Warm-Up

8. Copy and complete the table.

horizontal (change sign) ↓

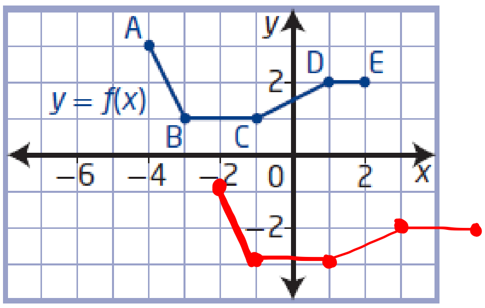
$y = f(x-h)+k$

↑ vertical

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \to (x, y + 5)$	$k=5$ (Up)
horizontal	$y = f(x \pm 7)$	$(x, y) \to (x - 7, y)$	$h=-7$ (Left)
horizontal	$y = f(x - 3)$	$(x,y) \to (x+3, y)$	$h=3$ (Right)
vertical	$y = f(x) - 6$	$(x,y) \to (x, y-6)$	$k=-6$ (Down)
horizontal and vertical	$y = f(x \pm 4) - 9$ $y + 9 = f(x + 4)$	$(x,y) \to (x-4, y-9)$	$h=-4$ Left, $k=-9$ Down
horizontal and vertical	$y = f(x-4)-6$	$(x, y) \to (x \pm 4, y - 6)$	$h=4$ Right, $k=-6$ Down
h + v	$y = f(x+2)+3$	$(x, y) \to (x - 2, y + 3)$	$h=-2$, $k=3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \to (x+h, y+k)$	

Questions from Homework

4.



b) $y = f(x-2) - 4$

$h = 2 \quad k = -4$

$(x, y) \rightarrow (x+2,\ y-4)$

$A(-4,3) \rightarrow (-2,-1)$

$B(-3,1) \quad (-1,-3)$

$C(-1,1) \quad (1,-3)$

$D(1,2) \quad (3,-2)$

$E(2,2) \quad (4,-2)$

Transformations:

New Functions From Old Functions

Translations

Stretches

✓Reflections

Reflections and Stretches

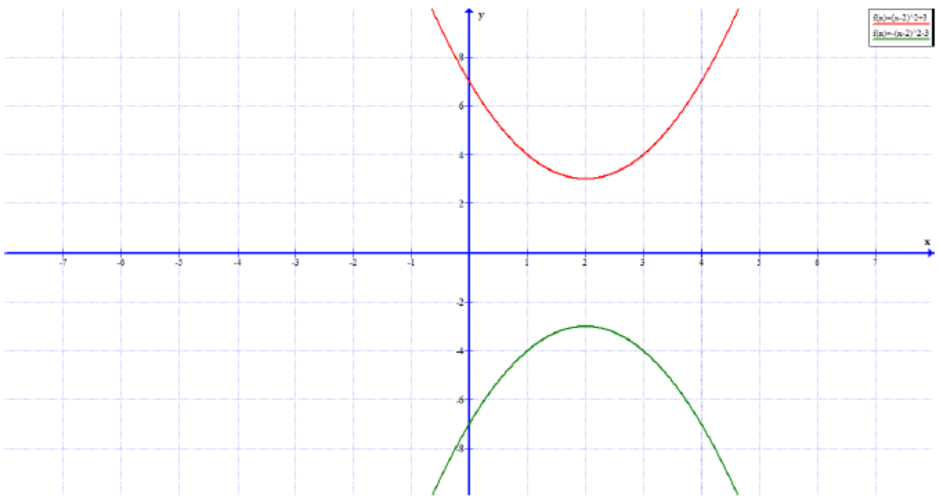
Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

