

b) ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

c) ${}_7C_0$ ${}_7C_1$ ${}_7C_2$ ${}_7C_3$ ${}_7C_4$ ${}_7C_5$ ${}_7C_6$ ${}_7C_7$

Section 11.3 Page 542 Question 3

a) $_3C_2 = \dfrac{3!}{(3-2)!2!}$
$= \dfrac{3!}{1!2!}$

b) $_6C_3 = \dfrac{6!}{(6-3)!3!}$
$= \dfrac{6!}{3!3!}$

c) $_1C_0 = \dfrac{1!}{(1-0)!0!}$
$= \dfrac{1!}{1!0!}$

Section 11.3 Page 542 Question 4

The expansion of $(x + y)^n$, contains $n + 1$ terms.

a) $4 + 1 = 5$

b) $7 + 1 = 8$

c) $q + 1$

Section 11.3 Page 542 Question 5

a) $(x + y)^2 = {}_2C_0(x)^2(y)^0 + {}_2C_1(x)^{2-1}(y)^1 + {}_2C_2(x)^{2-2}(y)^2$
$= 1x^2y^0 + 2x^1y^1 + 1x^0y^2$
$= x^2 + 2xy + y^2$

b) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$
Substitute $x = a$ and $y = 1$.
$(a + 1)^3 = {}_3C_0(a)^3(1)^0 + {}_3C_1(a)^{3-1}(1)^1 + {}_3C_2(a)^{3-2}(1)^2 + {}_3C_3(a)^{3-3}(1)^3$
$= 1a^3 + 3a^2 + 3a + 1a^0$
$= a^3 + 3a^2 + 3a + 1$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$
Substitute $x = 1$ and $y = -p$.
$(1 - p)^4 = {}_4C_0(1)^4(-p)^0 + {}_4C_1(1)^{4-1}(-p)^1 + {}_4C_2(1)^{4-2}(-p)^2 + {}_4C_3(1)^{4-3}(-p)^3 + {}_4C_4(1)^{4-4}(-p)^4$
$= 1(1)^4(-p)^0 + 4(1)^3(-p)^1 + 6(1)^2(-p)^2 + 4(1)^1(-p)^3 + 1(1)^0(-p)^4$
$= 1 + 4(-p) + 6p^2 + 4(-p^3) + p^4$
$= 1 - 4p + 6p^2 - 4p^3 + p^4$

Section 11.3 Page 542 Question 6

a) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$
Substitute $x = a$ and $y = 3b$.
$(a + 3b)^3 = {}_3C_0(a)^3(3b)^0 + {}_3C_1(a)^{3-1}(3b)^1 + {}_3C_2(a)^{3-2}(3b)^2 + {}_3C_3(a)^{3-3}(3b)^3$
$= 1(a)^3(1) + 3a^2(3b) + 3(a)^1(3b)^2 + 1a^0(3b)^3$
$= a^3 + 9a^2b + 3a(9b^2) + 1(27b^3)$
$= a^3 + 9a^2b + 27ab^2 + 27b^3$

b) $(x + y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^{5-1}(y)^1 + {}_5C_2(x)^{5-2}(y)^2 + {}_5C_3(x)^{5-3}(y)^3 + {}_5C_4(x)^{5-4}(y)^4 + {}_5C_5(x)^{5-5}(y)^5$

Substitute $x = 3a$ and $y = -2b$.

$(3a - 2b)^5$
$= {}_5C_0(3a)^5(-2b)^0 + {}_5C_1(3a)^{5-1}(-2b)^1 + {}_5C_2(3a)^{5-2}(-2b)^2 + {}_5C_3(3a)^{5-3}(-2b)^3 + {}_5C_4(3a)^{5-4}(-2b)^4 + {}_5C_5(3a)^{5-5}(-2b)^5$
$= 1(3a)^5(1) + 5(3a)^4(-2b) + 10(3a)^3(-2b)^2 + 10(3a)^2(-2b)^3 + 5(3a)^1(-2b)^4 + 1(3a)^0(-2b)^5$
$= 243a^5 + 5(81a^4)(-2b) + 10(27a^3)(4b^2) + 10(9a^2)(-8b^3) + 15a(16b^4) - 32b^5$
$= 243a^5 - 810a^4b + 1080a^3b^2 - 720a^2b^3 + 240ab^4 - 32b^5$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$

Substitute $x = 2x$ and $y = -5$.

$(2x - 5)^4$
$= {}_4C_0(2x)^4(-5)^0 + {}_4C_1(2x)^{4-1}(-5)^1 + {}_4C_2(2x)^{4-2}(-5)^2 + {}_4C_3(2x)^{4-3}(-5)^3 + {}_4C_4(2x)^{4-4}(-5)^4$
$= 1(2x)^4(1) + 4(2x)^3(-5) + 6(2x)^2(-5)^2 + 4(2x)^1(-5)^3 + 1(2x)^0(-5)^4$
$= 16x^4 - 20(8x^3) + 6(4x^2)(25) + 8x(-125) + (1)(625)$
$= 16x^4 - 160x^3 + 6400x^2 - 1000x + 625$

Section 11.3 Page 542 Question 7

a) sixth term of $(a + b)^9$: ${}_9C_5(a)^4(b)^5 = 126a^4b^5$

b) fourth term of $(x - 3y)^6$: ${}_6C_3(x)^3(-3y)^3 = 20x^3(-27y^3)$
$= -540x^3y^3$

c) seventh term of $(1 - 2t)^{14}$: ${}_{14}C_6(1)^8(-2t)^6 = 3003(64t^6)$
$= 192\ 192t^6$

d) middle term of $(4x + y)^4$: ${}_4C_2(4x)^2(y)^2 = 6(16x^2)(y^2)$
$= 96x^2y^2$

e) second last term of $(3w^2 + 2)^8$: ${}_8C_7(3w^2)^1(2)^7 = 8(3w^2)(128)$
$= 3072w^2$

Section 11.3 Page 542 Question 8

All outside numbers of Pascal's triangle are 1s. The middle values are determined by adding the number to the left and right in the row above.

Section 11.3 Page 542 Question 9

a) 1; 1 + 1 = 2; 1 + 2 + 1 = 4; 1 + 3 + 3 + 1 = 8; 1 + 4 + 6 + 4 + 1 = 16

b) 2^8 or 256

c) 2^{n-1}, where n is the row number

Section 11.3 Page 542 Question 10

a) The sum of the numbers on the handle equals the number on the blade of each hockey stick.

b) No; the hockey stick handle must begin with 1 from the outside of the triangle and move diagonally down the triangle with each value being in a different row. The number on the blade must be diagonally below the last number on the handle of the hockey stick.

Section 11.3 Page 542 Question 11

a) The expansion of $(x + y)^{12}$ contains 12 + 1 = 13 terms.

b) The coefficient of the fourth term involves ${}_{12}C_3$.

fourth term:
$$\begin{aligned} {}_{12}C_3(x)^{12-3}(y)^3 &= \frac{12!}{(12-3)!3!}(x)^9y^3 \\ &= \frac{12!}{9!3!}x^9y^3 \\ &= \frac{12(11)(10)\cancel{(9!)}^{1}}{\cancel{9!}_{1}(3)(2)(1)}x^9y^3 \\ &= 220x^9y^3 \end{aligned}$$

c) The greatest coefficient is in the middle of the row.

$r = \frac{12}{2}$ or 6

$$\begin{aligned} {}_{12}C_6 &= \frac{12!}{(12-6)!6!} \\ &= \frac{12!}{6!6!} \\ &= \frac{12(11)(10)(9)(8)(7)\cancel{(6!)}^{1}}{\cancel{6!}_{1}(6)(5)(4)(3)(2)(1)} \\ &= 924 \end{aligned}$$

Section 11.3 Page 543 Question 12

a) number of terms = $n + 1$
$$\begin{aligned} 5 &= n + 1 \\ 4 &= n \end{aligned}$$
Substitute $a = x$, $b = y$ and $n = 4$:
$(a + b)^n = (x + y)^4$

b) number of terms = $n + 1$
$$\begin{aligned} 6 &= n + 1 \\ 5 &= n \end{aligned}$$
Substitute $a = 1$, $b = -y$ and $n = 5$:
$(a + b)^n = (1 - y)^5$

Section 11.3 Page 543 Question 13

a) No. While $11^0 = 1$, $11^1 = 11$, $11^2 = 121$, $11^3 = 1331$, and $11^4 = 14\ 641$, this pattern only works for the first five rows of Pascal's triangle.

b) m could represent the row number minus 1, $m \leq 4$.

Section 11.3 Page 543 Question 14

a) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
The signs for the second and fourth terms are negative in the expansion of $(x - y)^3$.

b)
$$\begin{aligned}\text{Left Side} &= (x + y)^3 + (x - y)^3\\ &= x^3 + 3x^2y + 3xy^2 + y^3 + x^3 - 3x^2y + 3xy^2 - y^3\\ &= 2x^3 + 6xy^2\\ &= 2x(x^2 + 3y^2)\\ &= \text{Right Side}\end{aligned}$$

c)
$$\begin{aligned}(x + y)^3 - (x - y)^3 &= x^3 + 3x^2y + 3xy^2\ y^3 - (x^3 - 3x^2y + 3xy^2 - y^3)\\ &= x^3 + 3x^2y + 3xy^2\ y^3 - x^3 + 3x^2y - 3xy^2 + y^3\\ &= 6x^2y + 2y^3\\ &= 2y(3x^2 + y^2)\end{aligned}$$

The expansion of $(x + y)^3 - (x - y)^3$ has coefficients for x^2 and y^2 that are reversed from the expansion of $(x + y)^3 + (x - y)^3$; also the common factors $2x$ and $2y$ are reversed.

Section 11.3 Page 543 Question 15

a) Case 1: no one attends, Case 2: one person attends, Case 3: two people attend, Case 4: three people attend, Case 5: four people attend, Case 6: all five people attend

b) Case 1: 1 way

Case 2: 5 ways

Case 3:
$$\begin{aligned}{}_5C_2 &= \frac{5!}{(5-2)!2!}\\ &= \frac{5!}{3!2!}\\ &= \frac{5(4)\,\cancel{(3!)}}{\cancel{3!}(2)(1)}\\ &= 10\end{aligned}$$

Case 4:
$$\begin{aligned}{}_5C_3 &= \frac{5!}{(5-3)!3!}\\ &= \frac{5!}{2!3!}\\ &= \frac{5(4)\,\cancel{(3!)}}{(2)(1)\,\cancel{3!}}\\ &= 10\end{aligned}$$

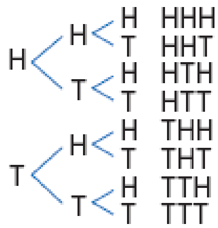
Case 5:
$$\begin{aligned}{}_5C_4 &= \frac{5!}{(5-4)!4!}\\ &= \frac{5!}{1!4!}\\ &= \frac{5\,\cancel{(4!)}}{\cancel{4!}}\\ &= 5\end{aligned}$$

Case 6: 1 way
Total number of combinations: 1 + 5 +10 + 10 + 5 + 1 = 32

c) The answer is the sum of the terms in the sixth row of Pascal's triangle.

Section 11.3 Page 543 Question 16

a)



b) $(H + T)^3 = HHH + HHT + HTH + HTT + THH + THT + TTH + TTT$
$= H^3 + H^2T + H^2T + HT^2 + H^2T + HT^2 + HT^2 + T^3$
$= H^3 + 3H^2T + 3HT^2 + T^3$

c) H^3 represents getting a head for each of three coin tosses and is the first term of the expansion of $(H + T)^3$. $3H^2T$ represents the possibility of getting two heads and one tail during three coin tosses and is the second term of the expansion of $(H + T)^3$.

