Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a=3 \rightarrow$ vertical stretch by a factor of 3

$(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow \boxed{(-2,15)}$

(2) $y = f\left(\left(-\frac{1}{3}\right)x\right)$

$b=-\frac{1}{3} \rightarrow$ horizontal stretch by a factor of 3 & a horizontal reflection in the y-axis

$(x,y) \rightarrow (-3x, y)$

$(-2,5)$ $\boxed{(6,5)}$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x+5)\right] - 3$

$a=4$ vertical stretch by a factor of 4

$b=\frac{1}{2}$ horizontal stretch by a factor of 2

$h=-5$ translated 5 units left

$k=-3$ translated 3 units down

$(x,y) \rightarrow (2x-5, 4y-3)$

$(-2,5) \rightarrow \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$

factor

$y = -2f(-2x+6)+5$

$y = -2f[-2(x-3)]+5$

$a=-2$ vertical stretch by a factor of 2 and a vertical reflection in the x-axis

$b=-2$ horizontal stretch by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis

$h=3$ translated 3 units right

$k=5$ translated 5 units up

$(x,y) \rightarrow (-\frac{1}{2}x+3, -2y+5)$

$(-2,5) \rightarrow \boxed{(4,-5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

factor

$$g(x) = -3f(4x-16)-10$$

$$g(x) = -3f[4(x-4)]-10$$

$a=-3$ $b=4$ $h=4$ $k=-10$

Summary of Transformations...

Transformations of the graphs of functions			
$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\rightarrow(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units	" "	$(x,y)\rightarrow(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans	$(x,y)\rightarrow(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units	" "	$(x,y)\rightarrow(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\rightarrow(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref.	$(x,y)\rightarrow(x,-y)$
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\rightarrow(x,ay)$
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**		$(x,y)\rightarrow(\frac{1}{b}x,y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

Mapping Rule:

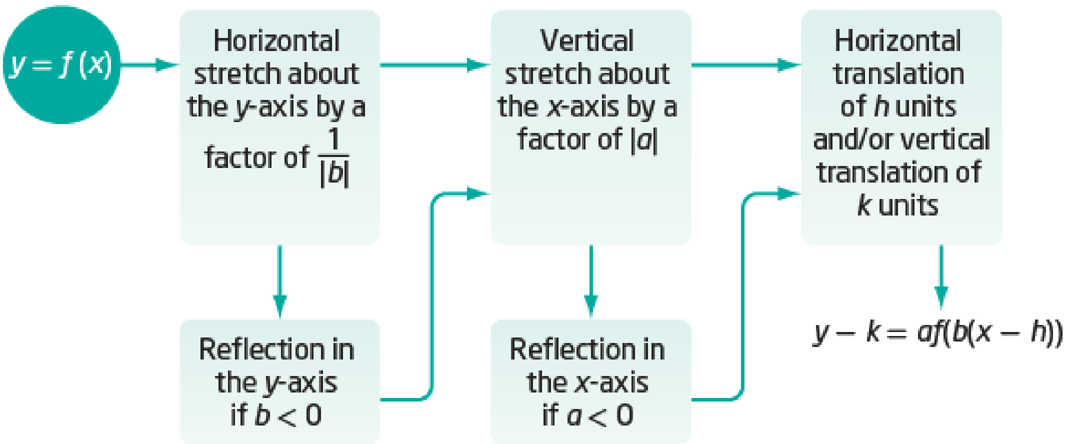
$$(x,y)\rightarrow\left(\frac{1}{b}x+h, ay+k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

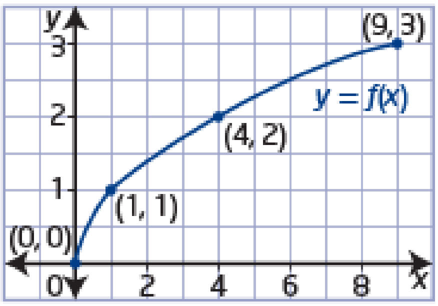
Remember....RST



Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.



