10.9

3 c) $81+27+9+\ldots$ $S_6=\dfrac{(81)\left[\left(\frac{1}{3}\right)^6-1\right]}{\left(\frac{1}{3}\right)-1}$

$S_6=?$
$a=81$
$r=\frac{1}{3}$

$$S_6=\frac{81\left[\frac{1}{729}-\frac{1}{1}\right]}{\frac{1}{3}-\frac{1}{1}}$$

$$S_6=\frac{81\left[\frac{1}{729}-\frac{729}{729}\right]}{\frac{1}{3}-\frac{3}{3}}$$

$$S_6=81\left(-\frac{728}{729}\right)\div-\frac{2}{3}$$

$$S_6=81\left(-\frac{728}{729}\right)\left(-\frac{3}{2}\right)$$

$$S_6=\frac{176904}{1458}=\frac{364}{3}$$

5 b) $1+\frac{5}{2}+\frac{25}{4}+\ldots+\frac{15625}{64}$

$a=1$
$r=\frac{5}{2}$
$t_n=\frac{15625}{64}$

① Find n:

$t_n=ar^{n-1}$

$$\frac{\frac{15625}{64}}{1}=\frac{(\cancel{1})\left(\frac{5}{2}\right)^{n-1}}{\cancel{1}}$$

$$\frac{15625}{64}=\left(\frac{5}{2}\right)^{n-1}$$

$$\frac{\log\left(\frac{15625}{64}\right)}{\log\left(\frac{5}{2}\right)}=6$$

$$\left(\frac{5}{2}\right)^6=\left(\frac{5}{2}\right)^{n-1}$$

$$6\,{}^{+1}=n-1\,{}^{+1}$$

$$\boxed{7=n}$$

② Find S_7

$$S_7=\frac{(1)\left[\left(\frac{5}{2}\right)^7-1\right]}{\left(\frac{5}{2}\right)-1}$$

$$S_7=\frac{1\left[\frac{78125}{128}-\frac{1}{1}\right]}{\frac{5}{2}-\frac{1}{1}}$$

$$S_7=\frac{1\left[\frac{78125}{128}-\frac{128}{128}\right]}{\frac{5}{2}-\frac{2}{2}}$$

$$S_7=1\left(\frac{77997}{128}\right)\div\frac{3}{2}$$

$$S_7=1\left(\frac{77997}{128}\right)\left(\frac{2}{3}\right)$$

$$S_7=\frac{155994}{384}=\boxed{\frac{25999}{64}}$$

6 $S_7=1093$ $S_7=\dfrac{a\left[\left(\frac{1}{3}\right)^7-1\right]}{\left(\frac{1}{3}\right)-1}$

$r=\frac{1}{3}$
$a=?$

$$1093=\frac{a\left[\frac{1}{2187}-\frac{2187}{2187}\right]}{\frac{1}{3}-\frac{3}{3}}$$

$$1093=a\left[-\frac{2186}{2187}\right]\div-\frac{2}{3}$$

$$1093=a\left(-\frac{2186}{2187}\right)\left(-\frac{3}{2}\right)$$

$$4374\cdot 1093=\frac{6558a}{\cancel{4374}}\cdot\cancel{4374}$$

$$\frac{4780782}{6558}=\frac{6558a}{6558}$$

$$\boxed{729=a}$$

① b) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

$a = 2$

$r = \frac{2}{3} \div \frac{2}{1}$

$= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}}$

$= \frac{1}{3}$

$n = 4$

$$S_4 = \frac{(2)\left[\left(\frac{1}{3}\right)^4 - 1\right]}{\left[\left(\frac{1}{3}\right) - 1\right]}$$

$$= \frac{2\left[\frac{1}{81} - \frac{81}{81}\right]}{\left[\frac{1}{3} - \frac{3}{3}\right]}$$

$$= \frac{2\left(-\frac{80}{81}\right)}{\left(\frac{-2}{3}\right)}$$

$$= \cancel{2}^{1}\left(\frac{-80}{\cancel{81}_{27}}\right)\left(-\frac{\cancel{3}^{1}}{\cancel{2}_{1}}\right)$$

$$= \frac{480}{162}$$

$$= \frac{80}{27}$$

⑦ Given:

$t_7 = 192$

$a = 3$

$S_8 = ?$

(1) Find r:

$t_n = ar^{n-1}$

$192 = (3)r^{7-1}$

$\frac{192}{3} = \frac{3r^6}{3}$

$64 = r^6$

$\pm 2 = r$

if $r = 2$

$S_8 = \frac{(3)[(2)^8 - 1]}{(2-1)}$

$= \frac{3(256-1)}{1}$

$= 3(255)$

$= 765$

if $r = -2$

$S_8 = \frac{(3)[(-2)^8 - 1]}{(-2-1)}$

$= \frac{3(256-1)}{-3}$

$= \frac{\cancel{3}(255)}{-\cancel{3}}$

$= -255$

② b) $16 + 8 + 4 + 2 + \dots + \frac{1}{32}$

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

$a = 16$

$r = \frac{1}{2}$

$t_n = \frac{1}{32}$

(1) Find n:

$t_n = ar^{n-1}$

$\frac{1}{32} \div 16 = (16)\left(\frac{1}{2}\right)^{n-1} \div 16$

$\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\cancel{\frac{1}{2}}\right)^{9} = \left(\cancel{\frac{1}{2}}\right)^{n-1}$ $\quad \frac{\log \frac{1}{512}}{\log \frac{1}{2}} = 9$

$9 = n - 1$

$10 = n$

⑤ a)

$S_6 = 1365$

$r = \frac{1}{4}$

$n = 6$

$a = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1365 = \frac{a\left(\frac{1}{4}^6 - 1\right)}{\frac{1}{4} - 1}$$

$$1365 = \frac{a\left(\frac{1}{4096} - \frac{4096}{4096}\right)}{\frac{1}{4} - \frac{4}{4}}$$

$$1365 = a\left(-\frac{\overset{1365}{\cancel{4095}}}{\underset{1024}{\cancel{4096}}}\right)\left(-\frac{\cancel{4}^{1}}{\cancel{3}_{1}}\right)$$

$$1365 = \frac{1365a}{1024}$$

$$1024 = a$$

10.9

5 b) $1 + \frac{5}{2} + \frac{25}{4} + \ldots + \frac{15625}{64}$ ← last term

$a = 1$

$r = \frac{5}{2}$

$t_n = \frac{15625}{64}$

① Find n:

$t_n = ar^{n-1}$

$\frac{15625}{64} = (1)\left(\frac{5}{2}\right)^{n-1}$

$\frac{15625}{64} = \left(\frac{5}{2}\right)^{n-1}$

$\left(\frac{5}{2}\right)^6 = \left(\frac{5}{2}\right)^{n-1}$

$6 = n-1$

$7 = n$

② Find S_7

$S_n = \frac{a(r^n - 1)}{r-1}$

$S_7 = \frac{(1)\left(\left(\frac{5}{2}\right)^7 - 1\right)}{\left(\frac{5}{2}\right) - 1}$

$S_7 = \frac{1\left(\frac{78125}{128} - \frac{128}{128}\right)}{\frac{5}{2} - \frac{2}{2}}$

$S_7 = \left(\frac{77997}{128}\right) \div \left(\frac{3}{2}\right)$

$S_7 = \left(\frac{77997}{128}\right)\left(\frac{2}{3}\right)$

$S_7 = \frac{155994}{384} = \boxed{\frac{25999}{64}}$

10.9

(2) $30 - 5 + \frac{5}{6} \ldots$

$S_7 = ?$

$r = -\frac{1}{6}$

$a = 30$

$n = 7$

$$S_7 = \frac{30\left(\left(-\frac{1}{6}\right)^7 - 1\right)}{-\frac{1}{6} - 1}$$

$$S_7 = \frac{30\left(\frac{-1}{279936} - 1\right)}{-\frac{1}{6} - \frac{6}{6}}$$

$$S_7 = \frac{30\left(\frac{-1}{279\,936} - \frac{279\,936}{279\,936}\right)}{-\frac{7}{6}}$$

$$S_7 = \frac{30}{1}\left(\frac{-279\,937}{279\,936}\right) \times -\frac{6}{7}$$

$$S_7 = \frac{50\,388\,660}{1\,959\,552}$$

$$\boxed{S_7 = \frac{199\,955}{7776}} \text{ or } 25\,\frac{5555}{7776}$$

10.9

6) $S_7 = 1093$

$r = \frac{1}{3}$

$n = 7$

$a = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1093 = \frac{a\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\frac{1}{3} - 1}$$

$$1093 = \frac{a\left(\frac{1}{2187} - \frac{2187}{2187}\right)}{\frac{1}{3} - \frac{3}{3}}$$

$$1093 = a\left(\frac{-2186}{2187}\right) \times -\frac{3}{2}$$

$$1093 = \frac{6558a}{4374}$$

$$6558a = 4780782$$

$$\boxed{a = 729}$$

b) $t_4 = ?$

$a = 729$

$r = \frac{1}{3}$

$n = 4$

$$t_4 = (729)\left(\frac{1}{3}\right)^{4-1}$$

$$t_4 = 729\left(\frac{1}{27}\right)$$

$$t_4 = \frac{729}{27}$$

$$\boxed{t_4 = 27}$$

10.9

1 b) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$

$a = 2$

$r = \frac{1}{3}$

$$S_n = \frac{2\left(\frac{1}{3}^n - 1\right)}{\frac{1}{3} - 1}$$

$$= \frac{2\left(\frac{1}{3}^n - 1\right)}{-\frac{2}{3}}$$

$$= \cancel{2}\left(\frac{1}{3}^n - 1\right) \times \frac{3}{-\cancel{2}}$$

$$= -3\left(\frac{1}{3}^n - 1\right)$$

10.9

3 c) $81 + 27 + 9 \ldots$

$a = 81$

$r = \frac{1}{3}$

$n = 6$

$$S_6 = \frac{81\left(\left(\frac{1}{3}\right)^6 - 1\right)}{\frac{1}{3} - 1}$$

$$= \frac{81\left(\frac{1}{729} - \frac{729}{729}\right)}{\frac{1}{3} - \frac{3}{3}}$$