Section 11.3 Page 543 Question 17

a) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^{3-1}(y)^1 + {}_3C_2(x)^{3-2}(y)^2 + {}_3C_3(x)^{3-3}(y)^3$
$= {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(x)^0(y)^3$
$= {}_3C_0(x)^3 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(y)^3$

Substitute $x = \frac{a}{b}$ and $y = 2$.

$$\left(\frac{a}{b}+2\right)^3 = {}_3C_0\left(\frac{a}{b}\right)^3 + {}_3C_1\left(\frac{a}{b}\right)^2(2)^1 + {}_3C_2\left(\frac{a}{b}\right)^1(2)^2 + {}_3C_3(2)^3$$
$$= 1\left(\frac{a}{b}\right)^3 + (3)\left(\frac{a}{b}\right)^2(2) + 3\left(\frac{a}{b}\right)(4) + 1(8)$$
$$= \frac{a^3}{b^3} + 6\frac{a^2}{b^2} + 12\frac{a}{b} + 8$$

b) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$
$= {}_4C_0(x)^4 + {}_4C_1(x)^3(y) + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$

Substitute $x = \frac{a}{b}$ and $y = –a$.

$$\left(\frac{a}{b}-a\right)^4 = {}_4C_0\left(\frac{a}{b}\right)^4 + {}_4C_1\left(\frac{a}{b}\right)^3(-a) + {}_4C_2\left(\frac{a}{b}\right)^2(-a)^2 + {}_4C_3\left(\frac{a}{b}\right)^1(-a)^3 + {}_4C_4\left(\frac{a}{b}\right)^0(-a)^4$$

$$= 1\left(\frac{a^4}{b^4}\right) + 4\left(\frac{a^3}{b^3}\right)(-a) + 6\left(\frac{a^2}{b^2}\right)(a^2) + 4\left(\frac{a}{b}\right)(-a^3) + (1)(a^4)$$

$$= \frac{a^4}{b^4} - 4\frac{a^4}{b^3} + 6\frac{a^4}{b^2} - 4\frac{a^4}{b} + a^4$$

$$= a^4\left(\frac{1}{b^4} - \frac{4}{b^3} + \frac{6}{b^2} - \frac{4}{b} + 1\right)$$

c) $(x + y)^6 = {}_6C_0(x)^6(y)^0 + {}_6C_1(x)^{6-1}(y)^1 + {}_6C_2(x)^{6-2}(y)^2 + {}_6C_3(x)^{6-3}(y)^3 + {}_6C_4(x)^{6-4}(y)^4 + {}_6C_5(x)^{6-5}(y)^5 + {}_6C_6(x)^{6-6}(y)^6$

$= {}_6C_0(x)^6(1) + {}_6C_1(x)^5(y) + {}_6C_2(x)^4(y)^2 + {}_6C_3(x)^3(y)^3 + {}_6C_4(x)^2(y)^4 + {}_6C_5(x)^1(y)^5 + {}_6C_6(x)^0(y)^6$

Substitute $x = 1$ and $y = -\frac{x}{2}$.

$$\left(1-\frac{x}{2}\right)^6 = {}_6C_0(1)^6 + {}_6C_1(1)^5\left(-\frac{x}{2}\right) + {}_6C_2(1)^4\left(-\frac{x}{2}\right)^2 + {}_6C_3(1)^3\left(-\frac{x}{2}\right)^3 + {}_6C_4(1)^2\left(-\frac{x}{2}\right)^4 + {}_6C_5(1)^1\left(-\frac{x}{2}\right)^5 + {}_6C_6(1)^0\left(-\frac{x}{2}\right)^6$$

$$= 1(1) + 6(1)\left(-\frac{x}{2}\right) + 15(1)\left(\frac{x^2}{2^2}\right) + 20(1)\left(-\frac{x^3}{2^3}\right) + 15(1)\left(\frac{x^4}{2^4}\right) + 6(1)\left(-\frac{x^5}{2^5}\right) + 1(1)\left(\frac{x^6}{2^6}\right)$$

$$= 1 + \overset{3}{\cancel{6}}\left(-\frac{x}{\underset{1}{\cancel{2}}}\right) + 15\left(\frac{x^2}{4}\right) + \overset{5}{\cancel{20}}\left(-\frac{x^3}{\underset{2}{\cancel{8}}}\right) + 15\left(\frac{x^4}{16}\right) + \overset{3}{\cancel{6}}\left(-\frac{x^5}{\underset{16}{\cancel{32}}}\right) + \left(\frac{x^6}{64}\right)$$

$$= 1 - 3x + \frac{15}{4}x^2 - \frac{5}{2}x^3 + \frac{15}{16}x^4 - \frac{3}{16}x^5 + \frac{x^6}{64}$$

d) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^{4-1}(y)^1 + {}_4C_2(x)^{4-2}(y)^2 + {}_4C_3(x)^{4-3}(y)^3 + {}_4C_4(x)^{4-4}(y)^4$

$= {}_4C_0(x)^4 + {}_4C_1(x)^3(y) + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$

Substitute $x = 2x^2$ and $y = -\frac{1}{x}$.

$$\left(2x^2-\frac{1}{x}\right)^4 = {}_4C_0(2x^2)^4 + {}_4C_1(2x^2)^3\left(-\frac{1}{x}\right) + {}_4C_2(2x^2)^2\left(-\frac{1}{x}\right)^2 + {}_4C_3(2x^2)^1\left(-\frac{1}{x}\right)^3 + {}_4C_4(2x^2)^0\left(-\frac{1}{x}\right)^4$$

$$= 1(2^4x^8) + 4(2^3x^6)\left(-\frac{1}{x}\right) + 6(2^2x^4)\left(\frac{1^2}{x^2}\right) + 4(2x^2)\left(-\frac{1^3}{x^3}\right) + 1(1)\left(\frac{1^4}{x^4}\right)$$

$$= 16x^8 + 4(8x^6)\left(-\frac{1}{x}\right) + 6(4x^4)\left(\frac{1^2}{x^2}\right) + 8x^2\left(-\frac{1^3}{x^3}\right) + \frac{1}{x^4}$$

$$= 16x^8 - \frac{32x^{\overset{5}{\cancel{6}}}}{x^{\overset{0}{\cancel{1}}}} + 24x^4\left(\frac{1}{x^2}\right) - \frac{8x^2}{x^3} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^{\overset{2}{\cancel{4}}}\left(\frac{1}{x^{\overset{0}{\cancel{2}}}}\right) - \frac{8x^{\overset{0}{\cancel{2}}}}{x^{\overset{1}{\cancel{3}}}} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^2 - \frac{8}{x} + \frac{1}{x^4}$$

$$= 16x^8 - 32x^5 + 24x^2 - 8x^{-1} + x^{-4}$$

Section 11.3 Page 543 Question 18

a) middle term of $(a - 3b^3)^8$: ${}_8C_4(a)^{8-4}(-3b^3)^4 = 70(a)^4(81b^{12})$
$= 5670a^4b^{12}$

b) $\left(x^2-\frac{1}{x}\right)^{10}$

$$= {}_{10}C_0(x^2)^{10} + {}_{10}C_1(x^2)^9\left(-\frac{1}{x}\right) + {}_{10}C_2(x^2)^8\left(-\frac{1}{x}\right)^2 + {}_{10}C_3(x^2)^7\left(-\frac{1}{x}\right)^3 + {}_{10}C_4(x^2)^6\left(-\frac{1}{x}\right)^4 + {}_{10}C_5(x^2)^5\left(-\frac{1}{x}\right)^5 + {}_{10}C_6(x^2)^4\left(-\frac{1}{x}\right)^6$$

$$+ {}_{10}C_7(x^2)^3\left(-\frac{1}{x}\right)^7 + {}_{10}C_8(x^2)^2\left(-\frac{1}{x}\right)^8 + {}_{10}C_9(x^2)^1\left(-\frac{1}{x}\right)^9 + {}_{10}C_{10}(x^2)^0\left(-\frac{1}{x}\right)^{10}$$

$$= 1(x^{20}) + 10(x^{18})\left(-\frac{1}{x}\right) + 45(x^{16})\left(\frac{1^2}{x^2}\right) + 120(x^{14})\left(-\frac{1^3}{x^3}\right) + 210(x^{12})\left(\frac{1^4}{x^4}\right) +$$

$$252(x^{10})\left(-\frac{1^5}{x^5}\right) + 210(x^8)\left(\frac{1^6}{x^6}\right) + 120(x^6)\left(-\frac{1^7}{x^7}\right) + 45(x^4)\left(\frac{1^8}{x^8}\right) + 10(x^2)\left(-\frac{1^9}{x^9}\right) + 1(1)\left(\frac{1^{10}}{x^{10}}\right)$$

$$= x^{20} - 10(x^{\overset{17}{\cancel{18}}})\left(\frac{1}{x^{\overset{0}{\cancel{1}}}}\right) + 45(x^{\overset{14}{\cancel{16}}})\left(\frac{1}{x^{\overset{0}{\cancel{2}}}}\right) - 120(x^{\overset{11}{\cancel{14}}})\left(\frac{1}{x^{\overset{0}{\cancel{3}}}}\right) + 210(x^{\overset{8}{\cancel{12}}})\left(\frac{1}{x^{\overset{0}{\cancel{4}}}}\right) -$$

$$252(x^{\overset{5}{\cancel{10}}})\left(\frac{1}{x^{\overset{0}{\cancel{5}}}}\right) + 210(x^{\overset{2}{\cancel{8}}})\left(\frac{1}{x^{\overset{0}{\cancel{6}}}}\right) - 120(x^{\overset{0}{\cancel{6}}})\left(\frac{1}{x^{\overset{1}{\cancel{7}}}}\right) + 45(x^{\overset{0}{\cancel{4}}})\left(\frac{1}{x^{\overset{4}{\cancel{8}}}}\right) - 10(x^{\overset{0}{\cancel{2}}})\left(\frac{1}{x^{\overset{7}{\cancel{9}}}}\right) + \frac{1}{x^{10}}$$

$$= x^{20} - 10x^{17} + 45x^{14} - 120x^{11} + 210x^8 - 252x^5 + 210x^2 - \frac{120}{x} + \frac{45}{x^4} - \frac{10}{x^7} + \frac{1}{x^{10}}$$

The fourth term is $-120x^{11}$.

Section 11.3 Page 543 Question 19

a) $\left(x^2-\dfrac{2}{x}\right)^{12}$

$$\begin{aligned}
&= {}_{12}C_0(x^2)^{12} + {}_{12}C_1(x^2)^{11}\left(-\frac{2}{x}\right) + {}_{12}C_2(x^2)^{10}\left(-\frac{2}{x}\right)^2 + {}_{12}C_3(x^2)^9\left(-\frac{2}{x}\right)^3 + {}_{12}C_4(x^2)^8\left(-\frac{2}{x}\right)^4 + {}_{12}C_5(x^2)^7\left(-\frac{2}{x}\right)^5 + {}_{12}C_6(x^2)^6\left(-\frac{2}{x}\right)^6 \\
&\quad + {}_{12}C_7(x^2)^5\left(-\frac{2}{x}\right)^7 + {}_{12}C_8(x^2)^4\left(-\frac{2}{x}\right)^8 + {}_{12}C_9(x^2)^3\left(-\frac{2}{x}\right)^9 + {}_{12}C_{10}(x^2)^2\left(-\frac{2}{x}\right)^{10} + {}_{12}C_{11}(x^2)^1\left(-\frac{2}{x}\right)^{11} + {}_{12}C_{12}(x^2)^0\left(-\frac{2}{x}\right)^{12} \\
&= 1(x^{24}) + 12(x^{22})\left(-\frac{2}{x}\right) + 66(x^{20})\left(\frac{2^2}{x^2}\right) + 220(x^{18})\left(-\frac{2^3}{x^3}\right) + 495(x^{16})\left(\frac{2^4}{x^4}\right) + 792(x^{14})\left(-\frac{2^5}{x^5}\right) + 924(x^{12})\left(\frac{2^6}{x^6}\right) \\
&\quad + 792(x^{10})\left(-\frac{2^7}{x^7}\right) + 495(x^8)\left(\frac{2^8}{x^8}\right) + 220(x^6)\left(-\frac{29^9}{x^9}\right) + 66(x^4)\left(\frac{2^{10}}{x^{10}}\right) + 12x^2\left(-\frac{2^{11}}{x^{11}}\right) + 1\left(\frac{2^{12}}{x^{12}}\right)
\end{aligned}$$

By inspection, the fifth term is a constant.

$$\begin{aligned}
495\,\cancel{x^8}^{\,1}\left(\frac{2^8}{\cancel{x^8}_{\,1}}\right) &= 495(256) \\
&= 126\ 720
\end{aligned}$$

The value of the constant is 126 720.

b) $\left(y-\dfrac{1}{y^2}\right)^{12}$

$$\begin{aligned}
&= {}_{12}C_0(y)^{12} + {}_{12}C_1(y)^{11}\left(-\frac{1}{y^2}\right) + {}_{12}C_2(y)^{10}\left(-\frac{1}{y^2}\right)^2 + {}_{12}C_3(y)^9\left(-\frac{1}{y^2}\right)^3 + {}_{12}C_4(y)^8\left(-\frac{1}{y^2}\right)^4 + {}_{12}C_5(y)^7\left(-\frac{1}{y^2}\right)^5 + {}_{12}C_6(y)^6\left(-\frac{1}{y^2}\right)^6 \\
&\quad + {}_{12}C_7(y)^5\left(-\frac{1}{y^2}\right)^7 + {}_{12}C_8(y)^4\left(-\frac{1}{y^2}\right)^8 + {}_{12}C_9(y)^3\left(-\frac{1}{y^2}\right)^9 + {}_{12}C_{10}(y)^2\left(-\frac{1}{y^2}\right)^{10} + {}_{12}C_{11}(y)^1\left(-\frac{1}{y^2}\right)^{11} + {}_{12}C_{12}(y)^0\left(-\frac{1}{y^2}\right)^{12} \\
&= 1(y)^{12} + 12(y)^{11}\left(-\frac{1}{y^2}\right) + 66(y)^{10}\left(\frac{1^2}{y^4}\right) + 220(y)^9\left(-\frac{1^3}{y^6}\right) + 495(y)^8\left(\frac{1^8}{y^8}\right) + 792(y)^7\left(-\frac{1}{y^2}\right)^5 + 924(y)^6\left(-\frac{1}{y^2}\right)^6 \\
&\quad + 792(y)^5\left(-\frac{1}{y^2}\right)^7 + 495(y)^4\left(-\frac{1}{y^2}\right)^8 + 220(y)^3\left(-\frac{1}{y^2}\right)^9 + 66(y)^2\left(-\frac{1}{y^2}\right)^{10} + 12(y)^1\left(-\frac{1}{y^2}\right)^{11} + 1(y)^0\left(-\frac{1}{y^2}\right)^{12}
\end{aligned}$$

By inspection, the fifth term is a constant.

$$495\,\cancel{y^8}^{\,1}\left(\frac{1}{\cancel{y^8}_{\,1}}\right) = 495$$

The value of the constant is 495.

