Questions from Homework

Find the derivative of each function.

Remember!

1. $f(x) = 8x^2 - 10$

$$f'(x) = 16x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

2. $f(x) = 2x^2 + 14x - 7$

$$f'(x) = 4x + 14$$

3. $f(x) = x^3$

$$f'(x) = 3x^2$$

4. $f(x) = \dfrac{x + 4}{2x + 3}$

$$f'(x) = \frac{-5}{(2x+3)^2}$$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

① Find $f(x+h)$
② Fill in formula

2. $f(x) = 2x^2 + 14x - 7$

① $f(x+h) = 2(x+h)^2 + 14(x+h) - 7$

$= 2(x^2 + 2xh + h^2) + 14x + 14h - 7$

$= 2x^2 + 4xh + 2h^2 + 14x + 14h - 7$

$f'(x) = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 14x + 14h - 7 - (2x^2 + 14x - 7)}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 14x + 14h - 7 - 2x^2 - 14x + 7}{h}$

common factor

$= \lim_{h \to 0} \frac{4xh + 2h^2 + 14h}{h}$

$= \lim_{h \to 0} \frac{h(4x + 2h + 14)}{h} = 4x + 14$

$f'(x) = 4x + 14$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

③ $f(x) = x^3$

$f(x+h) = (x+h)^3$
$= (x+h)(x+h)(x+h)$
$= (x+h)(x^2 + 2xh + h^2)$
$= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$
$= x^3 + 3x^2h + 3xh^2 + h^3$

$f'(x) = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$f'(x) = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$

$f'(x) = \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2$

$$\boxed{f'(x) = 3x^2}$$

$f(x) = x^3$ $f(x+h) = (x+h)^3$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$= \lim_{h \to 0} \frac{[(x+h) - x][(x+h)^2 + x(x+h) + x^2]}{h}$

$= \lim_{h \to 0} \frac{h[(x+h)^2 + x(x+h) + x^2]}{h} = x^2 + x^2 + x^2$

$$\boxed{f'(x) = 3x^2}$$

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$f(x) = \boxed{\frac{x+4}{2x+3}}$$

$$f(x+h) = \frac{x+h+4}{2(x+h)+3}$$

$$= \boxed{\frac{x+h+4}{2x+2h+3}}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{x+h+4}{2x+2h+3} - \left(\frac{x+4}{2x+3}\right)}{h}$$

* Multiply each term by $(2x+3)(2x+2h+3)$

$$= \lim_{h \to 0} \frac{(2x+3)(x+h+4) - (x+4)(2x+2h+3)}{h(2x+3)(2x+2h+3)}$$

$$= \lim_{h \to 0} \frac{2x^2+2xh+8x+3x+3h+12 - (2x^2+2xh+3x+8x+8h+12)}{h(2x+3)(2x+2h+3)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2}+\cancel{2xh}+\cancel{8x}+\cancel{3x}+3h+\cancel{12} - \cancel{2x^2}-\cancel{2xh}-\cancel{3x}-\cancel{8x}-8h-\cancel{12}}{h(2x+3)(2x+2h+3)}$$

$$= \lim_{h \to 0} \frac{-5h}{h(2x+3)(2x+2h+3)} = \boxed{\frac{-5}{(2x+3)^2}}$$

Slope of the tangent

Try this one!

Remember!

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Differentiate the following function using the *Limit Definition of the Derivative*

$$f(x) = \sqrt{x + 3} \qquad f(x+h) = \sqrt{(x+h)+3} \ = \sqrt{x+h+3}$$

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{x+h+3} - \sqrt{x+3}\right)\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}{h \left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

$$= \lim_{h \to 0} \frac{x+h+3 - (x+3)}{h \left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

Homework