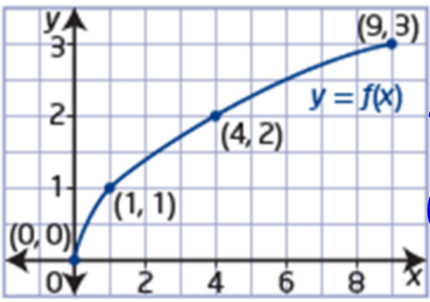
a) $y = 3f(2x)$

b) $y = f(3x + 6)$

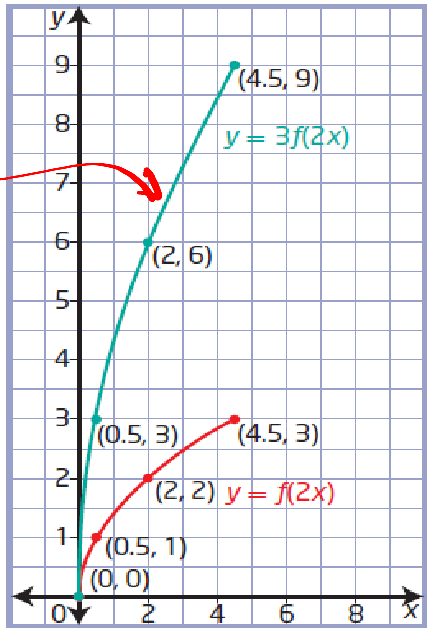
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \to \left(\frac{1}{2}x + 0, 3y + 0\right)$



$(0, 0)$	$(0, 0)$
$(1, 1)$	$(\frac{1}{2}, 3)$
$(4, 2)$	$(2, 6)$
$(9, 3)$	$(\frac{9}{2}, 9)$



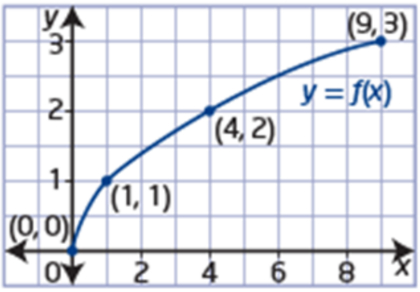
factor

b) $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$

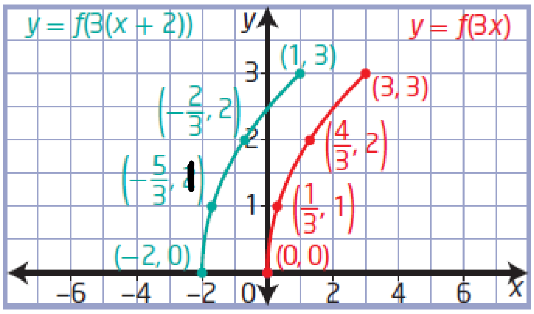
$y=1f[3(x+2)]+0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x,y) \to \left(\frac{1}{3}x - 2, y\right)$



$(0,0)$	$(-2, 0)$
$(1,1)$	$(-\frac{5}{3}, 1)$
$(4,2)$	$(-\frac{2}{3}, 2)$
$(9,3)$	$(1, 3)$



$\frac{1}{3}\left(\frac{1}{1}\right) - \frac{2}{1}$ $\frac{1}{3}\left(\frac{4}{1}\right) - 2$ $\frac{1}{3}\left(\frac{9}{1}\right) - 2$

$\frac{1}{3} - \frac{6}{3}$ $\frac{4}{3} - \frac{6}{3}$ $\frac{9}{3} - 2$

$\frac{-5}{3}$ $\frac{-2}{3}$ $3 - 2$

1

Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

	Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
(i)	$y - 4 = f(x - 5)$	–	–	–	4	5
(ii)	$y + 5 = 2f(3x)$	–	2	$\frac{1}{3}$	-5	–
(iii)	$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	–	$\frac{1}{2}$	2	–	4
(iv)	$y + 2 = -3f(2(x + 2))$	↑	3	$\frac{1}{2}$	-2	-2

vertical reflection in x-axis

(i) $y = f(x-5) + 4$
$a=1$ $b=1$ $h=5$ $k=4$

(ii) $y = 2f(3x) - 5$
$a=2$ $b=3$ $h=0$ $k=-5$

(iii) $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$
$a=\frac{1}{2}$ $b=\frac{1}{2}$ $h=4$ $k=0$

(iv) $y = -3f(2(x+2)) - 2$
$a=-3$ $b=2$ $h=-2$ $k=-2$

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

e) $y + 3 = -\frac{1}{3}f[2(x+6)]$

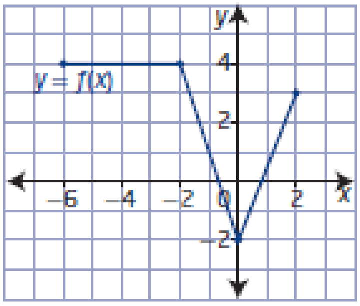
$y = -\frac{1}{3}f[2(x+6)] - 3$

$a = -\frac{1}{3}$ $b = 2$ $h = -6$ $k = -3$

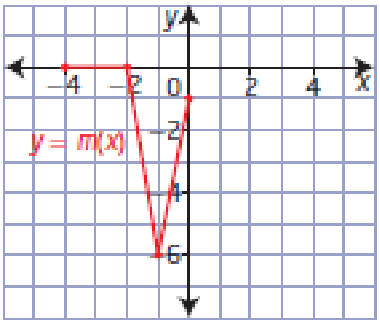
$(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$