Section 11.3 Page 543 Question 20

$$
\begin{aligned}
&(2x - m)^7 \\
&= {}_7C_0(2x)^7(-m)^0 + {}_7C_1(2x)^6(-m)^1 + {}_7C_2(2x)^5(-m)^2 + {}_7C_3(2x)^4(-m)^3 + {}_7C_4(2x)^3(-m)^4 \\
&\quad + {}_7C_5(2x)^2(-m)^5 + {}_7C_6(2x)^1(-m)^6 + {}_7C_7(2x)^0(-m)^7 \\
&= 1(2^7x^7)(1) + 7(2^6x^6)(-m) + 21(2^5x^5)(m^2) + 35(2^4x^4)(-m^3) + 35(2^3x^3)(m^4) \\
&\quad + 21(2^2x^2)(-m^5) + 7(2x)(m^6) + 1(1)(-m^7) \\
&= 128x^7 - 448x^6m + 672x^5m^2 - 560x^4m^3 + 280x^3m^4 - 84x^2m^5 + 14xm^6 - m^7
\end{aligned}
$$

The term in the expansion that contains x^4 is $-560x^4m^3$.
Solve:

$$
\begin{aligned}
-560x^4m^3 &= -15\,120x^4y^3 \\
m^3 &= \frac{\overset{27}{\cancel{-15\,120}}\ \overset{1}{\cancel{x^4}}\ y^3}{\underset{1}{\cancel{-560}}\ \underset{1}{\cancel{x^4}}} \\
m^3 &= 27y^3 \\
\sqrt[3]{m^3} &= \sqrt[3]{3^3y^3} \\
m &= 3y
\end{aligned}
$$

Section 11.3 Page 543 Question 21

Example:
Step 1: The numerators start with the second value, 4, and decrease by ones, while the denominators start at 1 and increase by ones to 4.
For the sixth row:

$\times\frac{5}{1}$ $\times\frac{4}{2}$ $\times\frac{3}{3}$ $\times\frac{2}{4}$ $\times\frac{1}{5}$

1 → 5 → 10 → 10 → 5 → 1

Step 2: The second element in the row is equal to the row number minus 1.

Step 3: The first two terms in the 21st row of Pascal's triangle are 1 and 20. The multipliers for successive terms are $\times\frac{20}{1}$, $\times\frac{19}{2}$, $\times\frac{18}{3}$, ... $\times\frac{3}{18}$, $\times\frac{2}{19}$, $\times\frac{1}{20}$.

Section 11.3 Page 544 Question 22

a) Each entry is the sum of the two values in the row below it to the left and the right.

b) $\frac{1}{6}\quad \frac{1}{30}\quad \frac{1}{60}\quad \frac{1}{60}\quad \frac{1}{30}\quad \frac{1}{6}$

$\frac{1}{7}\quad \frac{1}{42}\quad \frac{1}{105}\quad \frac{1}{140}\quad \frac{1}{105}\quad \frac{1}{42}\quad \frac{1}{7}$

c) Example: Outside values are the reciprocal of the row number. The product of two consecutive outside rows values gives the value of the second term in the lower row.

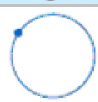
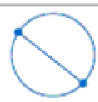
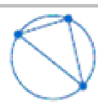
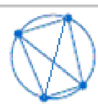
Section 11.3 Page 544 Question 23

Consider $a + b = x$ and $c = y$, and substitute in $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

$$\begin{aligned}(a + b + c)^3 &= (a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 + 3c(a^2 + 2ab + b^2) + 3ac^2 + 3bc^2 + c^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3b^2c + 3ac^2 + 3bc^2 + c^3\end{aligned}$$

Section 11.3 Page 544 Question 24

a)

Diagram	Points	Line Segments	Triangles	Quadrilaterals	Pentagons	Hexagons
	1					
	2	1				
	3	3	1			
	4	6	4	1		
	5	10	10	5	1	
	6	15	20	15	6	1

b) The numbers are values from row 1 to row 6 of Pascal's triangle with the exception of the first term.

c) The numbers will be values from the 8th row of Pascal's triangle with the exception of the first term: 8 28 56 70 56 28 8 1

Section 11.3 Page 544 Question 25

a)
$$\begin{aligned} e &= \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots \\ &\approx \frac{1}{1} + \frac{1}{1} + \frac{1}{2(1)} + \frac{1}{3(2)(1)} + \frac{1}{4(3)(2)(1)} \\ &\approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} \\ &\approx \frac{24}{24} + \frac{24}{24} + \frac{12}{24} + \frac{4}{24} + \frac{1}{24} \\ &\approx \frac{24+24+12+4+1}{24} \\ &\approx \frac{65}{24} \\ &\approx 2.7083... \end{aligned}$$

The approximate value of *e* is 2.7083.

b) Using seven terms,

$$\begin{aligned} e &\approx \frac{65}{24} + \frac{1}{5!} + \frac{1}{6!} \\ &\approx \frac{65}{24} + \frac{1}{5(4)(3)(2)(1)} + \frac{1}{6(5)(4)(3)(2)(1)} \\ &\approx \frac{65}{24} + \frac{1}{120} + \frac{1}{720} \\ &\approx \frac{1950}{720} + \frac{6}{720} + \frac{1}{720} \\ &\approx \frac{1957}{720} \\ &\approx 2.7180... \end{aligned}$$

Using eight terms,

$$\begin{aligned} e &\approx \frac{1957}{720} + \frac{1}{7!} \\ &\approx \frac{1957}{720} + \frac{1}{7(6)(5)(4)(3)(2)(1)} \\ &\approx \frac{1957}{720} + \frac{1}{5040} \\ &\approx \frac{13\ 699}{5040} + \frac{1}{5040} \\ &\approx \frac{13\ 700}{5040} \\ &\approx 2.7182... \end{aligned}$$

The value of *e* becomes more precise for the seventh and eighth terms. The more terms used, the more accurate the approximation.

c) The value of *e* is approximately 2.718 281 828.

d) Using Stirling's approximation,

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$15! \approx \left(\frac{15}{e}\right)^{15} \sqrt{2\pi(15)}$$

$$\approx 1.3004 \times 10^{12}$$

Using a calculator,
15! = 15(14)(13)…(3)(2)(1)
= 1.3076… × 10^{12}

e) Using the formula from part d),

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

$$50! \approx \left(\frac{50}{e}\right)^{50} \sqrt{2\pi(50)}$$

$$\approx 3.036\ 344... \times 10^{64}$$

Using the formula from part e),

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}\left(1+\frac{1}{12n}\right)$$

$$50! \approx \left(\frac{50}{e}\right)^{50} \sqrt{2\pi(50)}\left(1+\frac{1}{12(50)}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(1+\frac{1}{600}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(\frac{600}{600}+\frac{1}{600}\right)$$

$$\approx 3.036\ 344... \times 10^{64}\left(\frac{601}{600}\right)$$

$$\approx 3.041\ 404... \times 10^{64}$$

Using a calculator,
50! = 50(49)(48)…(3)(2)(1)
= 3.041 409 32 × 10^{64}

The formula in part e) seems to give a more accurate approximation.

Section 11.3 Page 545 Question C1

The coefficients of the terms in the expansion of $(x + y)^n$ are the same as the numbers in row $n + 1$ of Pascal's triangle.
Examples: $(x + y)^2 = x^2$ and $2xy + y^2$ and row 3 of Pascal's triangle is 1 2 1;
$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^2$ and row 4 of Pascal's triangle is 1 3 3 1.

Section 11.3 Page 545 Question C2

Examples:
a) Permutation: In how many ways can four different chocolate bars be given to two people?

Combination: Steve has two Canadian dimes and two U.S. dimes in his pocket. In how many ways can he draw out two coins?

Binomial expansion: What is the coefficient of the middle term in the expansion of $(a + b)^4$?

b) All three problems have the same answer, 6, but they answer different questions.

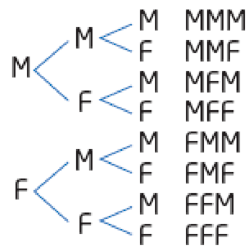
Section 11.3 Page 545 Question C3

Examples:
a), b) For small values of n, it is easier to use Pascal's triangle, but for large values of n, it is easier to use combinations to determine the coefficients in the expansion of $(a + b)^n$.

Chapter 11 Review

Chapter 11 Review Page 546 Question 1

a)



b) There are three outcomes that give them one boy and two girls: MFF, FMF, FFM.

Chapter 11 Review Page 546 Question 2

a) You can leave or enter using any of the gates.
9(9) = 81
There are 81 ways to enter and leave the stadium.

b) ways to enter through a north gate: 4 ways
ways to leave through any other gate: 9 – 1 = 8 ways
4(8) = 32
There are 32 ways to enter through a north gate and leave through any other gate.

Chapter 11 Review Page 546 Question 3

a) *Bite* has four different letters. There are 4! = 24 ways to arrange the letters.

b) There are six letters in *bitten*. There are 2! ways to arrange the two *t*'s.

$$\frac{6!}{2!} = \frac{6(5)(4)(3)\cancel{(2!)}^{1}}{\cancel{2!}_{1}}$$
$$= 360$$

The number of different six-letter arrangements is 360.

c) There are six letters in *mammal*. There are 2! ways to arrange the two *a*'s and 3! ways to arrange the three *m*'s.

$$\frac{6!}{2!3!} = \frac{6(5)(4)\cancel{(3!)}^{1}}{2(1)\cancel{(3!)}_{1}}$$
$$= 60$$

The number of different six-letter arrangements is 60.

d) There are 12 letters in *mathematical*. There are 2! ways to arrange the two *m*'s, 3! ways to arrange the three *a*'s, 2! ways to arrange the two *t*'s.

$$\frac{12!}{2!3!2!}$$

The number of different 12-letter arrangements is $\frac{12!}{2!3!2!}$.

Chapter 11 Review Page 546 Question 4

a) There are 2! = 2 ways to arrange Anna and Cleo together. Considering them as one object means there are four objects to arrange in 4! = 24 ways. Since Anna and Cleo must be seated together, there are 4!2! = 48 ways to arrange the people.

b) There are 2! = 2 ways to arrange Anna and Cleo together, and 2! ways to arrange Dina and Eric. Considering each pair as one object means there are three objects to arrange in 3! = 6 ways. Since each pair must be seated together, there are 3!2!2! = 24 ways to arrange the people.

c) Total arrangements of five people: 5! = 120

Ways of seating Anna and Cleo apart = total arrangements – ways of seating them together

= 120 – 48 (from part a))

= 72

There are 72 ways of seating Anna and Cleo apart.

Chapter 11 Review Page 546 Question 5

a) 7! = 5040

b) There are 4! ways to arrange the consonants, and 3! ways to arrange the vowels.
Case 1: start with a vowel
Arrangements with vowels and consonants alternating: 4!3! = 144

Case 2: start with a consonant
Arrangements with vowels and consonants alternating: 4!3! = 144

Total number of possibilities: 144 + 144 = 288

c) There are 3! = 6 ways to arrange the vowels in the middle. There are 4! = 24 ways to arrange two consonants on each end.
4!3! = 144

Chapter 11 Review Page 546 Question 6

a) Case 1: no digits
There are 26^4 = 456 976 ways to create four-digit passwords using letters that may repeat.

Case 2: one digit
There are 10(26)(26)(26) = 175 760 ways to choose the characters, and eight possible positions for the digit.

Total number of possibilities: $26^4 + 4(10)(26^4) = 1\ 160\ 016$

b) Case 1: no digits
There are 26^8 = 208 827 064 576 ways to create eight-digit passwords using letters that may repeat.

Case 2: one digit
There are $10(26^7)$ ways to choose the characters, and eight possible positions for the digit.

Total number of possibilities: $26^8 + 8(10)(26^7) = 851\ 371\ 878\ 656$

c) 851 371 878 656(10) – 1 160 016(10)

$$= (8\ 513\ 707\ 186\ 400 \text{ s})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ day}}{24 \text{ h}}\right)\left(\frac{1 \text{ year}}{365 \text{ days}}\right)$$

= 270 000 years

It would take about 270 000 years longer to check all of the eight-character passwords than to check the four-character passwords.

Chapter 11 Review Page 546 Question 7

a) $$\frac{n! + (n-1)!}{n! - (n-1)!} = \frac{n(n-1)! + (n-1)!}{n(n-1)! - (n-1)!}$$
$$= \frac{(n-1)!(n+1)}{(n-1)!(n-1)}$$
$$= \frac{n+1}{n-1}$$

b) $$\frac{(x+1)! + (x-1)!}{x!} = \frac{(x+1)(x)(x-1)! + (x-1)!}{x(x-1)!}$$
$$= \frac{(x-1)![(x+1)(x) + 1]}{x(x-1)!}$$
$$= \frac{(x-1)![x^2 + x + 1]}{x(x-1)!}$$
$$= \frac{x^2 + x + 1}{x}$$

Chapter 11 Review Page 546 Question 8

a) $${}_{10}C_4 = \frac{10!}{(10-4)!4!}$$
$$= \frac{10!}{6!4!}$$
$$= \frac{10(9)(8)(7)(6!)}{6!(4)(3)(2)(1)}$$
$$= 210$$
You can select four light bulbs in 210 ways.

b) There are 10 – 3 = 7 good light bulbs.
$${}_7C_2 \times {}_3C_2 = \frac{7!}{(7-2)!2!} \times \frac{3!}{(3-2)!2!}$$
$$= \frac{7!}{5!2!} \times \frac{3!}{1!2!}$$
$$= \frac{7(6)(5!)}{5!(2)(1)} \times \frac{3(2)(1)}{1(2)(1)}$$
$$= 63$$
There are 63 ways to select the light bulbs.

