

# Radical Functions and Transformations

## Focus on...

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- investigating the function  $y = \sqrt{x}$  using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

## radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$  and  $y = 4\sqrt[3]{5+x}$  are radical functions.

**Example 1**

**Graph Radical Functions Using Tables of Values**

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

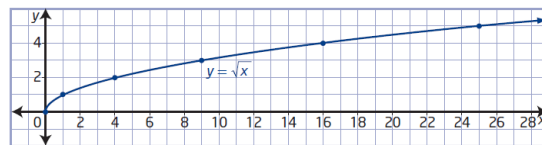
- a)  $y = \sqrt{x}$       b)  $y = \sqrt{x-2}$       c)  $y = \sqrt{x} - 3$

- a) For the function  $y = \sqrt{x}$ , the radicand  $x$  must be greater than or equal to zero,  $x \geq 0$ .

D:  $x \geq 0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of  $x$  that allow you to complete the table without using a calculator?



The graph has an endpoint at  $(0, 0)$  and continues up and to the right. The domain is  $\{x \mid x \geq 0, x \in \mathbb{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbb{R}\}$ .

$[0, \infty)$

- b) For the function  $y = \sqrt{x-2}$ , the value of the radicand must be greater than or equal to zero.

D:  $x-2 \geq 0$   
 $x \geq 2$

$h=2$

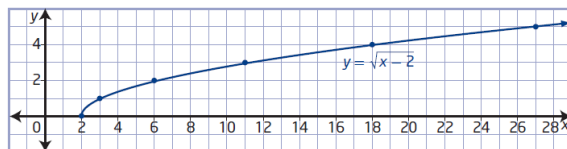
$(x,y) \rightarrow (x+2,y)$

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for  $y = \sqrt{x}$  in part a)?

translated 2 units right

How does the graph of  $y = \sqrt{x-2}$  compare to the graph of  $y = \sqrt{x}$ ?



The domain is  $\{x \mid x \geq 2, x \in \mathbb{R}\}$ . The range is  $\{y \mid y \geq 0, y \in \mathbb{R}\}$ .

$[2, \infty)$

$[0, \infty)$

- c) The radicand of  $y = \sqrt{x} - 3$  must be non-negative.

$k=-3$

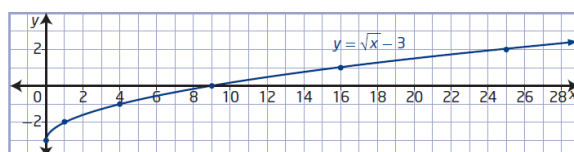
D:  $x \geq 0$

$(x,y) \rightarrow (x, y-3)$

x	y
0	-3
1	-2
4	-1
9	0
16	1
25	2

translated 3 units down

How does the graph of  $y = \sqrt{x} - 3$  compare to the graph of  $y = \sqrt{x}$ ?



The domain is  $\{x \mid x \geq 0, x \in \mathbb{R}\}$  and the range is  $\{y \mid y \geq -3, y \in \mathbb{R}\}$ .

$[0, \infty)$

$[-3, \infty)$

### Graphing Radical Functions Using Transformations

You can graph a radical function of the form  $y = a\sqrt{b(x - h)} + k$  by transforming the graph of  $y = \sqrt{x}$  based on the values of  $a$ ,  $b$ ,  $h$ , and  $k$ . The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter  $a$  results in a vertical stretch of the graph of  $y = \sqrt{x}$  by a factor of  $|a|$ . If  $a < 0$ , the graph of  $y = \sqrt{x}$  is reflected in the  $x$ -axis.
- Parameter  $b$  results in a horizontal stretch of the graph of  $y = \sqrt{x}$  by a factor of  $\frac{1}{|b|}$ . If  $b < 0$ , the graph of  $y = \sqrt{x}$  is reflected in the  $y$ -axis.
- Parameter  $h$  determines the horizontal translation. If  $h > 0$ , the graph of  $y = \sqrt{x}$  is translated to the right  $h$  units. If  $h < 0$ , the graph is translated to the left  $|h|$  units.
- Parameter  $k$  determines the vertical translation. If  $k > 0$ , the graph of  $y = \sqrt{x}$  is translated up  $k$  units. If  $k < 0$ , the graph is translated down  $|k|$  units.

$$y = a\sqrt{b(x-h)} + k$$

$$(x,y) \rightarrow \left(\frac{1}{b}x+h, ay+k\right)$$

## Example 2

### Graph Radical Functions Using Transformations

Sketch the graph of each function using transformations. Compare the domain and range to those of  $y = \sqrt{x}$  and identify any changes.

a)  $y = 3\sqrt{-(x - 1)}$

b)  $y - 3 = -\sqrt{2x}$

a)  $y = \underline{3}\sqrt{\underline{-}(x-1)}$        $y = a\sqrt{b(x-h)} + k$

$a=3 \rightarrow$  vertical stretch by a factor of 3.

