Fractional Exponents

$4^{\frac{1}{2}}$

Base = 4 exponent = $\frac{1}{2}$

Fractional Exponent?

$$4^{0.5} = 2$$

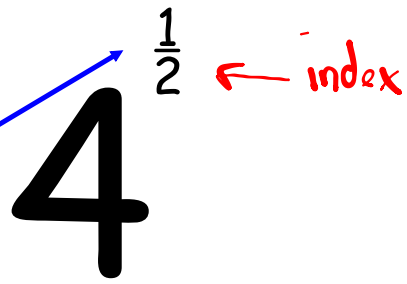
on calculator:

[^] or [y^x] or [xy]

Fractional Exponents

to

Radicals



Fractional Exponents

to

Radicals

$8^{\frac{1}{3}}$ ← index

$\sqrt[3]{8}$

$= \sqrt[3]{2 \times 2 \times 2}$

$= 2$

Fractional Exponents

to

Radicals

$81^{\frac{1}{4}}$ ← index

$\sqrt[4]{81}$

$= \sqrt[4]{3 \times 3 \times 3 \times 3}$

$= 3$

What if...

Fractional Exponents

to

Radicals

$$125^{\frac{2}{3}}$$ (exp. = 2, index = 3)

$$= \left(\sqrt[3]{125}\right)^2$$

$$= \left(\sqrt[3]{5 \times 5 \times 5}\right)^2$$

$$= (5)^2$$

$$= 5 \times 5$$

$$= \boxed{25}$$

Express the radical as a power.

$\sqrt[3]{15}^{1}$ $\sqrt[2]{25}^{7}$ $\sqrt[5]{9}^{2}$

$= 15^{\frac{1}{3}}$ $= 25^{\frac{7}{2}}$ $= 9^{\frac{2}{5}}$

Index → denominator

exponent → numerator

radicand → base

Making Connections

$100(0.87)^{1/2}$

Coffee, Tea, and Hot Chocolate contain caffeine.

The expression $100(0.87)^{1/2}$

represents the percent of caffeine left in your body 1/2 hour after you drink a caffeine beverage

How can you estimate the value of $0.87^{1/2}$

Let's Take a Closer Look!!

Fill in the chart. (You can use your calculator!!)

x	$x^{\frac{1}{2}}$
1	$1^{\frac{1}{2}} = \sqrt{1} = 1$
4	$4^{\frac{1}{2}} = \sqrt{4} = 2$
9	$9^{\frac{1}{2}} = \sqrt{9} = 3$
16	$16^{\frac{1}{2}} = \sqrt{16} = 4$
25	$25^{\frac{1}{2}} = \sqrt{25} = 5$

x	$x^{\frac{1}{3}}$
1	$1^{\frac{1}{3}} = \sqrt[3]{1} = 1$
8	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
27	$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$
64	$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
125	$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

What do you notice?

Our Conclusion

- Raising a number to an exponent of 1/2 is equivalent to taking the square root!
- Raising a number to an exponent of 1/3 is equivalent to taking the cube root!

* $x^{1/n} = \sqrt[n]{x}$

Practice Questions

Calculate each of the following without using a calculator:

$27^{1/3}$

$= \sqrt[3]{27}$

$= \sqrt[3]{3 \times 3 \times 3}$

$= 3$

$100^{1/2}$

$= \sqrt{100}$

$= \sqrt{2 \times 2 \times 5 \times 5}$

$= 2 \times 5$

$= 10$

$16^{1/4}$

$= \sqrt[4]{16}$

$= \sqrt[4]{2 \times 2 \times 2 \times 2}$

$= 2$

Calculate each of the following without using a calculator:

$36^{0.5}$

$= 36^{1/2}$

$= \sqrt{36}$

$= \sqrt{2 \times 2 \times 3 \times 3}$

$= 2 \times 3$

$= 6$

$32^{0.2}$

$= 32^{2/10}$

$= 32^{1/5}$

$= \sqrt[5]{32}$

$= \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

$= 2$

$625^{0.25}$

$= 625^{25/100}$

$= 625^{1/4}$

$= \sqrt[4]{625}$

$= \sqrt[4]{5 \times 5 \times 5 \times 5}$

$= 5$

Calculate each of the following without using a calculator:

$4^{3/2}$

$= (\sqrt{4})^3$

$27^{2/3}$

$= (\sqrt[3]{27})^2$

Therefore:

$$x^{m/n} = (\sqrt[n]{x})^m$$

Write as a power:

$$\left(\sqrt[4]{625}\right)^{9} = 625^{\frac{9}{4}}$$

radicand = base

index = denominator

exponent = numerator

Calculate the following without using a calculator:

$128^{1/7}$

$= \left(\sqrt[7]{128}\right)^{1}$

$= \left(\sqrt[7]{2\times2\times2\times2\times2\times2\times2}\right)^{1}$

$= (2)^{1}$

$= 2$

$343^{2/3}$

$= \left(\sqrt[3]{343}\right)^{2}$

$= \left(\sqrt[3]{7\times7\times7}\right)^{2}$

$= (7)^{2}$

$= 7\times7$

$= 49$

Calculate the following without using a calculator:

$289^{3/2}$

$= \left(\sqrt{289}\right)^3$

$= \left(\sqrt{17 \times 17}\right)^3$

$= (17)^3$

$= 17 \times 17 \times 17$

$= 4913$

$625^{3/4}$

$= \left(\sqrt[4]{625}\right)^3$

$= \left(\sqrt[4]{5 \times 5 \times 5 \times 5}\right)^3$

$= (5)^3$

$= 5 \times 5 \times 5$

$= 125$

Stop....

Check out page 227.

Questions:

5, 6,

7a,b, f

8,

10a,c,f,

11, 15

Anything to the power of 0 equals 1

Ex: $7^0 = 1$

$15^0 = 1$

$(-3)^0 = 1$

$\left(\frac{1}{8}\right)^0 = 1$

(10) c) $\left(\frac{3}{8}\right)^{2.5}$

$= \left(\frac{3}{8}\right)^{2\frac{1}{2}}$

$= \left(\frac{3}{8}\right)^{\frac{5}{2}}$

$= \left(\sqrt[2]{\frac{3}{8}}\right)^5$

Check out page 227 of your text book.

Questions:

5, 6, 7a,b, f 8,10a,c,f,11

To **multiply powers** with the same base you **add.**

$$a^m \times a^n = a^{m+n}$$

Examples:

x $5^2 = 5^5$

x $8^2 = 8^7$

x $4^2 = 4^6$

$$5^{1/2} \times 5^{1/2} = 5$$

$1/2 + 1/2 = 2/2 = 1$

This can also be written like:

$$\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5$$

$$2^{1/3} \times 2^{1/3} \times 2^{1/3} = 2$$

$1/3 + 1/3 + 1/3 = 3/3$
$= 1$

This can also be written like:

$$\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = \sqrt[3]{8}$$
$$= 2$$