Chapter 11 Review Page 546 Question 9

a)
$$
\begin{aligned}
{}_{10}C_3 &= \frac{10!}{(10-3)!3!} \\
&= \frac{10!}{7!3!} \\
&= \frac{10(9)(8)\cancel{(7!)}}{\cancel{7!}(3)(2)(1)} \\
&= 120
\end{aligned}
$$

b)
$$
\begin{aligned}
{}_{10}P_4 &= \frac{10!}{(10-4)!} \\
&= \frac{10!}{6!} \\
&= \frac{10(9)(8)(7)\cancel{(6!)}}{\cancel{6!}} \\
&= 5040
\end{aligned}
$$

c)
$$
\begin{aligned}
{}_5C_3 \times {}_5P_2 &= \frac{5!}{(5-3)!3!} \times \frac{5!}{(5-2)!} \\
&= \frac{5!}{2!3!} \times \frac{5!}{3!} \\
&= \frac{5(4)\cancel{(3!)}}{2(1)\cancel{3!}} \times \frac{5(4)\cancel{(3!)}}{\cancel{3!}} \\
&= 200
\end{aligned}
$$

d)
$$
\begin{aligned}
\left(\frac{15!}{4!11!}\right){}_6P_3 &= \left(\frac{15(14)(13)(12)\cancel{(11!)}}{4(3)(2)(1)\cancel{(11!)}}\right)\left(\frac{6!}{(6-3)!}\right) \\
&= 1365\left(\frac{6!}{3!}\right) \\
&= 1365\left(\frac{6(5)(4)\cancel{(3!)}}{\cancel{3!}}\right) \\
&= 163\ 800
\end{aligned}
$$

Chapter 11 Review Page 546 Question 10

a) Case 1: Use 1 coin

$$
\begin{aligned}
{}_4C_1 &= \frac{4!}{(4-1)!1!} \\
&= \frac{4!}{3!(1)} \\
&= \frac{4(3!)}{3!} \\
&= 4
\end{aligned}
$$

Case 2: Use 2 coins

$$
\begin{aligned}
{}_4C_2 &= \frac{4!}{(4-2)!2!} \\
&= \frac{4!}{2!2!} \\
&= \frac{4(3)(2!)}{2!(2)(1)} \\
&= 6
\end{aligned}
$$

Case 3: Use 3 coins

$$
\begin{aligned}
{}_4C_3 &= \frac{4!}{(4-3)!3!} \\
&= \frac{4!}{1!3!} \\
&= \frac{4(3!)}{(1)3!} \\
&= 4
\end{aligned}
$$

Case 4: Use 4 coins

$$
\begin{aligned}
{}_4C_4 &= \frac{4!}{(4-4)!4!} \\
&= \frac{4!}{0!4!} \\
&= \frac{(4!)}{(1)4!} \\
&= 1
\end{aligned}
$$

Total number of ways: 4 + 6 + 4 + 1 = 15

b) Amounts below are in cents.
From Case 1: 1, 5, 10, 25
From Case 2: 1 + 5 = 6, 1 + 10 = 11, 1 + 25 = 26, 5 + 10 = 15, 5 + 25 = 30, 10 + 25 = 35
From Case 3: 1 + 5 + 10 = 16, 1 + 5 + 25 = 31, 1 + 10 + 25 = 36, 5 + 10 + 25 = 40
From Case 4: 1 + 5 + 10 + 25 = 41

Chapter 11 Review Page 546 Question 11

a)

$$
\begin{aligned}
{}_nC_2 &= 28 \\
\frac{n!}{(n-2)!2!} &= 28 \\
\frac{n(n-1)(n-2)!}{(n-2)!} &= 28(2!) \\
\frac{n(n-1)(n-2)!}{(n-2)!} &= 28(2)(1) \\
n(n-1) &= 56 \\
8(7) &= 56
\end{aligned}
$$

By inspection, $n = 8$.

${}_8C_2 = 28$

Left Side = ${}_8C_2$ Right Side = 28

$$= \frac{8!}{(8-2)!2!}$$

$$= \frac{8!}{6!2!}$$

$$= \frac{8(7)\,\cancel{(6!)}^{1}}{\cancel{6!}_{1}(2)(1)}$$

$$= 28$$

Left Side = Right Side

b) $${}_nC_3 = 4({}_nP_2)$$

$$\frac{n!}{(n-3)!3!} = 4\left(\frac{n!}{(n-2)!}\right)$$

$$\frac{n(n-1)(n-2)\,\cancel{(n-3)!}^{1}}{\cancel{(n-3)!}_{1}\,3!} = 4\left(\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}}\right)$$

$$\frac{n(n-1)(n-2)}{3!} = 4n(n-1)$$

$$\frac{\cancel{n}^{1}\,\cancel{(n-1)}^{1}(n-2)}{\cancel{n}_{1}\,\cancel{(n-1)}_{1}} = 4(3!)$$

$$n-2 = 4(3)(2)(1)$$

$$n = 24+2$$

$$n = 26$$

${}_{26}C_3 = 4({}_nP_2)$

Left Side = ${}_{26}C_3$ Right Side = $4({}_{26}P_2)$

$$= \frac{26!}{(26-3)!3!} \qquad = 4\left(\frac{26!}{(26-2)!}\right)$$

$$= \frac{26!}{23!3!} \qquad = 4\left(\frac{26!}{24!}\right)$$

$$= \frac{26(25)(24)\,\cancel{(23!)}^{1}}{\cancel{3!}_{1}(3)(2)(1)} \qquad = 4\left(\frac{26(25)\,\cancel{(24!)}^{1}}{\cancel{24!}_{1}}\right)$$

$$= 2600 \qquad = 2600$$

Left Side = Right Side

Chapter 11 Review Page 547 Question 12

$$
\begin{aligned}
{}_{10}C_2 \times {}_8C_3 \times {}_5C_5 &= \frac{10!}{(10-2)!2!} \times \frac{8!}{(8-3)!3!} \times \frac{5!}{(5-5)!5!} \\
&= \frac{10!\,\cancel{8!}^{1}\,\cancel{5!}^{1}}{\cancel{8!}_{1}\,2!5!3!0!\,\cancel{5!}_{1}} \\
&= \frac{10!}{2!5!3!0!} \\
&= \frac{10(9)(8)(7)(6)\,\cancel{(5!)}}{2(1)\,\cancel{(5!)}_{1}\,(3)(2)(1)(1)} \\
&= 2520
\end{aligned}
$$

There are 2520 ways to break the students into groups.

Chapter 11 Review Page 547 Question 13

a) Example:
Permutation: How many arrangements of the letters AAABB are possible?
Combination: How many ways can you choose three students from a group of five?

b) Yes, since 2 + 3 = 5, the number of ways of selecting two out of a group of five is the same as the number of ways of 'rejecting' the remaining three.

$$
\begin{aligned}
{}_5C_2 &= \frac{5!}{(5-2)!2!} \\
&= \frac{5!}{3!2!}
\end{aligned}
\qquad
\begin{aligned}
{}_5C_3 &= \frac{5!}{(5-3)!3!} \\
&= \frac{5!}{2!3!}
\end{aligned}
$$

Chapter 11 Review Page 547 Question 14

a) The row below 1 2 1 is 1 3 3 1.

b) The row below 1 8 28 56 70 56 28 8 1 is 1 9 36 84 126 126 84 36 9 1.

Chapter 11 Review Page 547 Question 15

Examples:
Using multiplication, determine the coefficients by expanding, collecting like terms, and writing the answer in descending order of the exponent of x.

$$
\begin{aligned}
(x + y)^3 &= (x + y)(x + y)(x + y) \\
&= x^3 + 3x^2y + 3xy^2 + y^3
\end{aligned}
$$

Using, Pascal's triangle, coefficients are the terms from row $n + 1$ of Pascal's triangle.
For $(x + y)^3$, row 4 is 1 3 3 1.

Using combinations, coefficients correspond to the combinations as shown:
$(x + y)^3 = {}_3C_0x^3y^0 + {}_3C_1x^2y^1 + {}_3C_2\mathrm{C}1x^1y^2 + {}_3C_3x^0y^3$

Chapter 11 Review Page 547 Question 16

a) $(x + y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5$
$= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Substitute $x = a$ and $y = b$.
$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

b) $(x + y)^3 = {}_3C_0(x)^3(y)^0 + {}_3C_1(x)^2(y)^1 + {}_3C_2(x)^1(y)^2 + {}_3C_3(x)^0(y)^3$
$= 1(x)^3(1) + 3(x)^2(y) + 3(x)(y)^2 + 1(1)(y)^3$
$= (x)^3 + 3(x)^2(y) + 3(x)(y)^2 + (y)^3$

Substitute $y = –3$.
$(x – 3)^3 = (x)^3 + 3(x)^2(–3) + 3(x)(–3)^2 + (–3)^3$
$= x^3 – 9x^2 + 3x(9) + (–27)$
$= x^3 – 9x^2 + 27x – 27$

c) $(x + y)^4 = {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^3(y)^1 + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$
$= 1(x)^4(1) + 4(x)^3(y) + 6(x)^2(y)^2 + 4(x)(y)^3 + 1(1)(y)^4$
$= (x)^4 + 4(x)^3(y) + 6(x)^2(y)^2 + 4(x)(y)^3 + (y)^4$

Substitute $x = 2x^2$ and $y = -\frac{1}{x^2}$.

$$\left(2x^2 - \frac{1}{x^2}\right)^4 = (2x^2)^4 + 4(2x^2)^3\left(-\frac{1}{x^2}\right) + 6(2x^2)^2\left(-\frac{1}{x^2}\right)^2 + 4(2x^2)\left(-\frac{1}{x^2}\right)^3 + \left(-\frac{1}{x^2}\right)^4$$

$$= (2^4x^8) + 4(2^3x^6)\left(-\frac{1}{x^2}\right) + 6(2^2x^4)\left(\frac{1}{x^4}\right) + 4(2x^2)\left(-\frac{1}{x^6}\right) + \frac{1}{x^8}$$

$$= 16x^8 + 4(8x^{\overset{4}{\cancel{6}}})\left(-\frac{1}{x^{\overset{0}{\cancel{2}}}}\right) + 6(4\overset{1}{\cancel{x^4}})\left(\frac{1}{\underset{1}{\cancel{x^4}}}\right) + 4(2x^{\overset{0}{\cancel{2}}})\left(-\frac{1}{x^{\overset{4}{\cancel{6}}}}\right) + \frac{1}{x^8}$$

$$= 16x^8 - 32x^4 + 24 - \frac{8}{x^4} + \frac{1}{x^8}$$

Chapter 11 Review Page 547 Question 17

a) third term of $(a + b)^9$: ${}_9C_2(a)^{9-2}(b)^2 = 36a^7b^2$

b) sixth term of $(x – 2y)^6$: ${}_6C_5(x)^{6-5}(–2y)^5 = 6(x)^1(–2^5y^5)$
$= 6x(–32y^5)$
$= –192xy^5$

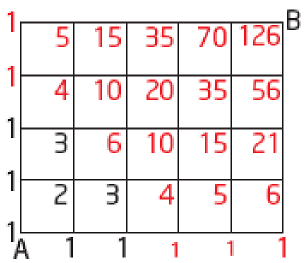
c) middle term of $\left(\frac{1}{x} - 2x^2\right)^6$: ${}_6C_3\left(\frac{1}{x}\right)^{6-3}(-2x^2)^3 = 20\left(\frac{1}{x}\right)^3(-2^3x^6)$

$$= 20\left(\frac{1^3}{\cancel{x^3}_{\,x^0}}\right)(-8x^{\cancel{6}\,3})$$

$$= -160x^3$$

Chapter 11 Review Page 547 Question 18

a)



b) Pascal's triangle values are shown with the top of the triangle at point A and the rows appearing diagonally up and right of point A.

c) From the diagram in part a), there are 126 pathways from A to B.

d) There are four identical moves up and five identical moves right.

Number of pathways: $\frac{(4+5)!}{4!5!} = \frac{9!}{4!5!}$

$$= \frac{9(8)(7)(6)\cancel{(5!)}^{1}}{4(3)(2)(1)\cancel{(5!)}_{1}}$$

$$= 126$$

There are 126 pathways from A to B.

Chapter 11 Review Page 547 Question 19

a) To move all of the green counters to the left side, each green counter must jump over the yellow counters that are to the left of it.
$0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
It takes 45 moves to separate the green and yellow counters into two groups.

b) Two counters: $0 + 1 = 1$ move; three counters: $0 + 1 + 2 = 3$ moves; four counters: $0 + 1 + 2 + 3 = 6$ moves, and so on up to 12 counters:
$0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 = 66$ moves
The pattern is the sum of the numbers up to one less than the number of counters of each colour.

c) 25 counters: $0 + 1 + 2 + 3 + \ldots + 22 + 23 + 24 = 300$
It takes 300 moves to separate 25 counters of each colour.

Chapter 11 Practice Test

Chapter 11 Practice Test Page 548 Question 1

There are four choices for the first digit (1, 2, 8, 9), four choices for the middle digit, and three choices for the last digit.
4(4)(3) = 48
The answer is C.

Chapter 11 Practice Test Page 548 Question 2

The word *sweepers* has eight letters, with 2 *s*'s and 3 *e*'s.

$$\frac{8!}{2!3!} = \frac{8(7)(6)(5)(4)(3)(2)(1)}{2(1)(3)(2)(1)}$$
$$= 3360$$

The answer is D.

Chapter 11 Practice Test Page 548 Question 3

$$_7C_2 \times {_6C_2} \times {_5C_1} = \frac{7!}{(7-2)!2!} \times \frac{6!}{(6-2)!2!} \times \frac{5!}{(5-1)!1!}$$
$$= \frac{7!}{\cancel{5!}\,2!} \times \frac{6!}{4!2!} \times \frac{\cancel{5!}}{4!1!}$$
$$= \frac{7(6)(5)(4)(3)(2)(1)(6)(5)(4)(3)(2)(1)}{2(1)(4)(3)(2)(1)(2)(1)(4)(3)(2)(1)(1)}$$
$$= 1575$$

The answer is C.

