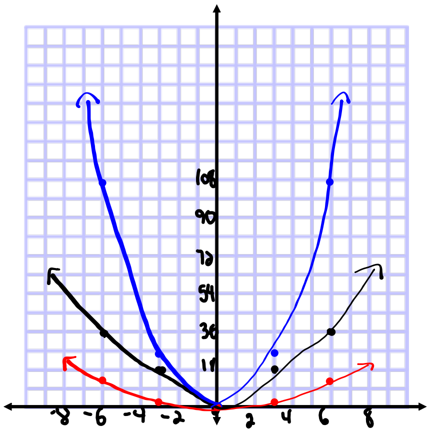
Questions from Homework

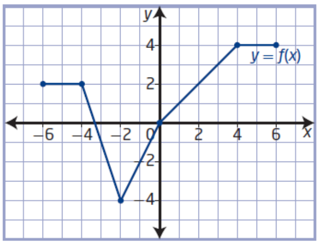
2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2.

$a = 2$



$(x, y) \rightarrow (x, 2y)$

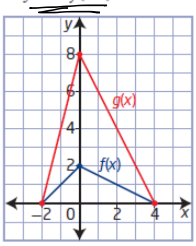
$f(x)$ $g(x)$

D: $[-6, 6]$ D: $[-6, 6]$

R: $[-4, 4]$ R: $[-8, 8]$

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



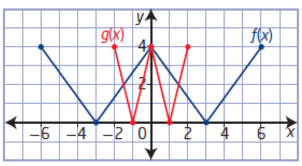
$(x, y) \rightarrow (x, 4y)$ A vertical stretch by a factor of 4

$f(x)$	$g(x)$
$(-2, 0)$	$(-2, 0)$
$(0, 2)$	$(0, 8)$
$(4, 0)$	$(4, 0)$

$a = 4$

$\boxed{y = 4f(x)}$

c)



$(x, y) \rightarrow (\frac{1}{3}x, y)$ A horizontal compression by a factor of $\frac{1}{3}$

$f(x)$	$g(x)$
$(-6, 4)$	$(-2, 4)$
$(-3, 0)$	$(-1, 0)$
$(0, 4)$	$(0, 4)$
$(3, 0)$	$(1, 0)$
$(6, 4)$	$(2, 4)$

$b = 3$

$\boxed{y = f(3x)}$

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3 \rightarrow$ vertical stretch by a factor of 3

$(x, y) \rightarrow (x, 3y)$

$(-2, 5) \rightarrow \boxed{(-2, 15)}$

(2) $y = f\left(-\frac{1}{3}x\right)$

$b = -\frac{1}{3} \rightarrow$ horizontal stretch by a factor of 3 & a horizontal reflection in the y-axis

$(x, y) \rightarrow (-3x, y)$

$(-2, 5)$ $\boxed{(6, 5)}$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$a = 4$ vertical stretch by a factor of 4

$b = \frac{1}{2}$ horizontal stretch by a factor of 2

$h = -5$ translated 5 units left

$k = -3$ translated 3 units down

$(x, y) \rightarrow (2x - 5, 4y - 3)$

$(-2, 5) \rightarrow \boxed{(-9, 17)}$

(4) $y - 5 = -2f(-2x + 6)$

factor

$y = -2f(-2x + 6) + 5$

$y = -2f[-2(x - 3)] + 5$

$a = -2$ vertical stretch by a factor of 2 and a vertical reflection in the x-axis

$b = -2$ horizontal stretch by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis

$h = 3$ translated 3 units right

$k = 5$ translated 5 units up

$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$

$(-2, 5) \rightarrow \boxed{(4, -5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

factor

$g(x) = -3f(4x-16)-10$

$g(x) = -3f[4(x-4)]-10$

$a=-3$ $b=4$ $h=4$ $k=-10$

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

Summary of Transformations...

Transformations of the graphs of functions			
$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\rightarrow(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units	" "	$(x,y)\rightarrow(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans	$(x,y)\rightarrow(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units	" "	$(x,y)\rightarrow(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\rightarrow(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref.	$(x,y)\rightarrow(x,-y)$
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\rightarrow(x,ay)$
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**		$(x,y)\rightarrow\left(\frac{1}{b}x,y\right)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

Mapping Rule:

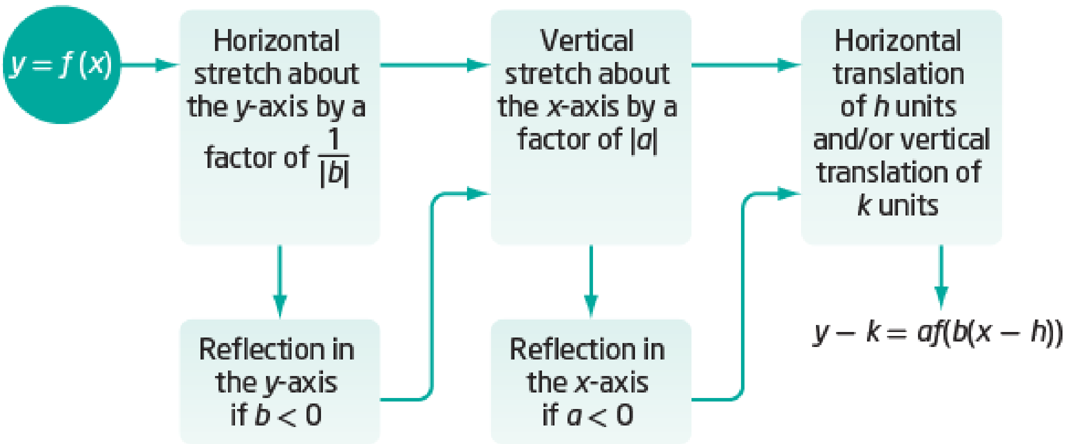
$$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