$b=-1 \rightarrow$  no horizontal stretch but there is a horizontal reflection in the y-axis

$h=1 \rightarrow$  translate 1 unit to the right.

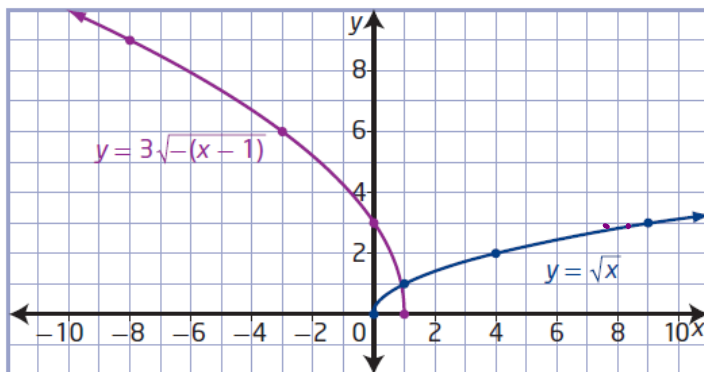
$k=0 \rightarrow$  no vertical translation

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \rightarrow \left( \frac{1}{-1}x+1, 3y+0 \right)$

x	y
1	0
0	3
-3	6
-8	9
-15	12
-24	15



D:  $-(x-1) \geq 0$

$-x+1 \geq 0$

$-x \geq -1$

$x \leq 1$

$\{x | x \leq 1, x \in \mathbb{R}\}$

$(-\infty, 1]$

R:  $\{y | y \geq 0, y \in \mathbb{R}\}$

$[0, \infty)$

$$b) y - 3 = -\sqrt{2x}$$

$$y = \underline{\underline{-\sqrt{2x}}} + \underline{\underline{3}}$$

$a = -1 \rightarrow$  no vertical stretch but vertical reflection in x-axis

$b = 2 \rightarrow$  horizontal stretch by a factor of  $\frac{1}{2}$ .

$h = 0 \rightarrow$  no horizontal translation

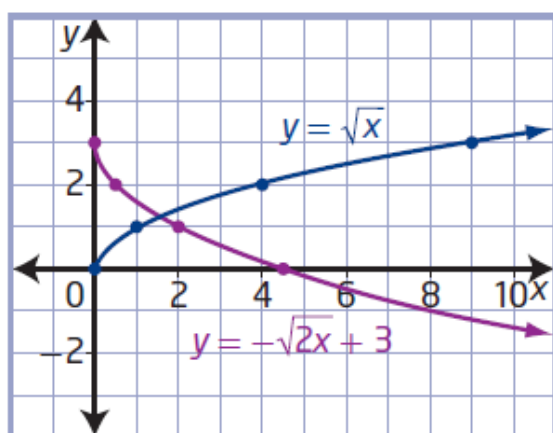
$k = 3 \rightarrow$  translated 3 units up.

