

Sketch the following function $f(x) = \frac{(x+2)^2}{(x^2+4)}$ if

$f'(x) = \frac{16-4x^2}{(x^2+4)^2}$ and $f''(x) = \frac{8x(x^2-12)}{(x^2+4)^3}$

- ✓ Intercepts
- ✓ Asymptotes
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values
- Intervals of Concavity $f''(x)$
- Points of Inflection

(i) y-int (x=0) $f(0) = \frac{(0+2)^2}{(0^2+4)} = \frac{4}{4} = 1$ (0,1)

x-int: (y=0) $0 = \frac{(x+2)^2}{(x^2+4)}$
 $0 = (x+2)^2$
 $0 = (x+2)(x+2)$
 $x+2=0$
 $x=-2$
 (-2,0)

(ii) VA (set denom = 0) $x^2+4=0$
 $x^2=-4$
 Not possible
 No VA

(iii) HA (compute degree) $\lim_{x \rightarrow \infty} \frac{x^2+4x+4}{x^2+4} = \frac{1}{1} = 1$
 HA: $y=1$

(iv) Intervals of Inc/Dec.

$f'(x) = \frac{16-4x^2}{(x^2+4)^2}$

CV: $16-4x^2=0$
 $(4-2x)(4+2x)=0$
 $4-2x=0 \Rightarrow 2x=4 \Rightarrow x=2$
 $4+2x=0 \Rightarrow 2x=-4 \Rightarrow x=-2$

$(x^2+4)^2=0$
 $x^2+4=0$
 $x^2=-4$
 Not possible

Decreasing on $(-\infty, -2) \cup (2, \infty)$
 Increasing on $(-2, 2)$

(v) local min (x=-2) $f(-2) = \frac{(-2+2)^2}{((-2)^2+4)} = \frac{0}{8} = 0$ (-2,0) Same as x-int

(vi) local max (x=2) $f(2) = \frac{(2+2)^2}{((2)^2+4)} = \frac{16}{8} = 2$ (2,2)

(vii) Concavity

$f''(x) = \frac{8x(x^2-12)}{(x^2+4)^3}$

CV: $8x(x^2-12)=0$
 $8x(x-\sqrt{12})(x+\sqrt{12})=0$
 $8x=0 \Rightarrow x=0$
 $x-\sqrt{12}=0 \Rightarrow x=\sqrt{12} \approx 3.46$
 $x+\sqrt{12}=0 \Rightarrow x=-\sqrt{12} \approx -3.46$

$(x^2+4)^3=0$
 $x^2+4=0$
 $x^2=-4$
 Not Possible

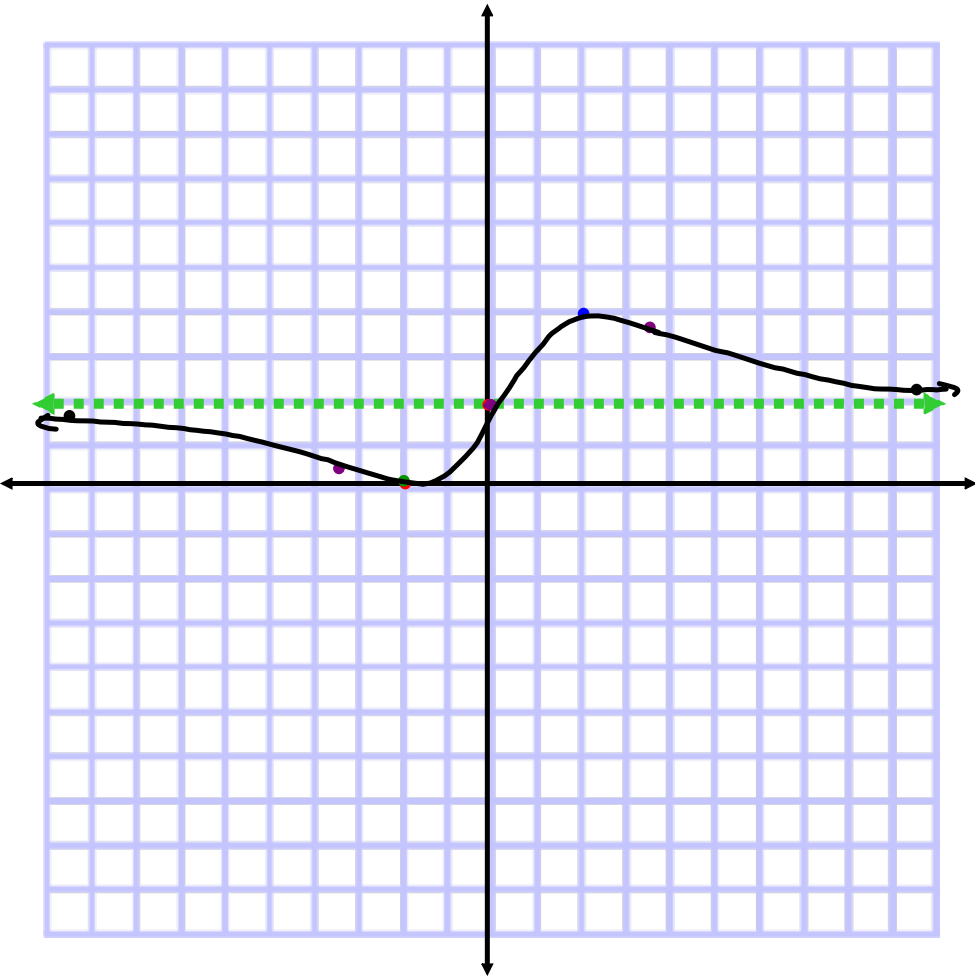
CU on $(-\sqrt{12}, 0)$ and $(\sqrt{12}, \infty)$
 CD on $(-\infty, -\sqrt{12})$ and $(0, \sqrt{12})$

(viii) IP.