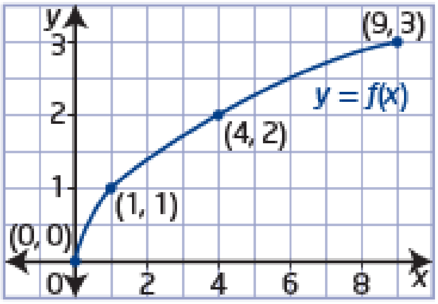
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

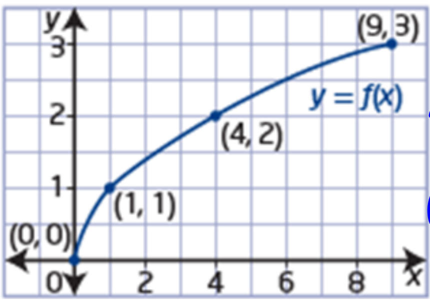
b) $y = f(3x + 6)$



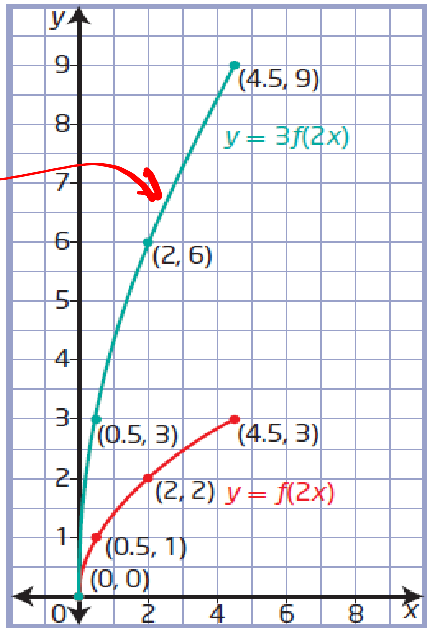
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \rightarrow \left(\frac{1}{2}x + 0, 3y + 0\right)$



(0,0)	(0,0)
(1,1)	(½,3)
(4,2)	(2,6)
(9,3)	(9/2,9)



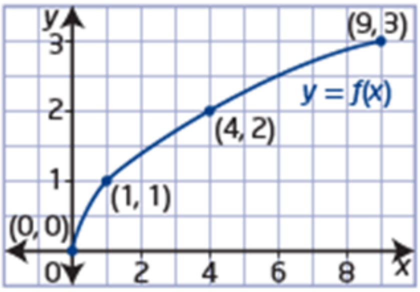
factor

b) $y = f(3x + 6)$ $a = 1$ $b = 3$ $h = -2$ $k = 0$

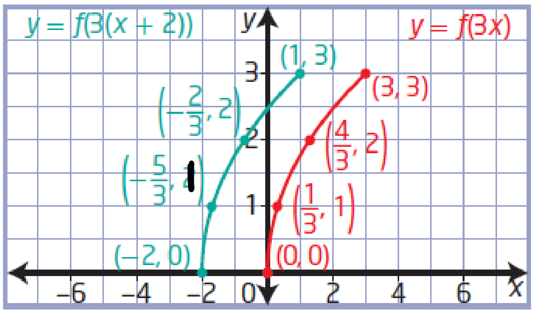
$y = 1f[3(x+2)] + 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \rightarrow (\frac{1}{3}x - 2, y)$



$(0,0)$	$(-2, 0)$
$(1,1)$	$(-\frac{5}{3}, 1)$
$(4,2)$	$(-\frac{2}{3}, 2)$
$(9,3)$	$(1, 3)$



$\frac{1}{3}\left(\frac{1}{1}\right) - \frac{2}{1}$ $\quad$ $\frac{1}{3}\left(\frac{4}{1}\right) - 2$ $\quad$ $\frac{1}{3}\left(\frac{9}{1}\right) - 2$

$\frac{1}{3} - \frac{6}{3}$ $\quad$ $\frac{4}{3} - \frac{6}{3}$ $\quad$ $\frac{9}{3} - 2$

$\frac{-5}{3}$ $\quad$ $\frac{-2}{3}$ $\quad$ $3 - 2$

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Homework

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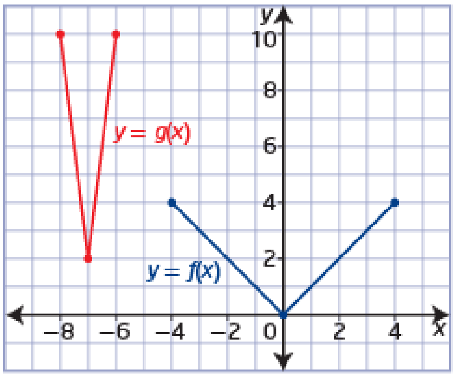
Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is [redacted]

[redacted]

How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.