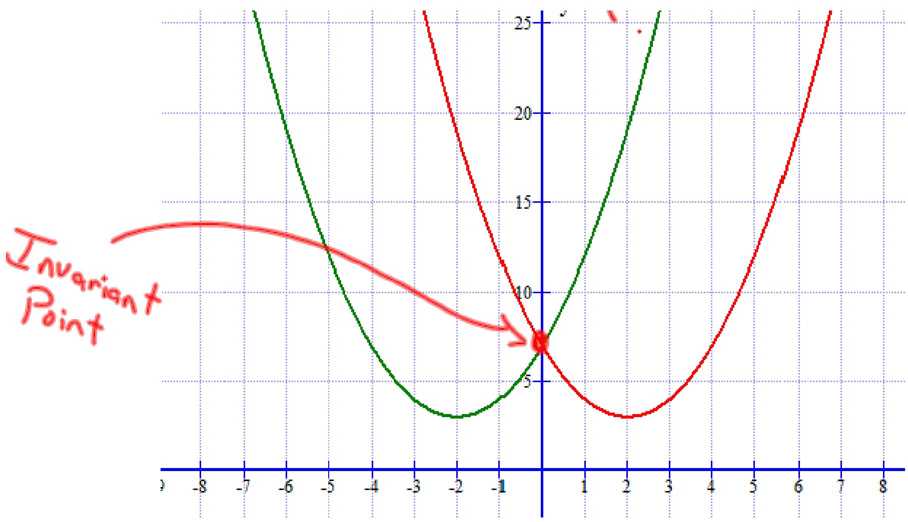
Vertical reflection $(x, y) \rightarrow (x, -y)$

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.



Horizontal Reflection $(x,y) \to (-x,y)$

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.



invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

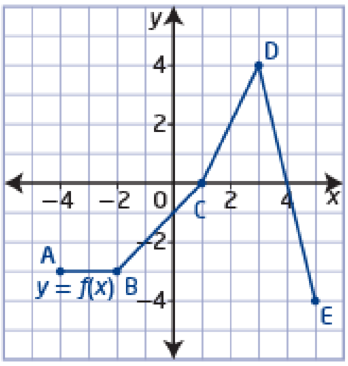
vertical horizontal

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?

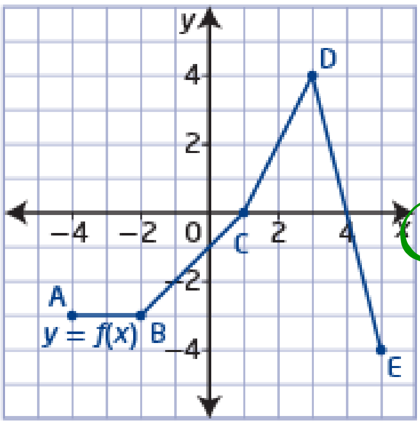


Remember...

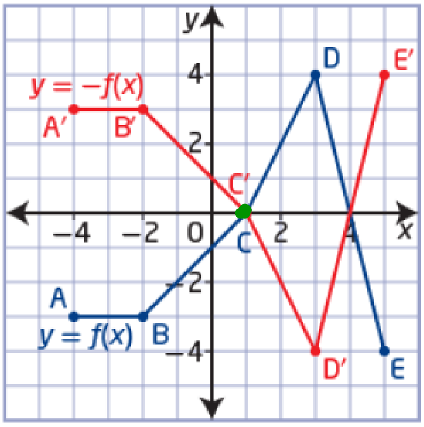
- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below (Vertical Reflection)

$(x, y) \rightarrow (x, -y)$



(-4,-3)	(-4,3)
(-2,-3)	(-2,3)
(1,0)	(1,0)
(3,4)	(3,-4)
(5,-4)	(5,4)



Invariant Point

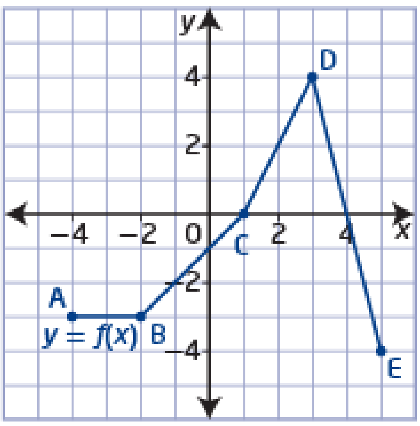
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

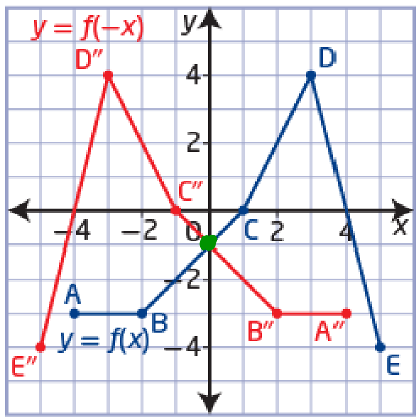
- Sketch $y = f(-x)$ on the axis below

Horizontal reflection

$(x, y) \rightarrow (-x, y)$



(-4,-3)	(4,-3)
(-2,-3)	(2,-3)
(1,0)	(-1,0)
(3,4)	(-3,4)
(5,-4)	(-5,-4)



