

Warm Up

Differentiate: $e^{xy^2} = 2x - (3y) + e^{\tan x}$

$$e^{xy^2} (1y^2 + 2xy \frac{dy}{dx}) = 2 - (3y + 3x \frac{dy}{dx}) + e^{\tan x} (\sec^2 x)$$

$$y^2 e^{xy^2} + 2xy e^{xy^2} \frac{dy}{dx} = 2 - 3y - 3x \frac{dy}{dx} + \sec^2 x e^{\tan x}$$

$$2xy e^{xy^2} \frac{dy}{dx} + 3x \frac{dy}{dx} = 2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}$$

$$\frac{dy}{dx} (2xy e^{xy^2} + 3x) = 2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}$$

$$\frac{dy}{dx} = \frac{2 - 3y + \sec^2 x e^{\tan x} - y^2 e^{xy^2}}{2xy e^{xy^2} + 3x}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$. $x_1 = 0$

$$y = 1 + xe^{2x}$$

(i) Find y_1 :

$$y = 1 + (0)e^{2(0)}$$

$$y = 1 + 0$$

$$y = 1$$

$$y_1 = 1$$

(ii) Find y'

$$y = 1 + (xe^{2x})$$

$$y' = 0 + 1e^{2x} + xe^{2x} \cdot 2$$

$$y' = e^{2x} + 2xe^{2x}$$

$$y' = e^{2x}(1 + 2x)$$

(iii) Find m :

$$y'(0) = e^{2(0)}(1 + 2(0))$$

$$y'(0) = 1(1 + 0)$$

$$y'(0) = 1$$

$$m = 1$$

$$(iv) y - y_1 = m(x - x_1)$$

$$y - 1 = (1)(x - 0)$$

$$y - 1 = x$$

$$y = x + 1 \quad \text{or} \quad x - y + 1 = 0$$

Questions from Homework

$$\textcircled{6} \quad e^{xy} = 2x + y$$

$$e^{xy} \left(1y + x \frac{dy}{dx} \right) = 2 + \frac{dy}{dx}$$

$$ye^{xy} + xe^{xy} \frac{dy}{dx} = 2 + \frac{dy}{dx}$$

$$xe^{xy} \frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$$

$$\frac{dy}{dx} (xe^{xy} - 1) = 2 - ye^{xy}$$

$$\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}$$

Questions from Homework

⑩ For $f(x) = (x)e^x$

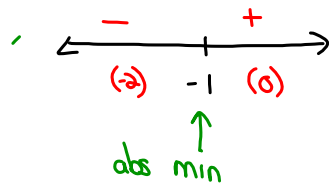
a) the abs min

$$f'(x) = 1e^x + x(e^x \cdot 1)$$

$$f'(x) = e^x + xe^x$$

$$f'(x) = e^x(1+x)$$

CV: $x = -1$



abs min: $(x = -1)$

$$f(x) = xe^x$$

$$f(-1) = (-1)e^{-1}$$

$$= -1\left(\frac{1}{e}\right)$$

$$= -\frac{1}{e} \quad (-1, -\frac{1}{e})$$

b) Intervals of Concavity

$$f(x) = xe^x$$

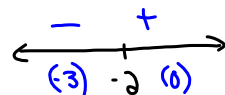
$$f'(x) = e^x + (x)e^x$$

$$f''(x) = e^x + e^x + xe^x$$

$$f''(x) = 2e^x + xe^x$$

$$f''(x) = e^x(2+x)$$

CV: $x = -2$



Concave Down on $(-\infty, -2)$

Concave Up on $(-2, \infty)$

c) Inflection Point $(x = -2)$

$$f(x) = xe^x$$

$$f(-2) = (-2)e^{-2}$$

$$= -2\left(\frac{1}{e^2}\right)$$

$$= -\frac{2}{e^2} \quad (-2, -\frac{2}{e^2})$$

⑪ $e^{xy} = 2x + y$ (Implicit diff.)

$$e^{xy} \left(1y + x \frac{dy}{dx} \right) = 2 + \frac{dy}{dx}$$

$$ye^{xy} + xe^{xy} \frac{dy}{dx} = 2 + \frac{dy}{dx}$$

$$xe^{xy} \frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$$

$$\frac{dy}{dx} (xe^{xy} - 1) = 2 - ye^{xy}$$

$$\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}$$

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$.

$$\log_e y = x$$

$$\ln y = x$$

What other function could this model?

$$\ln y = x$$

Try to differentiate $\longrightarrow y = \ln x$. $u = x$
 $du = 1$

$$y' = \frac{1}{x} \cdot 1 = \frac{1}{x}$$

Differentiate: $y = \ln x^3$ $u = x^3$
 $du = 3x^2$

$$y' = \frac{3x^2}{x^3} = \frac{3}{x}$$

$$y' = \frac{1}{x^3} \cdot 3x^2 = \frac{3x^2}{x^3} = \frac{3}{x}$$

$$\text{Rule: } d(\ln u) = \frac{1}{u} du = \frac{du}{u}$$

Ex: $u = x^7$ $du = 7x^6$
 $y = \ln x^7$
 $y' = \frac{1}{x^7} \cdot 7x^6 = \frac{7}{x}$

Practice Problem:

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Laws of Logarithms

$$\log_b M + \log_b N = \log_b (MN)$$

$$\log_b M - \log_b N = \log_b \left(\frac{M}{N} \right)$$

$$\log_b (N^p) = p \log_b (N)$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$$y = \log_6 x^3$$

$$y = \log(5x^4)$$

Rule: $d(\log_b u) = \frac{1}{u \ln b} du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Try this one... $y = \pi^{x^5}$

Rule:

$$d(b^u) = b^u (\ln b) du, \text{ where } b \in R$$

Practice Problems:

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#5 #6 #7 #8