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$y = \sqrt{x}$

$(x, y) \rightarrow (\frac{1}{2}x + 0, -|y + 3|)$

x	y
0	3
0.5	2
2	1
4.5	0
8	-1
12.5	-2



$$D: \begin{aligned} &2x \geq 0 \\ &x \geq 0 \end{aligned}$$

$$\{x \mid x \geq 0, x \in \mathbb{R}\}$$

$$[0, \infty)$$

$$R: \{y \mid y \leq 3, y \in \mathbb{R}\}$$

$$(-\infty, 3]$$

# Homework

#2-5 on page 72-73



5. Sketch the graph of each function using transformations. State the domain and range of each function.

a)  $f(x) = \sqrt{-x} - 3$

b)  $r(x) = 3\sqrt{x+1}$

c)  $p(x) = -\sqrt{x-2}$

d)  $y - 1 = -\sqrt{-4(x-2)}$

e)  $m(x) = \sqrt{\frac{1}{2}x + 4}$

f)  $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

$$y = \left(\frac{1}{3}\right)\sqrt{-(x+2)} - 1 \quad y = a\sqrt{b(x-h)} + k$$

$a = \frac{1}{3} \rightarrow$  vertical stretch by a factor of  $\frac{1}{3}$

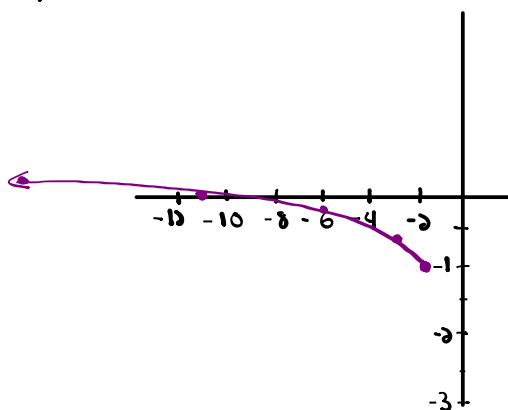
$b = -1 \rightarrow$  no horizontal stretch but there is a horizontal reflection in the y-axis

$h = -2 \rightarrow$  horizontal translation 2 units left.

$k = -1 \rightarrow$  vertical translation 1 unit down

$y = \sqrt{x}$	
x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \rightarrow \left[\frac{1}{3}x-2, \frac{1}{3}y-1\right]$	
x	y
-2	-1
-3	$-\frac{2}{3}$ or $-0.\bar{6}$
-6	$-\frac{1}{3}$ or $-0.\bar{3}$
-11	0
-18	$\frac{1}{3}$ or $0.\bar{3}$
-27	$\frac{2}{3}$ or $0.\bar{6}$



D:  $\{x \mid x \leq -2, x \in \mathbb{R}\}$   
or  $(-\infty, -2]$

R:  $\{y \mid y \geq -1, y \in \mathbb{R}\}$   
or  $[-1, \infty)$

5. Sketch the graph of each function using transformations. State the domain and range of each function.

- a)  $f(x) = \sqrt{-x} - 3$
- b)  $r(x) = 3\sqrt{x+1}$
- c)  $p(x) = -\sqrt{x-2}$
- d)  $y - 1 = -\sqrt{-4(x-2)}$
- e)  $m(x) = \sqrt{\frac{1}{2}x + 4}$
- f)  $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

d)  $y - 1 = -\sqrt{-4(x-2)}$        $y = a\sqrt{b(x-h)} + k$   
 $y = \underline{-1}\sqrt{\underline{-4}(x-\underline{2})} + \underline{1}$

$a = -1 \rightarrow$  no vertical stretch but there is a vertical reflection in the x-axis.

$b = -4 \rightarrow$  a horizontal stretch by a factor of  $\frac{1}{4}$  and a horizontal reflection in the y-axis

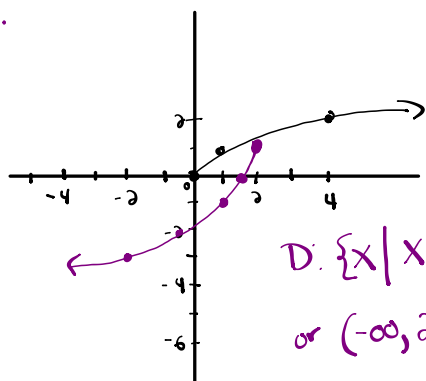
$h = 2 \rightarrow$  a horizontal translation 2 units right

$k = 1 \rightarrow$  a vertical translation 1 unit up

$y = \sqrt{x}$        $(x, y) \rightarrow \left(\frac{1}{-4}x + 2, -1y + 1\right)$

x	y
0	0
1	1
4	2
9	3
16	4

x	y
2	1
(1.75) 24	0
1	-1
(-0.25) -1/4	-2
-2	-3



$D: \{x \mid x \leq 2, x \in \mathbb{R}\}$

or  $(-\infty, 2]$

$R: \{y \mid y \leq 1, y \in \mathbb{R}\}$

or  $(-\infty, 1]$

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a)  $f(x) = \sqrt{-x} - 3$

b)  $r(x) = 3\sqrt{x+1}$

c)  $p(x) = -\sqrt{x-2}$

d)  $y - 1 = -\sqrt{-4(x-2)}$

e)  $m(x) = \sqrt{\frac{1}{2}x} + 4$

f)  $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

e)  $m(x) = \sqrt{\frac{1}{2}x} + 4$

$a=1 \rightarrow$  no vertical stretch or reflection

$b=\frac{1}{2} \rightarrow$  horizontal stretch by a factor of 2 and no horizontal reflection

