Warm Up

Differentiate: $e^{xy^2} = 2x - 3xy + e^{\tan x}$

$$e^{xy^2}\left(1y^2 + 2xy\frac{dy}{dx}\right) = 2 - \left(3y + 3x\frac{dy}{dx}\right) + e^{\tan x}(\sec^2 x)$$

$$y^2e^{xy^2} + 2xye^{xy^2}\frac{dy}{dx} = 2 - 3y - 3x\frac{dy}{dx} + \sec^2 x\, e^{\tan x}$$

$$2xye^{xy^2}\frac{dy}{dx} + 3x\frac{dy}{dx} = 2 - 3y + \sec^2 x e^{\tan x} - y^2e^{xy^2}$$

$$\frac{dy}{dx}\left(2xye^{xy^2} + 3x\right) = 2 - 3y + \sec^2 x e^{\tan x} - y^2e^{xy^2}$$

$$\frac{dy}{dx} = \frac{2 - 3y + \sec^2 x\, e^{\tan x} - y^2e^{xy^2}}{2xye^{xy^2} + 3x}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$. $x_1 = 0$

$y = 1 + xe^{2x}$

(i) Find y_1:

$y = 1 + (0)e^{2(0)}$

$y = 1 + 0$

$y = 1$

$y_1 = 1$

(ii) Find y'

$y = 1 + (xe^{2x})$

$y' = 0 + 1e^{2x} + xe^{2x}\cdot 2$

$y' = e^{2x} + 2xe^{2x}$

$y' = e^{2x}(1+2x)$

(iii) Find m:

$y'(0) = e^{2(0)}(1 + 2(0))$

$y'(0) = 1(1+0)$

$y'(0) = 1$

$m = 1$

(iv) $y - y_1 = m(x - x_1)$

$y - 1 = (1)(x - 0)$

$y - 1 = x$

$y = x + 1$ or $x - y + 1 = 0$

Questions from Homework

⑥ $e^{(x)(y)} = 2x + y$

$e^{xy}\left(1y + x\frac{dy}{dx}\right) = 2 + \frac{dy}{dx}$

$ye^{xy} + xe^{xy}\frac{dy}{dx} = 2 + \frac{dy}{dx}$

$xe^{xy}\frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$

$\frac{dy}{dx}\left(xe^{xy} - 1\right) = 2 - ye^{xy}$

$\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}$

Questions from Homework

⑩ For $f(x) = (x e^x)$

a) the abs min

$f'(x) = 1e^x + x(e^x \cdot 1)$

$f'(x) = e^x + xe^x$

$f'(x) = e^x(1+x)$

CV: $x = -1$

$-$ $+$

(-2) -1 (0)

abs min

abs min: $(x=-1)$

$f(x) = xe^x$

$f(-1) = (-1)e^{-1}$

$= -1\left(\frac{1}{e}\right)$

$= \frac{-1}{e}$ $\left(-1, \frac{-1}{e}\right)$

b) Intervals of Concavity

$f(x) = xe^x$

$f'(x) = e^x + (x)(e^x)$

$f''(x) = e^x + e^x + xe^x$

$f''(x) = 2e^x + xe^x$

$f''(x) = e^x(2+x)$

CV: $x = -2$

$-$ $+$

(-3) -2 (0)

Concave Down on $(-\infty, -2)$

Concave Up on $(-2, \infty)$

c) Inflection Point $(x=-2)$

$f(x) = xe^x$

$f(-2) = (-2)e^{-2}$

$= -2\left(\frac{1}{e^2}\right)$

$= \frac{-2}{e^2}$ $\left(-2, \frac{-2}{e^2}\right)$

⑥ $e^{xy} = 2x + y$ (Implicit diff.)

$e^{xy}\left(1y + x\frac{dy}{dx}\right) = 2 + \frac{dy}{dx}$

$ye^{xy} + xe^{xy}\frac{dy}{dx} = 2 + \frac{dy}{dx}$

$xe^{xy}\frac{dy}{dx} - \frac{dy}{dx} = 2 - ye^{xy}$

$\frac{dy}{dx}(xe^{xy} - 1) = 2 - ye^{xy}$

$\frac{dy}{dx} = \frac{2 - ye^{xy}}{xe^{xy} - 1}$

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$.

$\log_e y = x$

$\ln y = x$

What other function could this model?

$$\ln y = x$$

Try to differentiate → $y = \ln x$.

$u = x$

$du = 1$

$y' = \frac{1}{x} \cdot 1 = \frac{1}{x}$

Differentiate: $y = \ln x^3$

$u = x^3$

$du = 3x^2$

$y' = \frac{3x^2}{x^3} = \frac{3}{x}$

$y' = \frac{1}{x^3} \cdot 3x^2 = \frac{3x^2}{x^3} = \frac{3}{x}$

Rule: $d(\ln u) = \frac{1}{u}du = \frac{du}{u}$

$u = x^7 \quad du = 7x^6$

Ex: $y = \ln x^7$

$y' = \frac{1}{x^7} \cdot 7x^6 = \frac{7}{x}$

Practice Problem:

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#1

Laws of Logarithms

$$\log_b M + \log_b N = \log_b(MN)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b(N^p) = p\log_b(N)$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$y = \log_6 x^3$

$y = \log\left(5x^4\right)$

Rule: $d\left(\log_b u\right) = \dfrac{1}{u \ln b} du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Try this one... $y = \pi^{x^5}$

Rule:

$$d(b^{u}) = b^{u}(\ln b)du \ , where \ \ b \in R$$

Practice Problems:

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#5 #6 #7 #8