$f(3.46) = \frac{(3.46+2)^2}{(3.46)^2+4} = \frac{21.816}{15.916} \approx 1.37$ (3.46, 1.37)

$f(0) = \frac{(0+2)^2}{(0)^2+4} = \frac{4}{4} = 1$ (0,1) Same as y-int

$f(-3.46) = \frac{(-3.46+2)^2}{(-3.46)^2+4} = \frac{2.816}{15.916} \approx 0.18$ (-3.46, 0.18)



$$f(x) = 3x^5 - 5x^3$$

$$f(x) = x^3(3x^2 - 5)$$

(i) x-int (y=0) $0 = x^3(3x^2 - 5)$ $x^3 = 0 \quad \quad 3x^2 - 5 = 0$ $x = 0 \quad \quad 3x^2 = 5$ $\quad \quad \quad \quad x = \pm\sqrt{5/3}$ (0,0) ($\pm 1.3, 0$)	y-int (x=0) $f(0) = 3(0)^5 - 5(0)^3 = 0$ (0,0)
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(ii) Intervals of Inc/Dec:

$f(x) = 3x^5 - 5x^3$ $f'(x) = 15x^4 - 15x^2$ $f'(x) = 15x^2(x^2 - 1)$ $f'(x) = 15x^2(x-1)(x+1)$ CV: $15x^2 = 0 \quad \quad x-1=0 \quad \quad x+1=0$ $\quad \quad \quad x=0 \quad \quad x=1 \quad \quad x=-1$	<p>Increasing on $(-\infty, -1) \cup (1, \infty)$ Decreasing on $(-1, 1)$</p>
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(iii) Local min (x=1) $f(1) = 3(1)^5 - 5(1)^3$ $f(1) = 3 - 5$ $f(1) = -2$ (1, -2)	(iv) Local max (x=-1) $f(-1) = 3(-1)^5 - 5(-1)^3$ $f(-1) = -3 + 5$ $f(-1) = 2$ (-1, 2)
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(v) Intervals of Concavity:

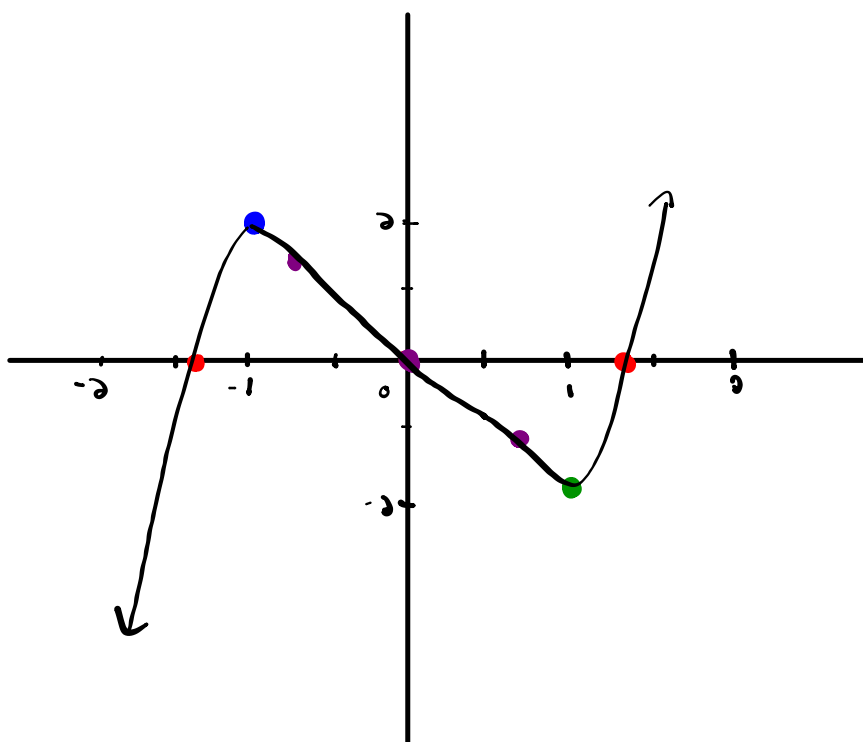
$f(x) = 3x^5 - 5x^3$ $f'(x) = 15x^4 - 15x^2$ $f''(x) = 60x^3 - 30x$ $f''(x) = 30x(2x^2 - 1)$ CV: $30x = 0 \quad \quad 2x^2 - 1 = 0$ $\quad \quad \quad x = 0 \quad \quad x^2 = 1/2$ $\quad \quad \quad \quad \quad \quad \quad x = \pm\sqrt{1/2}$ $\quad \quad \quad \quad \quad \quad \quad x \approx \pm 0.707$	
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(vi) Inflection Points:

$$f(0.71) = 3(0.71)^5 - 5(0.71)^3 = 1.24 \quad (-0.71, 1.24)$$

$$f(0) = 3(0)^5 - 5(0)^3 = 0 \quad (0, 0)$$

$$f(-0.71) = 3(-0.71)^5 - 5(-0.71)^3 = -1.24 \quad (0.71, -1.24)$$



1. Consider the function: $f(x) = \frac{2+x-x^2}{(x-1)^2}$
 CV: $x=1, 5$
 $f'(x) = \frac{x-5}{(x-1)^3}$ and $f''(x) = \frac{2(7-x)}{(x-1)^4}$ CV: $x=1, 7$

x-int: ($y=0$) y-int ($x=0$)
 $2+x-x^2=0$ $f(0) = \frac{2+(0)-(0^2)}{(0-1)^2}$
 $x^2+x+2=0$ $f(0) = \frac{2}{1} = 2$
 $-(x^2-x-2)=0$ $(0, 2)$
 $-(x+1)(x-2)=0$
 $x+1=0 \mid x-2=0$
 $x=-1 \mid x=2$
 $(-1, 0) \quad (2, 0)$

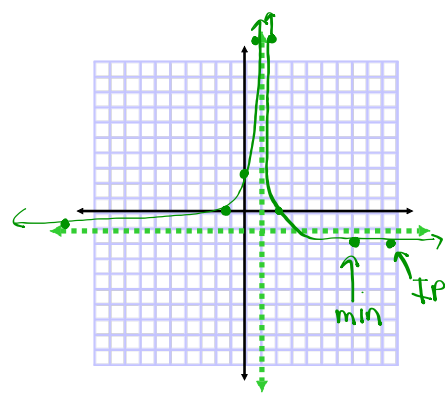
VA: (set denom = 0)
 $f(x) = \frac{2+x-x^2}{(x-1)^2} = -\frac{(x-2)(x+1)}{(x-1)^2}$
 $(x-1)^2=0$
 $x-1=0$
 $x=1$

Test behaviour near VA
 $\lim_{x \rightarrow 1^-} f(x) = \frac{-(-)(-)}{(+)} = +\infty$
 $x=0.99$
 $\lim_{x \rightarrow 1^+} f(x) = \frac{-(-)(+)}{+} = -\infty$
 $x=1.01$

HA: (compare the degree of the num. and denom.)
 $f(x) = \frac{2+x-x^2}{(x-1)^2} = \frac{2+x-x^2}{x^2-2x+1}$
 $\lim_{x \rightarrow \infty} \frac{2+x-x^2}{x^2-2x+1} = -1$ $y=-1$

$f'(x) = \frac{x-5}{(x-1)^3}$ $f(x) = \frac{2+x-x^2}{(x-1)^2}$
 CV: $x=1, 5$
 no max $x=1$ is VA min
 $f(5) = \frac{2+(5)-(5^2)}{(5-1)^2}$
 $f(5) = \frac{-18}{16} = -\frac{9}{8}$
 $(5, -\frac{9}{8})$ or $(5, -1.125)$

$f''(x) = \frac{2(7-x)}{(x-1)^4}$ $f(7) = \frac{2+(7)-(7^2)}{(7-1)^2}$
 CV: $x=1, 7$ IP
 $f(7) = \frac{-40}{36} = -\frac{10}{9}$
 $(7, -\frac{10}{9})$ or $(7, -1.11)$



$$f(x) = x^3 - 9x^2 + 24x - 10$$

① y-intercept (x=0)

$$f(0) = (0)^3 - 9(0)^2 + 24(0) - 10 = -10$$

(0, -10)

② Asymptotes: None
(Polynomial Function)

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f'(x) = 3x^2 - 18x + 24$$

$$f'(x) = 3(x^2 - 6x + 8)$$

$$f'(x) = 3(x-4)(x-2)$$

Increasing on $(-\infty, 2) \cup (4, \infty)$
Decreasing on $(2, 4)$

CV: x=2, 4

Max/Min

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f(2) = (2)^3 - 9(2)^2 + 24(2) - 10 = 10 \quad (2, 10) \text{ max}$$

$$f(4) = (4)^3 - 9(4)^2 + 24(4) - 10 = 6 \quad (4, 6) \text{ min}$$

Concavity:

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f'(x) = 3x^2 - 18x + 24$$

$$f''(x) = 6x - 18 \quad \text{CD on } (-\infty, 3)$$

$$f''(x) = 6(x-3) \quad \text{CU on } (3, \infty)$$

CV: x=3

IP:

$$f(x) = x^3 - 9x^2 + 24x - 10$$

$$f(3) = (3)^3 - 9(3)^2 + 24(3) - 10 = 8 \quad (3, 8) \text{ IP}$$