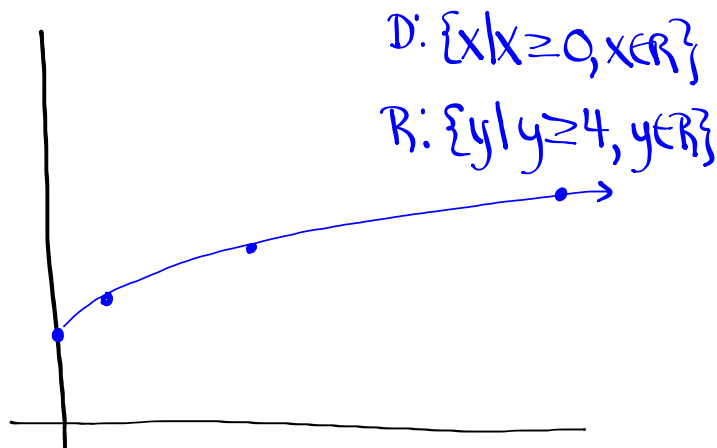
$h=0 \rightarrow$  no horizontal translation

$k=4 \rightarrow$  vertical translation 4 units up.

$$(x, y) \rightarrow (2x, y+4)$$

x	y
0	0
1	1
4	2
9	3

x	y
0	4
2	5
8	6
18	7



$$y - 4 = -2\sqrt{-3x - 9} + 4$$

$$y = -2\sqrt{-3x - 9} + 8$$

$$y = \underline{-2}\sqrt{\underline{-3}(x + \underline{3})} + \underline{8}$$

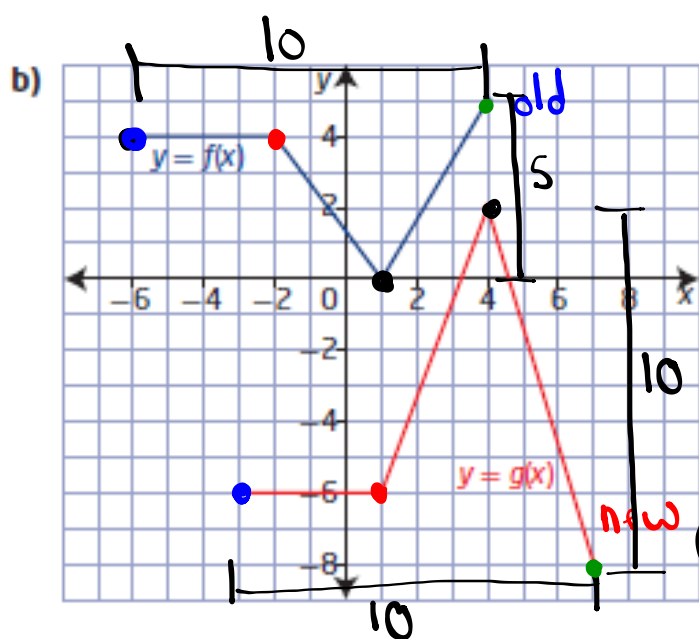
$$a = -2$$

$$b = -3$$

$$h = -3$$

$$k = 8$$

10. The graph of the function  $y = g(x)$  represents a transformation of the graph of  $y = f(x)$ . Determine the equation of  $g(x)$  in the form  $y = af(b(x - h)) + k$ .



① Reflection: vertical  
in x-axis  
( $a < 0$ )

② VSF:  $\frac{10}{5} = 2$   $\boxed{a = -2}$

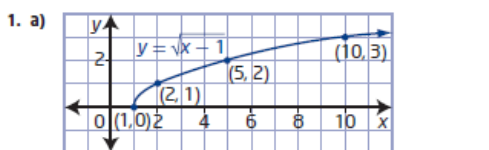
③ HSF:  $\frac{10}{10} = 1$   $\boxed{b = 1}$

④ VT:  $(1, 0) \rightarrow (4, -2)$   $\boxed{k = -2}$

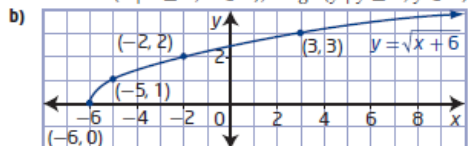
⑤ HT:  $(-6, 4) \rightarrow (-3, -6)$   $\boxed{h = 3}$