Homework

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*$f(-4) = 2(-4)+1$
$= -8+1$
$= -7$

$f(x) = 2x+1$

	x	y
*	-4	-7
	-2	-3
	0	1
	2	5
	4	9

Vertical

$g(x) = -f(x)$

x	y
-4	7
-2	3
0	-1
2	-5
4	-9

Horizontal

$h(x) = f(-x)$

x	y
4	-7
2	-3
0	1
-2	5
-4	9

③ a) $f(x) = 3x$

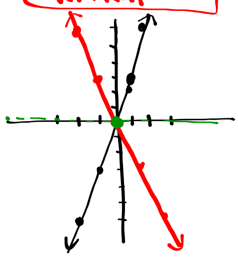
	x	y
	-2	-6
visible	-1	-3
visible	0	0
visible	1	3
	2	6

$f(x) = -3x$

$(x,y) \to (x,-y)$

reflection in x-axis is vertical

x	y
-2	6
-1	3
0	0
1	-3
2	-6



D: $\{x \mid x \in R\}$
R: $\{y \mid y \in R\}$

D: $\{x \mid x \in R\}$
R: $\{y \mid y \in R\}$

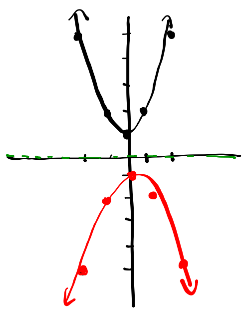
b) $g(x) = x^2+1$

x	y
-2	5
-1	2
0	1
1	2
2	5

$g(x) = -(x^2+1)$

$(x,y) \to (x,-y)$

x	y
-2	-5
-1	-2
0	-1
1	-2
2	-5



D: $\{x \mid x \in R\}$
R: $\{y \mid y \geq 1, y \in R\}$

D: $\{x \mid x \in R\}$
R: $\{y \mid y \leq 1, y \in R\}$

stretch

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

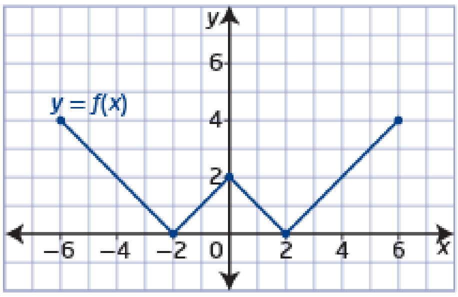
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

Graph $y = af(x)$

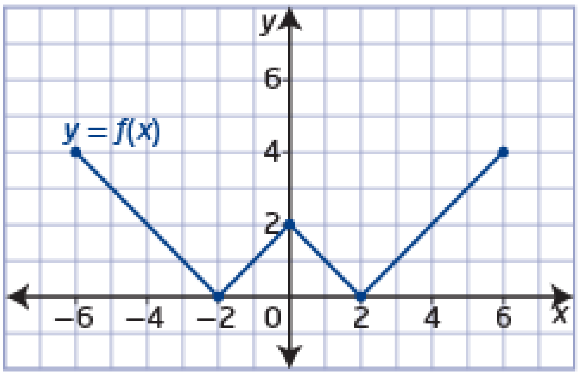
Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$

a) $g(x) = 2f(x)$



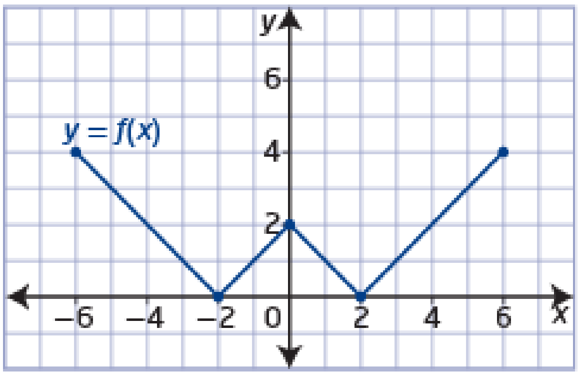
The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

b) $g(x) = \frac{1}{2}f(x)$



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The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