$$= \frac{\cancel{81}^{1}\left(\frac{-728}{\cancel{729}_{9}}\right)}{-\frac{2}{3}}$$

$$= -\frac{\overset{364}{\cancel{728}}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{-\cancel{2}_{1}}$$

$\frac{-2184}{-18}$

$$\boxed{= \frac{364}{3}} = \boxed{121\tfrac{1}{3}}$$

Ex 10.9

5 b) $1 + \frac{5}{2} + \frac{25}{4} + \ldots + \frac{15625}{64}$

$S_n = ?$

$a = 1$

$r = \frac{5}{2}$

$t_n = \frac{15625}{64}$

Solve for n:

$t_n = ar^{n-1}$

$\frac{15625}{64} = (\cancel{1})\left(\frac{5}{2}\right)^{n-1}$

$\frac{15625}{64} = \left(\frac{5}{2}\right)^{n-1}$

$\left(\frac{5}{2}\right)^{6} = \left(\frac{5}{2}\right)^{n-1}$

$6 = n-1$

$\boxed{7 = n}$

Find S_7:

$$S_7 = \frac{1\left(\left(\frac{5}{2}\right)^7 - 1\right)}{\frac{5}{2} - 1}$$

$$= \frac{1\left(\frac{78125}{128} - \frac{128}{128}\right)}{\frac{5}{2} - \frac{2}{2}}$$

$$= \left(\frac{77997}{128}\right) \div \left(\frac{3}{2}\right)$$

$$= \frac{77997}{128} \times \frac{2}{3}$$

$$= \frac{155994}{384} = \boxed{\frac{25999}{64}}$$

Review

②a) $n=?$ $\quad t_n = a+(n-1)d$

$a=3$ $\quad 39 = 3+(n-1)4$

$d=4$ $\quad 36 = 4n-4$

$t_n=39$ $\quad 40 = 4n$

$10 = n$

3, 7, 11, 15, 19, 23, 27, 31, 35, 39

① b) $t_9=-6$ $\quad t_{12}=-12$

$t_9 = a+8d$ $\quad t_{12}=a+11d$

$$\boxed{a+8d=-6 \qquad a+11d=-12}$$

$$\begin{array}{r} a+11d=-12 \\ (-)\ a+8d=-6 \\ \hline 3d=-6 \\ \boxed{d=-2} \end{array}$$

$a+8(-2)=-6$

$a-16=-6$

$\boxed{a=10}$

$t_n=a+(n-1)d$

$t_n=10+(n-1)(-2)$

$t_n=10-2n+2$

$t_n=12-2n$

⑥ b) $t_5=8$ $\quad t_{10}=\frac{1}{4}$ $\quad t_3=?$

$t_5=ar^4$ $\quad t_{10}=ar^9$

$$\boxed{ar^4=8 \qquad ar^9=\frac{1}{4}}$$

$$\frac{ar^9=\frac{1}{4}}{ar^4=8}$$

$r^5=\frac{1}{32}$

$\boxed{r=\frac{1}{2}}$

$a\left(\frac{1}{2}\right)^4=8$

$a\left(\frac{1}{16}\right)=8$

$\frac{a}{16}=8$

$\boxed{a=128}$

$t_3=(128)\left(\frac{1}{2}\right)^2$

$t_3=128\left(\frac{1}{4}\right)$

$t_3=32$

$$512 = 2(-2)^{n-1}$$

$$256 = (-2)^{n-1}$$

$$\cancel{(-2)}^{8} = \cancel{(-2)}^{n-1}$$

$$8 = n-1$$

$$9 = n$$

Review

⑩ $t_7 = 192$ $\boxed{a = t_1 = 3}$ $S_8 = ?$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$ar^6 = 192$

$3r^6 = 192$

$r^6 = 64$

$\boxed{r = \pm 2}$

$$S_8 = \frac{3(2^8 - 1)}{2 - 1} = \frac{3(256-1)}{1} = 3(255) = \boxed{765}$$

$$S_8 = \frac{3((-2)^8 - 1)}{(-2) - 1} = \frac{3(256-1)}{-3} = \frac{3(255)}{-3} = \boxed{-255}$$