Chapter 11 Practice Test Page 548 Question 4

The exponent is 11. So, there are 11 + 1 = 12 terms in the expansion.
The answer is B.

Chapter 11 Practice Test Page 548 Question 5

third term of $(2x^2 + 3y)^7$: $_7C_2(2x^2)^{7-2}(3y)^2 = 21(2x^2)^5(3y)^2$
$= 21(2^5x^{10})\,(3^2y^2)$
$= 21(32x^{10})\,(9y^2)$
$= 6048x^{10}y^2$

The answer is A.

Chapter 11 Practice Test Page 548 Question 6

The next row in Pascal's triangle is 1 7 21 35 35 21 7 1.
The sixth number in the row is 21.
The answer is C.

Chapter 11 Practice Test Page 548 Question 7

a) $\frac{6!}{2!2!} = \frac{6(5)(4)(3)(2)(1)}{2(1)2(1)}$

$= 180$

There are 180 possible answer keys.

b) AACBDB, ABCADB, ABCBDA, BACBDA, BACADB, BBCADA

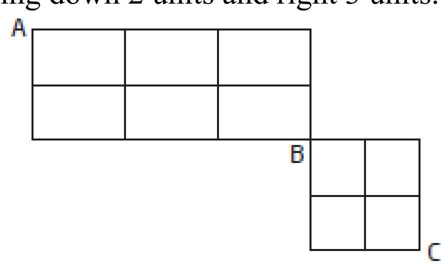
Chapter 11 Practice Test Page 548 Question 8

$$\begin{aligned} {}_nP_2 &= 72 \\ \frac{n!}{(n-2)!} &= 72 \\ \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} &= 72 \\ n(n-1) &= 72 \\ n(n-1) &= 9(8) \end{aligned}$$

By inspection, $n = 9$.
No, n does not equal –8.

Chapter 11 Practice Test Page 548 Question 9

a) Determine the number of arrangements of moving down 2 units and right 3 units.



$$\begin{aligned} \frac{(2+3)!}{2!3!} &= \frac{5!}{2!3!} \\ &= \frac{5(4)\cancel{(3!)}}{2(1)\cancel{(3!)}} \\ &= 10 \end{aligned}$$

There are 10 pathways from A to B.

b) The number of pathways from A to C is equal to the number of pathways from A to B multiplied by the number of pathways from B to C.

pathways from A to C: $\dfrac{5!}{2!3!} \times \dfrac{4!}{2!2!} = 10 \times \dfrac{4(3)\,(\cancel{2!})^{1}}{2(1)\,(\cancel{2!})_{1}}$

$= 10 \times 6$

$= 60$

There are 60 pathways from A to C.

Chapter 11 Practice Test Page 548 Question 10

Case 1: one digit number
5 ways

Case 2: two digit number
4 choices for first digit × 4 choices for last digit
4(4) = 16

Case 3: three digit number
4 choices for first digit × 4 choices for middle digit × 3 choices for last digit
4(4)(3) = 48

Total number of possibilities: 5 + 16 + 48 = 69
There are 69 possible numbers.

Chapter 11 Practice Test Page 548 Question 11

Permutations determine the number of arrangements of *n* items chosen *r* a t a time, when order is important. For example, the number of arrangements of 5 people chosen 2 at a time to ride on a motorcycle is $_5P_2 = 20$. A combination determines the number of different selections of *n* objects chosen *r* at a time, when order is not important. For example, the number of selections of 5 objects chosen 2 at a time when order is not important is $_5C_2 = 10$.

Chapter 11 Practice Test Page 548 Question 12

$$\left(x^2 + \frac{2}{x}\right)^9$$

$$= {}_9C_0(x^2)^9\left(\frac{2}{x}\right)^0 + {}_9C_1(x^2)^8\left(\frac{2}{x}\right) + {}_9C_2(x^2)^7\left(\frac{2}{x}\right)^2 + {}_9C_3(x^2)^6\left(\frac{2}{x}\right)^3 + {}_9C_4(x^2)^5\left(\frac{2}{x}\right)^4 + {}_9C_5(x^2)^4\left(\frac{2}{x}\right)^5 + {}_9C_6(x^2)^3\left(\frac{2}{x}\right)^6$$
$$+ {}_9C_7(x^2)^2\left(\frac{2}{x}\right)^7 + {}_9C_8(x^2)^1\left(\frac{2}{x}\right)^8 + {}_9C_9(x^2)^0\left(\frac{2}{x}\right)^9$$

$$= 1(x^{18})(1) + 9(x^{16})\left(\frac{2}{x}\right) + 36(x^{14})\left(\frac{2}{x}\right)^2 + 84(x^{12})\left(\frac{2}{x}\right)^3 + 126(x^{10})\left(\frac{2}{x}\right)^4 + 126(x^8)\left(\frac{2}{x}\right)^5 + 84(x^6)\left(\frac{2}{x}\right)^6$$
$$+ 36(x^4)\left(\frac{2}{x}\right)^7 + 9(x^2)\left(\frac{2}{x}\right)^8 + 1(1)\left(\frac{2}{x}\right)^9$$

$$= x^{18} + \frac{18x^{\cancel{16}\,15}}{\cancel{x}} + 36x^{\cancel{14}\,12}\left(\frac{2^2}{x^{\cancel{2}\,0}}\right) + 84(x^{\cancel{12}\,9})\left(\frac{2^3}{x^{\cancel{3}\,0}}\right) + 126(x^{10})\left(\frac{2}{x}\right)^4 + 126(x^8)\left(\frac{2}{x}\right)^5 + 84(x^6)\left(\frac{2}{x}\right)^6 + 36(x^4)\left(\frac{2}{x}\right)^7 + 9(x^2)\left(\frac{2}{x}\right)^8 + 1(1)\left(\frac{2}{x}\right)^9$$

The term that contains x^9 is $84x^9(2^3) = 672x^9$.

Chapter 11 Practice Test Page 548 Question 13

a) Case 1: Even Numbers That Start With 5 or 9
There are 2 choices for the first digit (5 or 9). There are 4 choices for the last digit (0, 2, 6, or 8). Since two numbers have been used, there are 6 and 5 choices for the second and third digits.
2(6)(5)(4) = 240

Case 2: Even Numbers That Start With 6 or 8
There are 2 choices for the first digit (6 or 8). There are 3 choices for the last digit (0, 2, and 6 or 8 since one of these numbers was used for the first digit). Since two numbers have been used, there are 6 and 5 choices for the second and third digits.
2(6)(5)(3) = 180

There are 240 + 180 = 420 4-digit even numbers greater than 5000 that can be formed.

b) Case 1: Numbers That Start With 5 or 9
There are 2 choices for the first digit (5 or 9). There is 1 choice for the last digit (0). There are 6 and 5 choices for the second and third digits.
2(6)(5)(1) = 60

Case 2: Numbers That Start With 6 or 8
There are 2 choices for the first digit (6 or 8). There is 1 choice for the last digit (0). There are 6 and 5 choices for the second and third digits.
2(6)(5)(1) = 60

You can form 60 + 60 = 120 numbers.

Chapter 11 Practice Test Page 548 Question 14

a)

$$\begin{aligned} {}_nP_3 &= 120 \\ \frac{n!}{(n-3)!} &= 120 \\ \frac{n(n-1)(n-2)\,\cancel{(n-3)!}^{\,1}}{\cancel{(n-3)!}_{\,1}} &= 120 \\ n(n-1)(n-2) &= 6(5)(4) \end{aligned}$$

By inspection, $n = 6$.

b)

$$\begin{aligned} 3({}_nC_2) &= 12({}_nC_1) \\ \frac{n!}{(n-2)!2!} &= \frac{12}{3}\left(\frac{n!}{(n-1)!1!}\right) \\ \frac{n(n-1)\,\cancel{(n-2)!}^{\,1}}{\cancel{(n-2)!}_{\,1}(2)(1)} &= 4\left(\frac{n\,\cancel{(n-1)!}^{\,1}}{\cancel{(n-1)!}_{\,1}(1)}\right) \\ n(n-1) &= 4(2)(n) \\ \frac{\cancel{n}^{\,1}(n-1)}{\cancel{n}_{\,1}} &= 8 \\ n-1 &= 8 \\ n &= 9 \end{aligned}$$

Chapter 11 Practice Test Page 548 Question 15

$$\begin{aligned}(x+y)^5 &= {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5\\ &= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5\\ &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5\end{aligned}$$

Substitute $x = y$ and $y = -\frac{2}{y^2}$.

$$\begin{aligned}\left(y-\frac{2}{y^2}\right)^5 &= (y)^5 + 5(y)^4\left(-\frac{2}{y^2}\right) + 10(y)^3\left(-\frac{2}{y^2}\right)^2 + 10(y)^2\left(-\frac{2}{y^2}\right)^3 + 5(y)\left(-\frac{2}{y^2}\right)^4 + \left(-\frac{2}{y^2}\right)^5\\ &= y^5 + 5y^{\not{4}\,2}\left(-\frac{2}{\not{y^2}\,y^0}\right) + 10y^{\not{3}\,0}\left(\frac{2^2}{\not{y^4}\,y^1}\right) + 10y^{\not{2}\,0}\left(-\frac{2^3}{\not{y^6}\,y^4}\right) + 5\not{y}\left(\frac{2^4}{\not{y^8}\,y^7}\right) + \left(-\frac{2^5}{y^{10}}\right)\\ &= y^5 - 10y^2 + 10\left(\frac{4}{y}\right) - 10\left(\frac{8}{y^4}\right) + 5\left(\frac{16}{y^7}\right) - \left(\frac{32}{y^{10}}\right)\\ &= y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}\end{aligned}$$

Chapter 11 Practice Test Page 548 Question 16

a) Considering the *a*'s as one object means there are four letters to arrange in 4! = 24 ways.

b) There are 2! = 2 ways of arranging the two *a*'s. The total number of arrangements of the letters in *aloha* is

$$\begin{aligned}\frac{5!}{2!} &= \frac{5(4)(3)(2)(1)}{2(1)}\\ &= 60\end{aligned}$$

If the *a*'s cannot be together, there are 60 – 24 = 36 ways of arranging the letters.

c) Determine the total arrangements with a vowel as the first letter.
There are 3 choices for the first vowel and 4! ways of arranging the remaining letters. Since there are 2! ways of arranging the two identical *a*'s, divide by 2!.

Total arrangements: $\frac{3(4!)}{2!} = \frac{3(4)(3)(2)(1)}{2(1)}$
$= 36$

Then, subtract the number of arrangements with a vowel as the first letter and the consonants together.
Consider the consonants together as one object. Since a vowel must be first, there are 3! ways of arranging the final letters. Multiply by 2 since there are 2! ways of arranging the consonants together, and divide by 2! since there are 2! ways of arranging the two identical *a*'s.

Arrangements with consonants together: $\frac{3(3!)\,\cancel{(2!)}^{1}}{\cancel{2!}_{1}} = 3(3)(2)(1)$

$= 18$

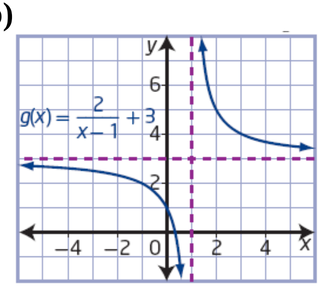
Arrangements beginning with a vowel and consonants not together: 36 – 18 = 18

Cumulative Review, Chapters 9–11

Cumulative Review, Chapters 9–11 Page 550 Question 1

a) The transformations involve a vertical stretch by a factor of 2 about the x-axis and a translation of 1 unit right and 3 units up.

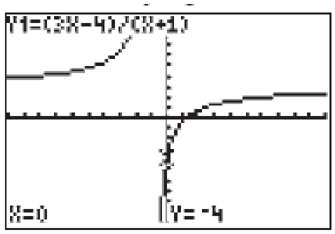
b)



c) domain: $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept: $\frac{1}{3}$, y-intercept: 1, horizontal asymptote: $y = 3$, vertical asymptote: $x = 1$

Cumulative Review, Chapters 9–11 Page 550 Question 2

a)



b) domain: $\{x \mid x \neq -1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept: $\frac{4}{3}$, y-intercept: –4, horizontal asymptote: $y = 3$, vertical asymptote: $x = –1$

Cumulative Review, Chapters 9–11 Page 550 Question 3

a) The graph of $y = \frac{x^2 - 3x}{x^2 - 9}$ has a vertical asymptote at $x = –3$, a point of discontinuity at (3, 0.5), and an x-intercept of 0. Therefore, graph C represents the function.

b) The graph of $y=\frac{x^2-1}{x+1}$ has no vertical asymptote, a point of discontinuity at (–1, –2), and an x-intercept of 1. Therefore, graph A represents the function.

c) The graph of $y=\frac{x^2+4x+3}{x^2+1}$ has no vertical asymptote, no point of discontinuity, and x-intercepts of –3 and –1. Therefore, graph B represents the function.