⑥  $y = -2f(1(x-3)) - 2$

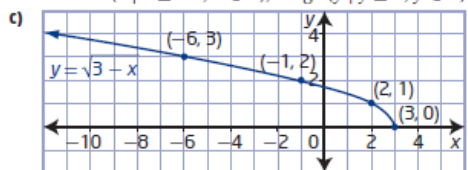
2.1 Radical Functions and Transformations, pages 72 to 77



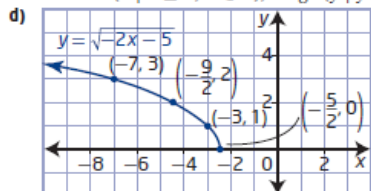
domain  $\{x \mid x \geq 1, x \in \mathbb{R}\}$ , range  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



domain  $\{x \mid x \geq -6, x \in \mathbb{R}\}$ , range  $\{y \mid y \geq 0, y \in \mathbb{R}\}$

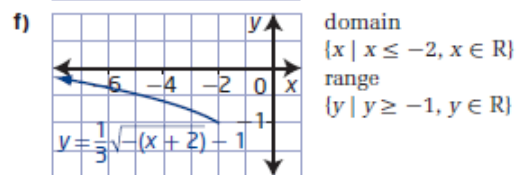
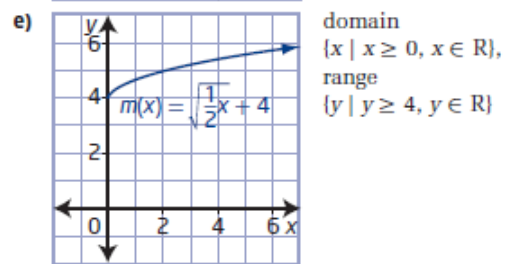
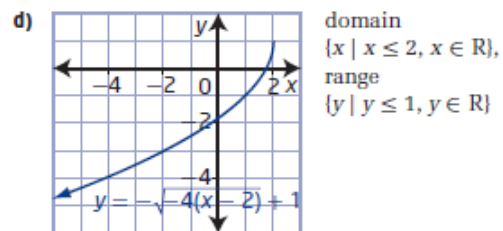
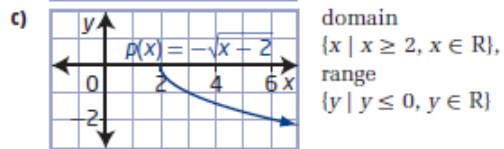
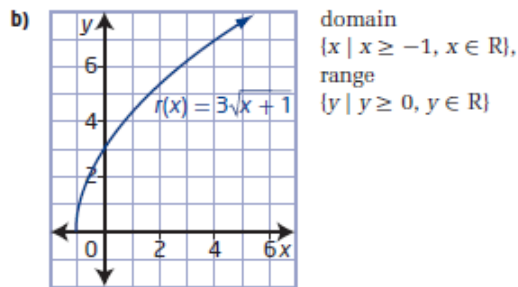
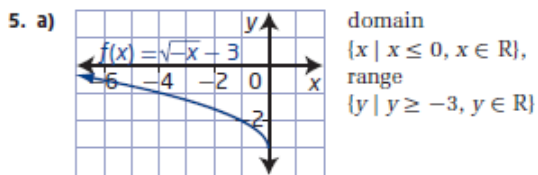


domain  $\{x \mid x \leq 3, x \in \mathbb{R}\}$ , range  $\{y \mid y \geq 0, y \in \mathbb{R}\}$



domain  $\{x \mid x \leq -\frac{5}{2}, x \in \mathbb{R}\}$ ,  
range  $\{y \mid y \geq 0, y \in \mathbb{R}\}$