$(-12, 18) \rightarrow \boxed{(-12, -9)}$

4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



b)



$f(x)$	$m(x)$
$(-6,4)$	$(-4,0)$
$(-2,4)$	$(-2,0)$
$(0,-2)$	$(-1,-6)$
$(2,3)$	$(0,-1)$

$$(x,y) \longrightarrow \left(\tfrac{1}{2}x-1,\ y-4\right)$$

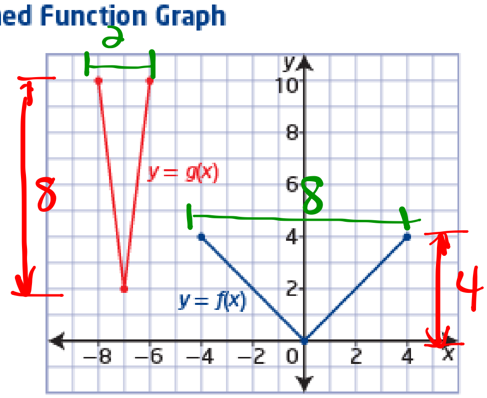
$a=1 \quad b=2 \quad h=-1 \quad k=-4$

$$m(x) = 1f(2(x+1))-4$$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: None

② Vertical stretch factor $= \frac{8}{4} = 2$ $a = 2$
(Compare Range $\frac{\text{New}}{\text{Old}}$)

③ horizontal stretch factor $= \frac{2}{8} = \frac{1}{4}$ $b = 4$
(Compare domain $\frac{\text{New}}{\text{Old}}$)

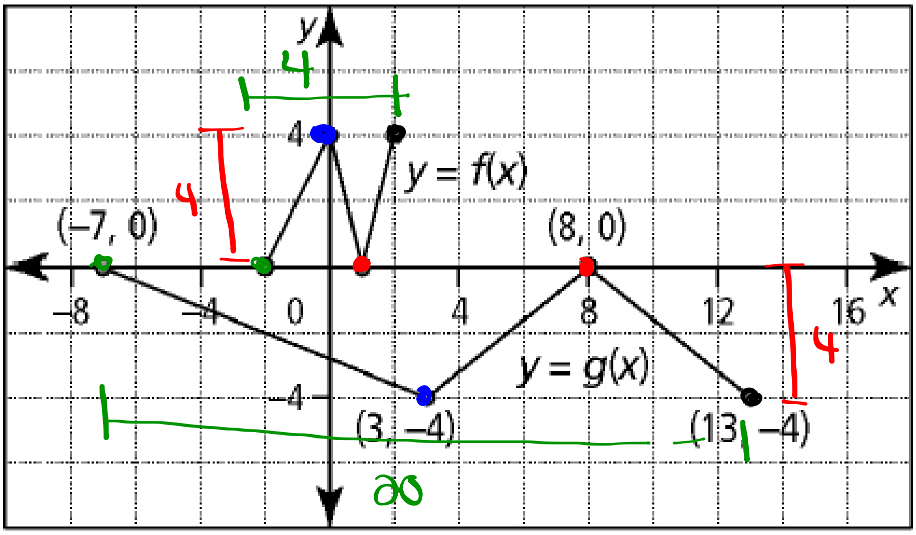
④ horizontal translation: $(\underline{0}, 0) \rightarrow (\underline{-7}, 2)$ left 7 units
(Pick a point on $f(x)$ with an x-value of 0) $h = -7$

⑤ vertical translation: $(0, \underline{0}) \rightarrow (-7, \underline{2})$ up 2 units
(Pick a point on $f(x)$ with an y-value of 0) $k = 2$

⑥ Equation: $g(x) = 2f(4(x+7)) + 2$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x-h)) + k$.

$y = -f\left(\frac{1}{5}(x-3)\right)$



① Reflections: vertical in x-axis $a<0$

② VSF $\frac{4}{4} = 1$ $a = -1$

③ HSF $\frac{20}{4} = 5$ $b = 1/5$

④ HT: $(\underline{0}, 4) \to (\underline{3}, -4)$ Right 3 $h = 3$

⑤ VT: $(-2, \underline{0}) \to (-7, \underline{0})$ $k = 0$

⑥ $g(x) = -1f\left(\frac{1}{5}(x-3)\right)$

Homework

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Plus 7, 8, 9 (a, c, e) and 10