Cumulative Review, Chapters 9–11 Page 550 Question 4

a)

$$\begin{aligned}
\frac{1}{x^2-9}+\frac{1}{x+3}&=0\\
\frac{1}{(x-3)(x+3)}+\frac{1}{x+3}&=0\\
\frac{1}{(x-3)(x+3)}+\frac{x-3}{(x-3)(x+3)}&=0\\
\frac{1+x-3}{(x-3)(x+3)}&=0\\
x-2&=0\\
x&=2
\end{aligned}$$

Check:

Left Side $=\frac{1}{x^2-9}+\frac{1}{x+3}$ Right Side = 0

$$\begin{aligned}
&=\frac{1}{(2)^2-9}+\frac{1}{2+3}\\
&=\frac{1}{4-9}+\frac{1}{5}\\
&=\frac{1}{-5}+\frac{1}{5}
\end{aligned}$$

Left Side = Right Side

b)

$$\begin{aligned}
\frac{8x}{x-3}&=x+3\\
8x&=(x+3)(x-3)\\
8x&=x^2-9\\
0&=x^2-8x-9\\
0&=(x-9)(x+1)
\end{aligned}$$

$x-9=0$ or $x+1=0$

$x=9$ $\quad$ $x=-1$

Check $x = 9$:

$$\text{Left Side} = \frac{8x}{x-3} = \frac{8(9)}{9-3} = \frac{72}{6} = 12$$

$$\text{Right Side} = x+3 = 9+3 = 12 = 12$$

Left Side = Right Side

Check $x = -1$:

$$\text{Left Side} = \frac{8x}{x-3} = \frac{8(-1)}{-1-3} = \frac{-8}{-4} = 2$$

$$\text{Right Side} = x+3 = -1+3 = 2$$

Left Side = Right Side

c)

$$\frac{x+4}{4} = \frac{x+5}{x^2+6x+5}$$

$$\frac{x+4}{4} = \frac{\overset{1}{\cancel{x+5}}}{\underset{1}{\cancel{(x+5)}}(x+1)}$$

$$\frac{x+4}{4} = \frac{1}{x+1}$$

$$(x+4)(x+1) = 4$$

$$x^2+4x+x+4 = 4$$

$$x^2+5x = 4-4$$

$$x(x+5) = 0$$

$x = 0$ or $x+5 = 0$

$x = -5$

Check $x = 0$:

$$\text{Left Side} = \frac{x+4}{4} = \frac{0+4}{4} = 1$$

$$\text{Right Side} = \frac{0+5}{(0)^2+6(0)+5} = \frac{5}{0+0+5} = 1$$

Left Side = Right Side

Check $x = -5$:

$$\text{Left Side} = \frac{x+4}{4} = \frac{-5+4}{4} = \frac{-1}{4}$$

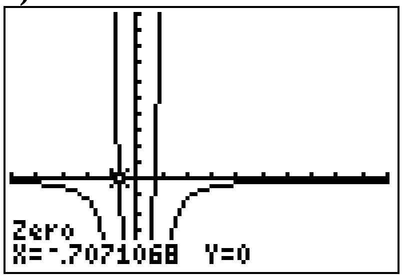
$$\text{Right Side} = \frac{-5+5}{(-5)^2+6(-5)+5} = \frac{5}{25-30+5} = \frac{5}{0}$$

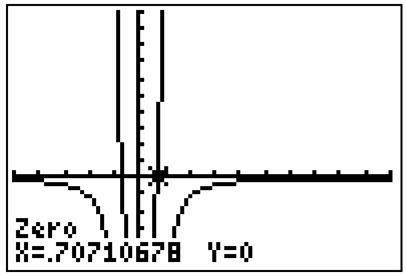
Left Side ≠ Right Side

The answer is $x = 0$.

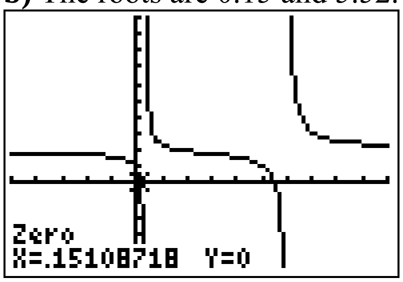
Cumulative Review, Chapters 9–11 Page 550 Question 5

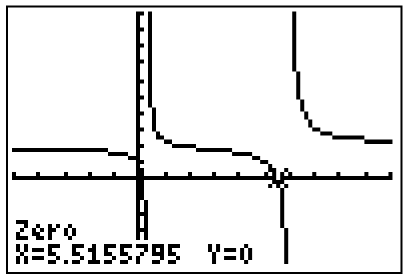
a) The roots are –0.71 and 0.71.





b) The roots are 0.15 and 5.52.





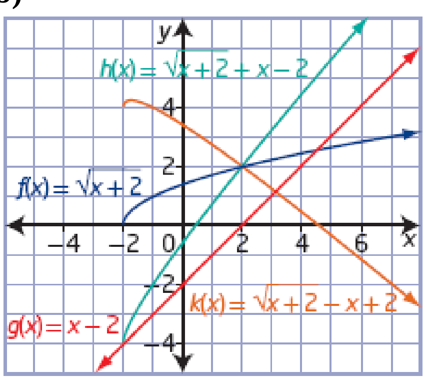
Cumulative Review, Chapters 9–11 Page 550 Question 6

a)

$$h(x) = (f + g)(x)$$
$$h(x) = f(x) + g(x)$$
$$h(x) = \sqrt{x+2} + x - 2$$

$$k(x) = (f - g)(x)$$
$$k(x) = f(x) - g(x)$$
$$k(x) = \sqrt{x+2} - (x - 2)$$
$$k(x) = \sqrt{x+2} - x + 2$$

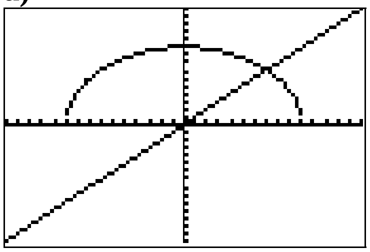
b)



c) $f(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$
$h(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \geq -2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 4.25, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 Page 550 Question 7

a)

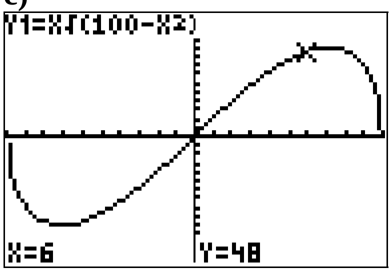


$f(x)$: domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$g(x)$: domain $\{x \mid -10 \le x \le 10, x \in \mathrm{R}\}$,
range $\{y \mid 0 \le y \le 10, y \in \mathrm{R}\}$

b) $h(x) = (f \cdot g)(x)$

$$h(x) = x\sqrt{100 - x^2}$$

c)



$h(x)$: domain $\{x \mid -10 \le x \le 10, x \in \mathrm{R}\}$, range $\{y \mid -50 \le y \le 50, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 Page 550 Question 8

a)

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(x) = \frac{x^2 + 3x + 2}{x^2 - 4}$$

$$h(x) = \frac{\overset{1}{\cancel{(x+2)}}(x+1)}{(x-2)\underset{1}{\cancel{(x+2)}}}$$

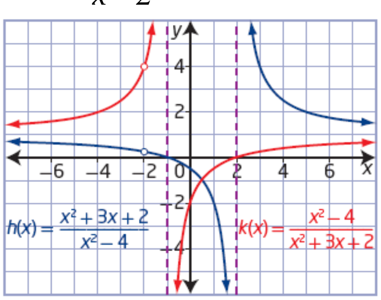
$$h(x) = \frac{x+1}{x-2},\ x \ne -2,\ 2$$

$$k(x) = \frac{g(x)}{f(x)}$$

$$k(x) = \frac{x^2 - 4}{x^2 + 3x + 2}$$

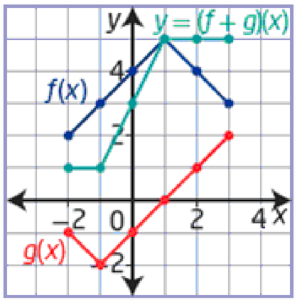
$$k(x) = \frac{(x-2)\overset{1}{\cancel{(x+2)}}}{\underset{1}{\cancel{(x+2)}}(x+1)}$$

$$k(x) = \frac{x-2}{x+1},\ x \ne -2,\ -1$$



b) The two functions have different domains but the same range.
$h(x)$: domain $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$
$k(x)$: domain $\{x \mid x \neq -2, -1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$

Cumulative Review, Chapters 9–11 Page 551 Question 9



Cumulative Review, Chapters 9–11 Page 551 Question 10

a) Determine the value of $h(3)$.
$h(x) = x^2 - 9$
$h(3) = (3)^2 - 9$
$h(3) = 9 - 9$
$h(3) = 0$

Substitute $h(3) = 0$ into $f(x)$.
$f(h(3)) = f(0)$
$f(h(3)) = 0 - 3$
$f(h(3)) = -3$

b) Determine the value of $f(5)$.
$f(x) = x - 3$
$f(5) = 5 - 3$
$f(5) = 2$

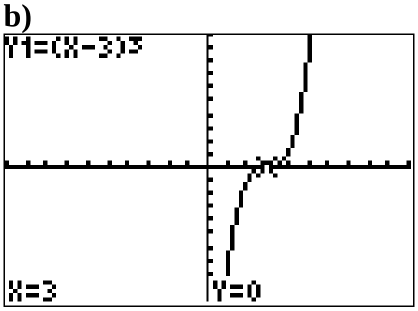
Substitute $f(5) = 2$ into $g(x)$.
$g(f(5)) = g(2)$
$g(f(5)) = \frac{1}{2}$

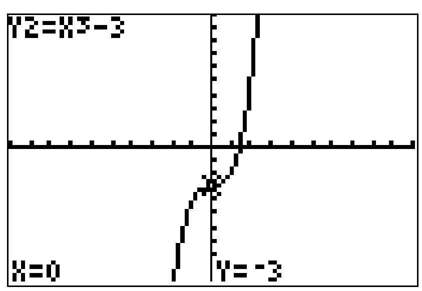
Cumulative Review, Chapters 9–11 Page 551 Question 11

a) $(f \circ g)(x) = f(x - 3)$
$(f \circ g)(x) = (x - 3)^3$

$(g \circ f)(x) = g(x^3)$
$(g \circ f)(x) = x^3 - 3$

b)





c) Both of the composite functions increase up into quadrant I and decrease down into quadrant III. The graph of $(f \circ g)(x) = (x - 3)^3$ is a translation of 3 units right of the graph of $f(x)$. The graph of $(g \circ f)(x) = x^3 - 3$ is a translation of 3 units down of the graph of $f(x)$.

Cumulative Review, Chapters 9–11 Page 551 Question 12

a) Substitute $g(x) = 10^x$ into $f(g(x))$.
$f(g(x)) = f(10^x)$
$f(g(x)) = \log 10^x$
$f(g(x)) = x$
The domain of the composite function is $\{x \mid x \in \mathrm{R}\}$.

b) Substitute $f(x) = \sin x$ into $g(f(x))$.
$g(f(x)) = g(\sin x)$
$g(f(x)) = \dfrac{1}{\sin x}$
$g(f(x)) = \csc x$
The domain of the composite function is $\{x \mid x \neq \pi n, n \in \mathrm{R}\}$.

c) Substitute $g(x) = x^2$ into $f(g(x))$.
$f(g(x)) = f(x^2)$
$f(g(x)) = \dfrac{1}{x^2 - 1}$
The domain of the composite function is $\{x \mid x \neq \pm 1, x \in \mathrm{R}\}$.

Cumulative Review, Chapters 9–11 Page 551 Question 13

total possibilities: 2(3)(2)(2)(4) = 96
There are 96 different meals possible.

Cumulative Review, Chapters 9–11 Page 551 Question 14

Total number of arrangements: 6! = 720
Arrangements with vowels together: Consider the vowels as one object. There are 5! ways to arrange the letters and 2! ways to arrange the vowels.
5!2! = 240
Since vowels cannot be together, there are 720 – 240 = 480 possible arrangements.

Cumulative Review, Chapters 9–11 Page 551 Question 15

$$\begin{aligned}
{}_7C_3 + {}_5P_2 &= \frac{7!}{(7-3)!3!} + \frac{5!}{(5-2)!} \\
&= \frac{7!}{4!3!} + \frac{5!}{3!} \\
&= \frac{7(6)(5)\cancel{(4!)}^{1}}{\cancel{4!}_{1}(3)(2)(1)} + \frac{5(4)\cancel{(3!)}^{1}}{\cancel{3!}_{1}} \\
&= 35 + 20 \\
&= 55
\end{aligned}$$

Cumulative Review, Chapters 9–11 Page 551 Question 16

There are 5 – 2 = 3 men on the committee.

$$\begin{aligned}
{}_6C_2 \times {}_7C_3 &= \frac{6!}{(6-2)!2!} \times \frac{7!}{(7-3)!3!} \\
&= \frac{6!}{4!2!} \times \frac{7!}{4!3!} \\
&= \frac{6(5)\cancel{(4!)}^{1}}{\cancel{4!}_{1}(2)(1)} \times \frac{7(6)(5)\cancel{(4!)}^{1}}{\cancel{4!}_{1}(3)(2)(1)} \\
&= 525
\end{aligned}$$

There are 525 ways to select the committee.

Cumulative Review, Chapters 9–11 Page 551 Question 17

a) There are 3! ways to arrange the three mathematics books, 5! ways to arrange the five history books, and 4! ways to arrange the four French books. The three sets of books can be arranged in 3! ways.

Total number of arrangements: 3!5!4!3! = 103 680

b) There are 3 ways of choosing the mathematics book for one end and 2 ways of choosing the mathematics book for the other end. There are 4! ways to arrange the four French books together. Consider the four French books as one object. There are 7 objects to arrange between the mathematics books, and this can be done in 7! ways.