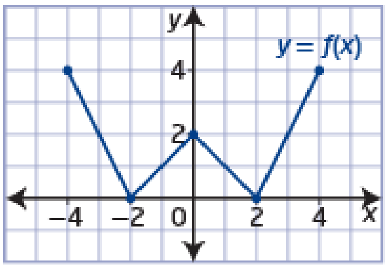
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

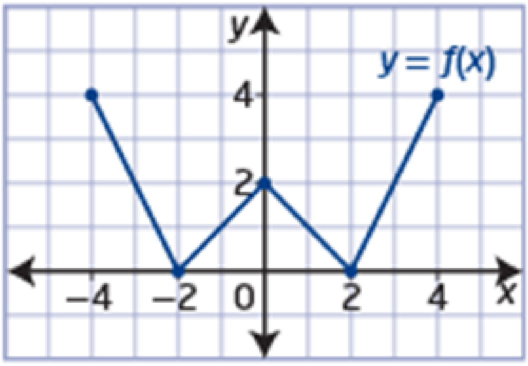
Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$



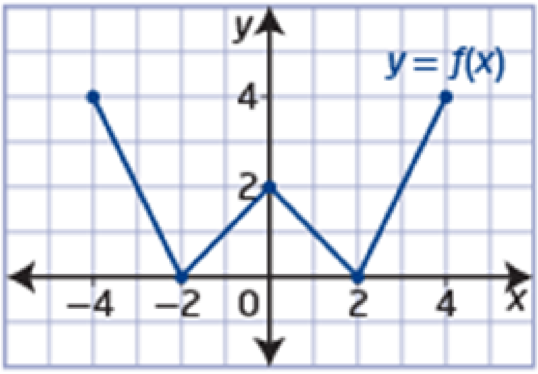
[redacted]

The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or and the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



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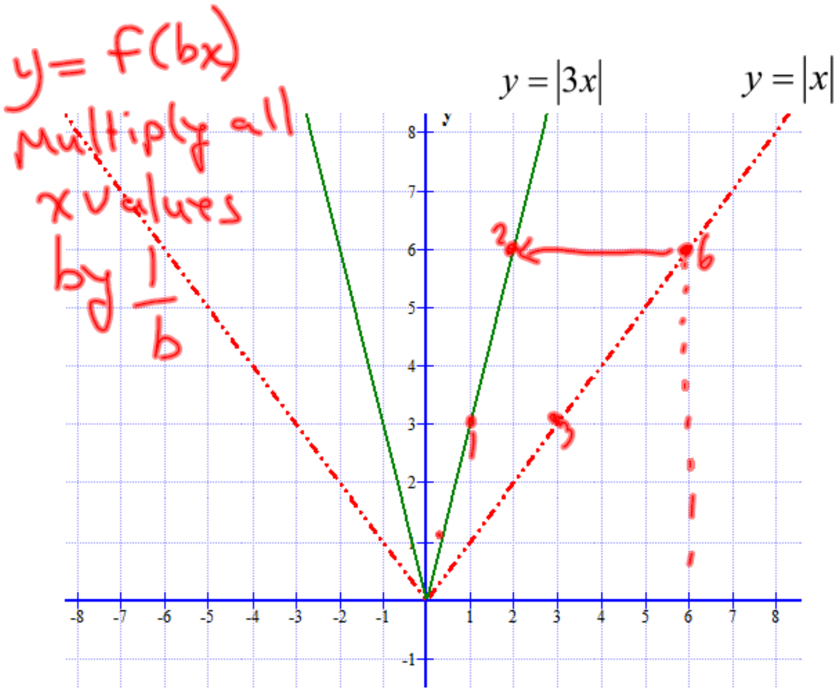
The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



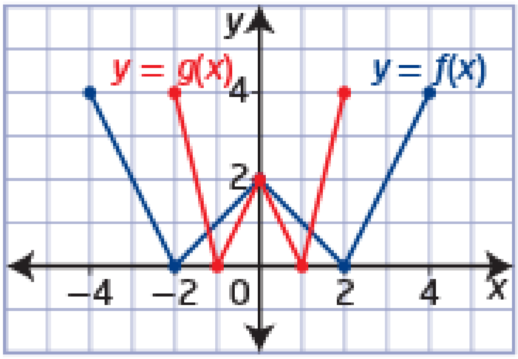
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

Homework

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x