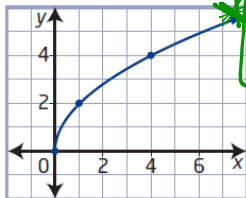
2. a)  $a = 7 \rightarrow$  vertical stretch by a factor of 7  
 $h = 9 \rightarrow$  horizontal translation 9 units right  
 domain  $\{x \mid x \geq 9, x \in \mathbb{R}\}$ , range  $\{y \mid y \geq 0, y \in \mathbb{R}\}$   
 b)  $b = -1 \rightarrow$  reflected in y-axis  
 $k = 8 \rightarrow$  vertical translation up 8 units  
 domain  $\{x \mid x \leq 0, x \in \mathbb{R}\}$ , range  $\{y \mid y \geq 8, y \in \mathbb{R}\}$   
 c)  $a = -1 \rightarrow$  reflected in x-axis  
 $b = \frac{1}{5} \rightarrow$  horizontal stretch factor of 5  
 domain  $\{x \mid x \geq 0, x \in \mathbb{R}\}$ , range  $\{y \mid y \leq 0, y \in \mathbb{R}\}$   
 d)  $a = \frac{1}{3} \rightarrow$  vertical stretch factor of  $\frac{1}{3}$   
 $h = -6 \rightarrow$  horizontal translation 6 units left  
 $k = -4 \rightarrow$  vertical translation 4 units down  
 domain  $\{x \mid x \geq -6, x \in \mathbb{R}\}$ ,  
 range  $\{y \mid y \geq -4, y \in \mathbb{R}\}$
3. a) B      b) A      c) D      d) C
4. a)  $y = 4\sqrt{x+6}$       b)  $y = \sqrt{8x} - 5$   
 c)  $y = \sqrt{-(x-4)} + 11$  or  $y = \sqrt{-x+4} + 11$   
 d)  $y = -0.25\sqrt{0.1x}$  or  $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$



**Example 3**

**Determine a Radical Function From a Graph**

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of  $y = \sqrt{x}$ . What are the equations of the four functions Mayleen needs to work with?



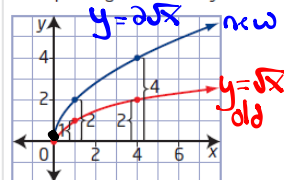
A radical function that involves a stretch can be obtained from either a vertical stretch or a horizontal stretch. Use an equation of the form  $y = a\sqrt{x}$  or  $y = \sqrt{bx}$  to represent the image function for each type of stretch

**Method 1: Compare Vertical or Horizontal Distances**

Superimpose the graph of  $y = \sqrt{x}$  and compare corresponding distances to determine the factor by which the function has been stretched.

**View as a Vertical Stretch ( $y = a\sqrt{x}$ )**

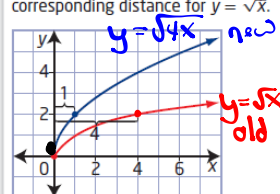
Each vertical distance is 2 times the corresponding distance for  $y = \sqrt{x}$ .



This represents a vertical stretch by a factor of 2, which means  $a = 2$ . The equation  $y = 2\sqrt{x}$  represents the function.

**View as a Horizontal Stretch ( $y = \sqrt{bx}$ )**

Each horizontal distance is  $\frac{1}{4}$  the corresponding distance for  $y = \sqrt{x}$ .



This represents a horizontal stretch by a factor of  $\frac{1}{4}$ , which means  $b = 4$ . The equation  $y = \sqrt{4x}$  represents the function.

Express the equation of the function as either  $y = 2\sqrt{x}$  or  $y = \sqrt{4x}$ .

① reflections: None

② VSF:  $\frac{2}{1} = 2$   $a=2$

③ HSF: Skip if  $y = \sqrt{x}$

④ HT:  $(0,0) \rightarrow (0,0)$   $h=0$

⑤ VT:  $(0,0) \rightarrow (0,0)$   $k=0$

⑥  $y = 2\sqrt{1(x-0)} + 0$

$y = 2\sqrt{x}$

① reflections: none

② HSF:  $\frac{1}{4}$   $b=4$

③ VSF: skip if  $y = \sqrt{x}$

④ HT:  $(0,0) \rightarrow (0,0)$   $h=0$

⑤ VT:  $(0,0) \rightarrow (0,0)$   $k=0$

⑥  $y = 1\sqrt{4(x-0)} + 0$

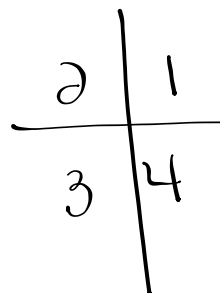
$y = \sqrt{4x}$

$y = 2\sqrt{x}$  (Quad 1)

$y = 2\sqrt{-x}$  (Quad 2)

$y = -2\sqrt{-x}$  (Quad 3)

$y = -2\sqrt{x}$  (Quad 4)





Homework  
#6, 9, 10  
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