Total number of arrangements: 3(2)(4!)(7!) = 725 760

Cumulative Review, Chapters 9–11 Page 551 Question 18

a)

$$\frac{(n+4)!}{(n+2)!} = 42$$

$$\frac{(n+4)(n+3)\cancel{(n+2)!}^{1}}{\cancel{(n+2)!}_{1}} = 42$$

$$(n+4)(n+3) = 7(6)$$

By inspection, $n + 4 = 7$ and $n + 3 = 6$. Therefore, $n = 3$.

b)

$$_{n}P_{3} = 20n$$

$$\frac{n!}{(n-3)!} = 20n$$

$$\frac{\overset{1}{\cancel{n}}(n-1)(n-2)\overset{1}{\cancel{(n-3)!}}}{\underset{1}{\cancel{(n-3)!}}} = 20\overset{1}{\cancel{n}}$$

$$(n-1)(n-2) = 20$$

$$(n-1)(n-2) = 5(4)$$

By inspection, $n - 1 = 5$ and $n - 2 = 4$. Therefore, $n = 6$.

c)

$$_{n+2}C_{n} = 21$$

$$\frac{(n+2)!}{(n+2-n)!n!} = 21$$

$$\frac{(n+2)!}{2!n!} = 21$$

$$\frac{(n+2)(n+1)\overset{1}{\cancel{(n)!}}}{2(1)\underset{1}{\cancel{(n)!}}} = 21$$

$$\frac{(n+2)(n+1)}{2} = 21$$

$$(n+2)(n+1) = 21(2)$$

$$(n+2)(n+1) = 42$$

$$(n+2)(n+1) = 7(6)$$

By inspection, $n + 2 = 7$ and $n + 1 = 6$. Therefore, $n = 5$.

Cumulative Review, Chapters 9–11 Page 551 Question 19

Examples:
Pascal's triangle:
$(x + y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$; the coefficients are values from the fifth row of Pascal's triangle.
$(x + y)^6 = 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$; the coefficients are values from the seventh row of Pascal's triangle.

Combinations:
$(x + y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3x^1y^3 + {}_4C_3x^0y^4$; the coefficients ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$, ${}_4C_4$ have the same values as in the fifth row of Pascal's triangle.
$(x + y)^6 = {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6x^0y^6$; the coefficients ${}_6C_0$, ${}_6C_1$, ${}_6C_2$, ${}_6C_3$, ${}_6C_4$, ${}_6C_5$, ${}_6C_6$ have the same values as in the seventh row of Pascal's triangle.

Cumulative Review, Chapters 9–11 Page 551 Question 20

a) $(x + y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Substitute $x = 3x$ and $y = –5$.
$(3x – 5)^4 = (3x)^4 + 4(3x)^3(–5) + 6(3x)^2(–5)^2 + 4(3x)(–5)^3 + (–5)^4$
$= 3^4x^4 – 20(3^3x^3) + 6(3^2x^2)(25) + 12x(–125) + 625$
$= 81x^4 – 20(27x^3) + 150(9x^2) – 1500x + 625$
$= 81x^4 – 540x^3 + 1350x^2 – 1500x + 625$

b) $(x + y)^5 = {}_5C_0(x)^5(y)^0 + {}_5C_1(x)^4(y)^1 + {}_5C_2(x)^3(y)^2 + {}_5C_3(x)^2(y)^3 + {}_5C_4(x)^1(y)^4 + {}_5C_5(x)^0(y)^5$
$= 1(x)^5(1) + 5(x)^4(y) + 10(x)^3(y)^2 + 10(x)^2(y)^3 + 5(x)(y)^4 + 1(1)(y)^5$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Substitute $x = \frac{1}{x}$ and $y = –2x$.

$$\left(\frac{1}{x}-2x\right)^5=\left(\frac{1}{x}\right)^5+5\left(\frac{1}{x}\right)^4(-2x)+10\left(\frac{1}{x}\right)^3(-2x)^2+10\left(\frac{1}{x}\right)^2(-2x)^3+5\left(\frac{1}{x}\right)(-2x)^4+(-2x)^5$$
$$=\frac{1}{x^5}+5\left(\frac{1}{x^4}\right)(-2x)+10\left(\frac{1}{x^3}\right)(4x^2)+10\left(\frac{1}{x^2}\right)(-2^3x^3)+5\left(\frac{1}{x}\right)(2^4x^4)+(-2^5x^5)$$
$$=\frac{1}{x^5}-10\cancel{x}\left(\frac{1}{x^{\cancel{4}\,3}}\right)+10\left(\frac{1}{x^{\cancel{3}\,1}}\right)(4x^{\cancel{2}\,0})+10\left(\frac{1}{x^{\cancel{2}\,0}}\right)(-8x^{\cancel{3}\,1})+5\left(\frac{1}{\cancel{x}}\right)(16x^{\cancel{4}\,3})-32x^5$$
$$=\frac{1}{x^5}-\frac{10}{x^3}+\frac{40}{x}-80x+80x^3-32x^5$$

Cumulative Review, Chapters 9–11 Page 551 Question 21

a) Fourth term of $(5x + y)^5$:
${}_5C_3(5x)^2(y)^3 = 10(25x^2)y^3$
$= 250x^2)y^3$
The coefficient of the fourth term is 250.

b) Fourth term of $\left(\frac{1}{x^2}-x^3\right)^8$:

$${}_8C_3\left(\frac{1}{x^2}\right)^5(-x^3)^3=56\left(\frac{1}{x^{10}}\right)(-x^9)$$
$$=-56\left(\frac{1}{x^{10}}\right)(x^9)$$

The coefficient of the fourth term is –56.

Cumulative Review, Chapters 9–11 Page 551 Question 22

a) In combination notation, ${}_nC_r$, the second value in the row is n, and r represents the term 0, 1, 2, ….
The next term is ${}_{25}C_4$.

b) The number of terms in this row is 25 + 1 = 26.

c) 2300 is the sum of the two terms in the row above, to the right and left of ${}_{25}C_3$. The values in the row above are ${}_{24}C_0$, ${}_{24}C_1$, ${}_{24}C_2$, ${}_{24}C_3$, ….
$${}_{25}C_3 = {}_{24}C_2 + {}_{24}C_3$$

Unit 4 Test

Unit 4 Test Page 552 Question 1

Factor the denominator.
$x^2 - 3x - 10 = (x - 5)(x + 2)$
$x + 2$ is a factor of both the numerator and denominator, so it does not correspond to a vertical asymptote. $x - 5$ is a factor of the denominator only, so it does correspond to a vertical asymptote and not a point of discontinuity. In statement C, the range should refer to y, not x.
The correct answer is **D**.

Unit 4 Test Page 552 Question 2

$y = \frac{-4}{x+2} + 3$ has a horizontal asymptote at $y = 3$, a vertical asymptote at $x = -2$, and a y-intercept of 1.
The answer is **B**.

Unit 4 Test Page 552 Question 3

$P(x) = R(x) - C(x)$
$P(x) = 10x - 0.001x^2 - (2x + 5000)$
$P(x) = 10x - 0.001x^2 - 2x - 5000$
$P(x) = -0.001x^2 + 8x - 5000$
$0 = -0.001x^2 + 8x - 5000$
$0 = 0.001x^2 - 8x + 5000$
Use the quadratic formula, with $a = 0.001$, $b = -8$, and $c = 5000$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(0.001)(5000)}}{2(0.001)}$$
$$x = \frac{8 \pm \sqrt{64 - 20}}{0.002}$$
$$x = \frac{8 + \sqrt{64 - 20}}{0.002} \quad \text{or} \quad x = \frac{8 - \sqrt{64 - 20}}{0.002}$$
$$x = 7316.624... \qquad x = 683.375...$$

Check $x \approx 683.375$:

Left Side = 0

Right Side $= 0.001x^2 - 8x + 5000$
$= 0.001(683.375)^2 - 8(683.375) + 5000$
$= 467.001\,39 - 5467 + 5000$
$= 0.00$

Left Side = Right Side

The answer is **A**.

Unit 4 Test Page 552 Question 4

$$h(x) = (f \cdot g)(x)$$
$$= \frac{1}{x}(x+1)^2$$

Since $x \neq 0$, the answer is **B**.

Unit 4 Test Page 552 Question 5

$g(x) = x^2 - 2$

$$f(g(x)) = f(x^2 - 2)$$
$$= \sqrt{(x^2 - 2) + 1}$$
$$= \sqrt{x^2 - 1}$$

$$m = \sqrt{x^2 - 1}$$

The answer is **B**.

Unit 4 Test Page 552 Question 6

A set of five desks can be assigned to two students in ${}_5P_2$ ways.
The answer is D.

Unit 4 Test Page 552 Question 7

$$\begin{aligned} {}_6C_2 + {}_4P_3 &= \frac{6!}{(6-2)!2!} + \frac{4!}{(4-3)!} \\ &= \frac{6!}{4!2!} + \frac{4!}{1!} \\ &= \frac{6(5)\cancel{(4!)}^{1}}{\cancel{4!}_{1}(2)(1)} + \frac{4(3)(2)(1)}{1} \\ &= 15 + 24 \\ &= 39 \end{aligned}$$

The answer is **C**.

Unit 4 Test Page 552 Question 8

Factor the denominator.
$2x^2 - 5x - 3 = (2x + 1)(x - 3)$
$x - 3$ is a factor of both the numerator and denominator. So, it corresponds to a point of discontinuity.
Find the x-coordinate:
$x - 3 = 0$
$x = 3$
Find the y-coordinate:

$$y = \frac{x-3}{2x^2 - 5x - 3}$$

$$y = \frac{\cancel{x-3}^{1}}{(2x+1)\cancel{(x-3)}_{1}}$$

$$y = \frac{1}{2x+1}$$

$$y = \frac{1}{2(3)+1}$$

$$y = \frac{1}{7}$$

The point of discontinuity is $\left(3, \frac{1}{7}\right)$.

Unit 4 Test Page 552 Question 9

$$\frac{x^2}{x^2+1}=\frac{x}{4}$$

$$4x^2 = x(x^2+1)$$

$$4x^2 = x^3 + x$$

$$0 = x^3 + x - 4x^2$$

$$0 = x(x^2 + 1 - 4x)$$

$x = 0$ or $x^2 - 4x + 1 = 0$

Use the quadratic formula, with $a = 1$, $b = -4$, and $c = 1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16-4}}{2}$$

$$x=\frac{4\pm\sqrt{12}}{2}$$

$$x=\frac{4+\sqrt{12}}{2} \quad \text{or} \quad x=\frac{4-\sqrt{12}}{2}$$

$$x = 3.7320\ldots \qquad x = 0.2679\ldots$$

The roots are $x = 0$, 3.73, and 0.27, to the nearest hundredth.

Unit 4 Test Page 552 Question 10

$(2x + 5y^2)^4 = {}_4C_0(2x)^4(5y^2)^0 + {}_4C_1(2x)^3(5y^2)^1 + {}_4C_2(2x)^2(5y^2)^2 + {}_4C_3(2x)^1(5y^2)^3 + {}_4C_4(2x)^0(5y^2)^4$

third term: ${}_4C_2(2x)^2(5y^2)^2 = 6(2^2x^2)(5^2y^4)$
$= 6(4x^2)(25y^4)$
$= 600x^2y^4$

Unit 4 Test Page 552 Question 11

$(x - 2)^5$
$= {}_5C_0(x)^5(-2)^0 + {}_5C_1(x)^4(-2)^1 + {}_5C_2(x)^3(-2)^2 + {}_5C_3(x)^2(-2)^3 + {}_5C_4(x)^1(-2)^4 + {}_5C_5(x)^0(-2)^5$
$= 1(x)^5(1) + 5(x)^4(-2) + 10(x)^3(4) + 10(x)^2(-8) + 5(x)(16) + 1(1)(-32)$
$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

The sum of the coefficients is $1 - 10 + 40 - 80 + 80 - 32 = -1$.

Unit 4 Test Page 552 Question 12

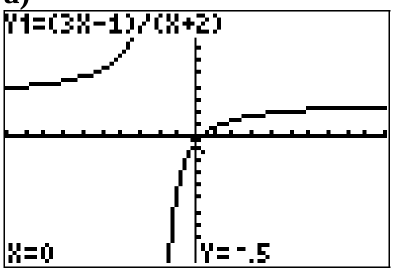
a) The transformations involve a vertical stretch by a factor of 2 and a translation of 1 unit left and 3 units down.

b) The equations of the asymptotes are $x = -1$ and $y = -3$.

c) For the graph of $g(x)$, as x approaches –1, $|y|$ becomes very large.

Unit 4 Test Page 553 Question 13

a)



b)
domain: $\{x \mid x \neq -2, x \in \mathrm{R}\}$,
range: $\{y \mid y \neq 3, y \in \mathrm{R}\}$, x-intercept: $\frac{1}{3}$,
y-intercept: $-\frac{1}{2}$

c)
$$0 = \frac{3x-1}{x+2}$$
$$0 = 3x - 1$$
$$1 = 3x$$
$$\frac{1}{3} = x$$
The root of the equation is $x = \frac{1}{3}$.

d) The x-intercept of the graph of the function $y = \frac{3x-1}{x+2}$ is the root of the equation $0 = \frac{3x-1}{x+2}$.

Unit 4 Test Page 553 Question 14

a) Factor the denominator.
$$f(x) = \frac{x-4}{x^2 - 2x - 8}$$
$$f(x) = \frac{x-4}{(x-4)(x+2)}$$
Since $x - 4$ is a factor of both the numerator and denominator, it corresponds to a point of discontinuity.
$$x - 4 = 0$$
$$x = 4$$
To determine the y-coordinate, substitute $x = 4$ into the simplified function.

$$f(x) = \frac{\cancel{x-4}^{\,1}}{\cancel{(x-4)}_{1}(x+2)}$$

$$f(x) = \frac{1}{(x+2)}$$

$$f(x) = \frac{1}{(4+2)}$$

$$f(x) = \frac{1}{6}$$

The point of discontinuity occurs at $\left(4,\ \frac{1}{6}\right)$.

Since $x + 2$ is a factor of only the denominator, it corresponds to a vertical asymptote.

$x + 2 = 0$

$x = -2$

The vertical asymptote occurs at $x = -2$.

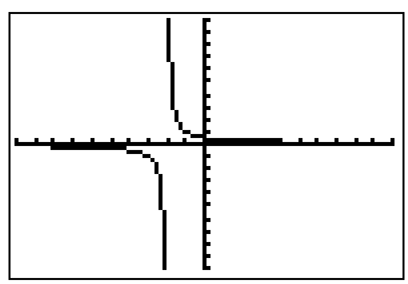
To determine y-intercepts, substitute $x = 0$.

$$f(x) = \frac{1}{x+2}$$

$$y = \frac{1}{0+2}$$

$$y = \frac{1}{2}$$

The y-intercept is $\left(0,\ \frac{1}{2}\right)$.



To determine x-intercepts, substitute $y = 0$.

$$0 = \frac{1}{x+2}$$

$0 \neq 1$

There are no x-intercepts.

b) Factor the numerator and denominator.

$$f(x) = \frac{x^2 + x - 6}{x^2 + 2x - 3}$$

$$f(x) = \frac{(x+3)(x-2)}{(x+3)(x-1)}$$

Since $x + 3$ is a factor of both the numerator and denominator, it corresponds to a point of discontinuity.

$x + 3 = 0$

$x = -3$

To determine the y-coordinate, substitute $x = -3$ into the simplified function.

$$f(x) = \frac{\overset{1}{\cancel{(x+3)}}(x-2)}{\underset{1}{\cancel{(x+3)}}(x-1)}$$

$$f(x) = \frac{x-2}{x-1}$$

$$f(x) = \frac{-3-2}{-3-1}$$

$$f(x) = \frac{-5}{-4}$$

$$f(x) = \frac{5}{4}$$

The point of discontinuity occurs at $\left(-3, \frac{5}{4}\right)$.

Since $x - 1$ is a factor of only the denominator, it corresponds to a vertical asymptote.

$x - 1 = 0$

$x = 1$

The vertical asymptote occurs at $x = 1$.

To determine y-intercepts, substitute $x = 0$.

$$f(x) = \frac{x-2}{x-1}$$

$$y = \frac{0-2}{0-1}$$

$$y = \frac{-2}{-1}$$

$$y = 2$$

The y-intercept is (0, 2).

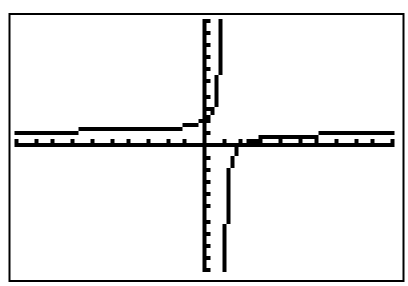
To determine x-intercepts, substitute $y = 0$.

$$0 = \frac{x-2}{x-1}$$

$$0 = x - 2$$

$$2 = x$$

The x-intercept is (2, 0).



c) Factor the numerator and denominator.

$$f(x) = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

$$f(x) = \frac{x(x-5)}{(x-3)(x+1)}$$

Since the numerator and denominator do not have a common factor, there is no point of discontinuity.

Since $x - 3$ and $x + 1$ are a factors of the denominator, they correspond to vertical asymptotes.

$x - 3 = 0$ $\quad\quad$ $x + 1 = 0$

$x = 3$ $\quad\quad$ $x = -1$

The vertical asymptotes occur at $x = 3$ and $x = -1$.

To determine y-intercepts, substitute $x = 0$.

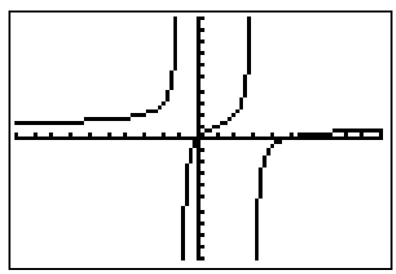
$$f(x) = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

$$y = \frac{0^2 - 5(0)}{0^2 - 2(0) - 3}$$

$$y = 0$$

The y-intercept is (0, 0).

To determine x-intercepts, substitute $y = 0$.



$$0 = \frac{x^2 - 5x}{x^2 - 2x - 3}$$

$$0 = x^2 - 5x$$

$$0 = x(x-5)$$

$x = 0$ or $x - 5 = 0$

$x = 5$

The x-intercepts are at (0, 0) and (5, 0).

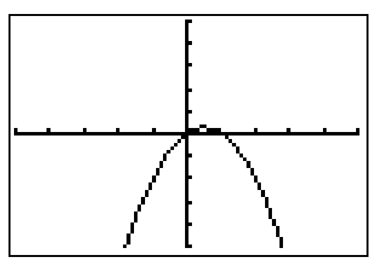
Unit 4 Test Page 553 Question 15

$f(x) = 1 - x^2$, $g(x) = x - 1$

a) $y = (f + g)(x)$

$y = 1 - x^2 + x - 1$

$y = x - x^2$



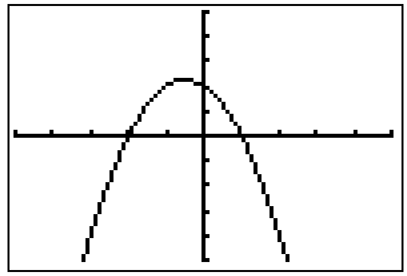
domain: $\{x \mid x \in \mathrm{R}\}$,
range: $\{y \mid y \le 0.25, y \in \mathrm{R}\}$

b) $y = (f - g)(x)$

$y = 1 - x^2 - (x - 1)$

$y = 1 - x^2 - x + 1$

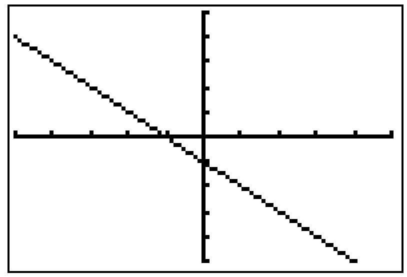
$y = 2 - x^2 - x$



domain: $\{x \mid x \in \mathrm{R}\}$,
range: $\{y \mid y \leq 2.25, y \in \mathrm{R}\}$

c) $y = \left(\frac{f}{g}\right)(x)$

$y = \frac{1 - x^2}{x - 1}$



Restrictions on domain and range:

$x - 1 \neq 0$

$x \neq 1$

$$y = \frac{1 - x^2}{x - 1}$$

$$y = \frac{(1 - x)(1 + x)}{x - 1} \times \frac{-1}{-1}$$

$$y = \frac{\overset{1}{\cancel{(1 - x)}}(-1 - x)}{\underset{1}{\cancel{-x + 1}}}$$

$$y = -1 - x$$

$$y = -1 - 1$$

$$y = -2$$

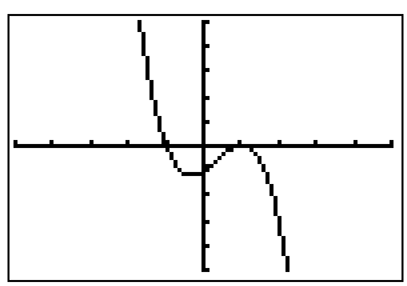
domain: $\{x \mid x \neq 1, x \in \mathrm{R}\}$, range: $\{y \mid y \neq -2, y \in \mathrm{R}\}$

d) $y = (f \cdot g)(x)$

$y = (1 - x^2)(x - 1)$

$y = x - x^3 - 1 + x^2$ or

$y = -x^3 + x^2 + x - 1$



domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$

Unit 4 Test Page 553 Question 16

$f(x) = x - 3,\ g(x) = \sqrt{x-1}$

a) $h(x) = f(x) + g(x)$

$h(x) = x - 3 + \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

b) $h(x) = f(x) - g(x)$

$h(x) = x - 3 - \sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

c) $h(x) = \left(\dfrac{f}{g}\right)(x)$

$h(x) = \dfrac{x-3}{\sqrt{x-1}}$

$\sqrt{x-1} > 0$

$x - 1 > 0$

$x > 1$

domain: $\{x \mid x > 1, x \in \mathrm{R}\}$

d) $h(x) = (f \cdot g)(x)$

$h(x) = (x-3)\sqrt{x-1}$

$x - 1 \geq 0$

$x \geq 1$

domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

Unit 4 Test Page 553 Question 17

a) $g(x) = |x|$

$g(2) = |2|$

$g(2) = 2$

$f(g(2)) = f(2)$

$f(g(2)) = 2^2 - 3$

$f(g(2)) = 4 - 3$

$f(g(2)) = 1$

b) $(f \circ g)(2) = f(g(2))$

From part a), $(f \circ g)(2) = 1$.

c) $f(g(x)) = f(|x|)$

$f(g(x)) = |x|^2 - 3$

$f(g(x)) = x^2 - 3$

d) $(g \circ f)(x) = g(f(x))$

$(g \circ f)(x) = g(x^2 - 3)$

$(g \circ f)(x) = |x^2 - 3|$

Unit 4 Test Page 553 Question 18

a) $f(g(x)) = 2^{3x+2}$

Let $g(x)$ represent the inner function.

$g(x) = 3x + 2$

Therefore, $f(x) = 2^x$.

b) $f(g(x)) = \sqrt{\sin x + 2}$

Let $g(x)$ represent the inner function.

$g(x) = \sin x + 2$

Therefore, $f(x) = \sqrt{x}$.

Unit 4 Test Page 553 Question 19

a)

$$\frac{n!}{(n-2)!} = 420$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}} = 420$$

$$n(n-1) = 420$$

$$n(n-1) = 21(20)$$

By inspection, $n = 21$.

b)

$${}_nC_2 = 78$$

$$\frac{n!}{(n-2)!2!} = 78$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{\cancel{(n-2)!}_{1}(2)(1)} = 78$$

$$\frac{n(n-1)}{2} = 78$$

$$n(n-1) = 156$$

$$n(n-1) = 13(12)$$

By inspection, $n = 13$.

c)

$${}_nC_{n-2} = 45$$

$$\frac{n!}{(n-(n-2))!(n-2)!} = 45$$

$$\frac{n!}{(n-n+2)!(n-2)!} = 45$$

$$\frac{n!}{2!(n-2)!} = 45$$

$$\frac{n(n-1)\,\cancel{(n-2)!}^{1}}{2(1)\,\cancel{(n-2)!}_{1}} = 45$$

$$\frac{n(n-1)}{2} = 45$$

$$n(n-1) = 90$$

$$n(n-1) = 10(9)$$

By inspection, $n = 10$.

Unit 4 Test Page 553 Question 20

a) There are 4! = 24 arrangements possible without repeating letters.

b) With repeating letters, there are $4^4 = 256$ possible arrangements. Therefore, there are 256 – 24 = 232 more arrangements.

c) No, there are fewer ways. Because the letter C is repeated, half of the arrangements will be repeats.

Unit 4 Test   Page 553   Question 21

a) The sub-committee will have 5 – 3 = 2 boys.

$$
\begin{aligned}
{}_4C_2 \times {}_5C_3 &= \frac{4!}{(4-2)!2!} \times \frac{5!}{(5-3)!3!} \\
&= \frac{4!}{2!2!} \times \frac{5!}{2!3!} \\
&= \frac{4(3)\cancel{(2!)}}{2(1)\cancel{(2)!}} \times \frac{5(4)\cancel{(3!)}}{2(1)\cancel{(3)!}} \\
&= 60
\end{aligned}
$$

There are 60 ways the sub-committee can be formed.

b) Case 1: Sub-committee has three girls
From part a), 60 arrangements

Case 2: Sub-committee has four girls
The sub-committee will have 5 – 4 = 1 boy.

$$
\begin{aligned}
{}_4C_1 \times {}_5C_4 &= \frac{4!}{(4-1)!1!} \times \frac{5!}{(5-4)!4!} \\
&= \frac{4!}{3!(1)} \times \frac{5!}{1!4!} \\
&= \frac{4\cancel{(3!)}}{\cancel{(3)!}} \times \frac{5\cancel{(4!)}}{1\cancel{(4)!}} \\
&= 20
\end{aligned}
$$

Case 3: Sub-committee has five girls
The sub-committee has zero boys.

$$
\begin{aligned}
{}_4C_0 \times {}_5C_5 &= \frac{4!}{(4-0)!0!} \times \frac{5!}{(5-5)!5!} \\
&= \frac{4!}{4!(1)} \times \frac{5!}{0!5!} \\
&= 1
\end{aligned}
$$

Total possibilities: 60 + 20 + 1 = 81

Unit 4 Test Page 553 Question 22

$(x + y)^7 = {}_7C_0(x)^7(y)^0 + {}_7C_1(x)^6(y)^1 + {}_7C_2(x)^5(y)^2 + {}_7C_3(x)^4(y)^3 + {}_7C_4(x)^3(y)^4 + {}_7C_5(x)^2(y)^5 + {}_7C_6(x)^1(y)^6 + {}_7C_7(x)^0(y)^7$

The term that contains x^5 is ${}_7C_2(x)^5(y)^2$.

$${}_7C_2(x)^5(y)^2 = 81\,648x^5$$
$$21(3x)^5(a)^2 = 81\,648x^5$$
$$21(3^5x^5)a^2 = 81\,648x^5$$
$$21(243x^5)a^2 = 81\,648x^5$$
$$a^2 = \frac{81\,648\,\cancel{x^5}^{1}}{21(243\,\cancel{x^5}_{1})}$$
$$a^2 = 16$$
$$a = \pm 4$$